A New Class of Sets in Nano topological Spaces with an Application in Medical Diagnosis

A. Jayalakshmi 1 and Dr. C. Janaki 2

1Research Scholar, Department of Mathematics, L. R. G. Government Arts College for Women, Tirupur-8, Tamil Nadu, India.
Orcid Id: 0000-0002-7235-1956

2Assistant Professor, Department of Mathematics, L. R. G. Government Arts College for Women, Tirupur-8, Tamil Nadu, India.

Abstract

The paper intends to introduce Ngρα-closed set and discuss some of its properties. Also, we identified the risk factors for the cause of heart attack through the concept of Nano topology.

Keywords: Nano topological space, Nrα-open, Ngρα-closed, TNgρα-space and Ngρα T_{1/2}-space, core, lower approximation and upper approximation.

2010 AMS Subject Classification: 54B05

Introduction:

L. Thivagar et al [1] introduced the concept of nano topological spaces which was defined in terms of approximations and boundary region of a subset of a universe U using an equivalence relation on it and also defined nano closed sets, nano interior and nano closure. In this paper we introduce and study Nrα-open, Ngρα-closed, TNgρα-space and Ngρα T_{1/2}-space are introduced. The researchers conducted a case study to identify the key factors for the cause of heart failure. Heart failure is a condition in which the heart cannot pump enough blood to meet the body’s needs. A ‘heart attack’ occurs when the flow of oxygen-rich blood to a section of heart muscle is blocked suddenly and when the heart does not receive the required oxygen. If blood flow is not restored quickly, the section of heart muscle begins to die. It is possible to eliminate many of these risk factors in order to reduce the chance of having a first or subsequent heart attack. High blood pressure is caused due to obesity, smoking, high cholesterol or diabetes that increases the risk of heart failure. If siblings, parents or grandparents would have had early heart attacks we may be at increased risk. Human beings are forced to undergo stress in ways that can increase the risk of heart attack. The paper highlights the risk factors for cause of heart attack using nano topology.

Preliminaries:

Definition 2.1[5]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The Pair (U, R) is said to be the approximation space. Let X⊆U

(i). The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by L_R(X).

That is L_R(X) = ⋃_{x∈U} R(x) : R(x) ⊆ X, where R(X) denotes the equivalence class determined by X∈U.

(ii). The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by U_R(X).

That is U_R(X) = ⋃_{x∈U} R(x) : R(x) ∩ X ≠ φ

(iii). The boundary of the region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by B_R(X). That is B_R(X) = U_R(X) – L_R(X).

Properties 2.2[5]

(i) L_R(X) ⊆ X ⊆ U_R(X).

(ii) L_R(φ) = U_R(φ) = φ and L_R(U) = U_R(U) = U

(iii) U_R(X∪Y) = U_R(X) ∪ U_R(Y)

(iv) U_R(X∩Y) ⊆ U_R(X) ∩ U_R(Y)

(v) L_R(X∪Y) ⊇ L_R(X) ∪ L_R(Y)

(vi) L_R(X∩Y) = L_R(X) ∩ L_R(Y)

(vii) L_R(X) ⊆ L_R(Y) and U_R(X) ⊆ U_R(Y) whenever X ⊆ Y

(viii) U_R(X^c) = [L_R(X)]^c and L_R(X^c) = [U_R(X)]^c
(i) $U_{R}U_{R}(X) = L_{R}U_{R}(X) = U_{R}(X)$

(ii) $L_{R}L_{R}(X) = U_{R}L_{R}(X) = L_{R}(X)$

**Definition 2.3 [1]**

Let $U$ be the Universe and $R$ be an equivalence relation on $U$. Then for $X \subseteq U$, $\tau_{R}(X) = \{U, \varphi, L_{R}(X), U_{R}(X), B_{R}(X)\}$ is called the nano topology on $U$. By property 2.2, $\tau_{R}(X)$ satisfies the following axioms:

(i) $U$ and $\varphi \in \tau_{R}(X)$.

(ii) The union of the elements of any subcollection of $\tau_{R}(X)$ is in $\tau_{R}(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_{R}(X)$ is in $\tau_{R}(X)$.

We call $(U, \tau_{R}(X))$ is a nano topological space. The elements of $\tau_{R}(X)$ are called nano open sets and the complement of a nano open sets is called nano closed sets.

Throughout this paper $(U, \tau_{R}(X))$ is a nano topological space with respect to $X$ where $X \subseteq U$. $R$ is an equivalence relation on $U$, $U/R$ denotes the family of equivalence classes of $U$ by $R$.

**Definition 2.4 [1]**

Let $(U, \tau_{R}(X))$ be a nano topological space, the set $\beta = \{U, L_{R}(X), B_{R}(X)\}$ is called a bases for the nano topology $\tau_{R}(X)$ on $U$ with respect to $X$.

**Definition 2.5 [1]**

If $(U, \tau_{R}(X))$ is a nano topological space with respect to $X$, where $X \subseteq U$ and if $A \subseteq U$, then (i) The nano interior of the set $A$ is defined as the union of all nano open subsets contained in $A$ and is denoted by $N_{\text{int}}(A)$.

(ii) The nano closure of the set $A$ is defined as the intersection of all nano closed subsets containing $A$ and is denoted by $N_{\text{cl}}(A)$.

**Definition 2.6**

Let $(U, \tau_{R}(X))$ be a nano topological space and $A \subseteq U$. Then $A$ is said to be

(i) nano regular open [1] if $A = N_{\text{int}}(N_{\text{cl}}(A))$

(ii) nano $\alpha$-open [1] if $A \subseteq N_{\text{int}}(N_{\text{cl}}(N_{\text{int}}(A)))$

(iii) nano preopen [1] if $A \subseteq N_{\text{int}}(N_{\text{cl}}(A))$

(iv) nano regular generalized closed set [6] if $N_{\text{cl}}(A) \subseteq V$ whenever $A \subseteq V$ and $V$ is Nano regular open in $(U, \tau_{R}(X))$.

**Definition 2.7 [1]**

Let $(U, \tau_{R}(X))$ be a nano topological space and $A \subseteq U$. Then $A$ is said to be nano $\alpha$-closed(respectively, nano regular closed) if its complement is nano $\alpha$-open (nano regular open).

**Ngrα-closed Sets**

**Definition 3.1**

A subset $A$ of a nano topological space $(U, \tau_{R}(X))$ is called Ngrα-open if there is a nano regular open set $V$ such that $V \subseteq A \subseteq N_{\text{cl}}(V)$.

**Definition 3.2**

A subset $A$ of a nano topological space $(U, \tau_{R}(X))$ is called Ngrα-closed if $N_{\text{cl}}(N_{\text{int}}(A)) \subseteq V$ whenever $A \subseteq V$ and $V$ is Ngrα-open in $(U, \tau_{R}(X))$.

**Theorem 3.3**

(i) Every nano-closed set is Ngrα-closed.

(ii) Every nano regular-closed set is Ngrα-closed.

(iii) Every nano-$\alpha$ closed set is Ngrα-closed.

**Remark 3.4**

Converse of the above theorem need not be true as seen from the following example.

**Example 3.5**

Let $U = \{a, b, c, d\}$, $X = \{a, b\}$, $\tau_{R}(X) = \{U, \varphi, \{a\}, \{b, d\}, \{a, b, d\}\}$. The nano closed sets are $\{U, \varphi, \{a\}, \{a, c\}, \{b, c, d\}\}$. The nano regular closed sets are $\{U, \varphi, \{a, c\}, \{b, c, d\}\}$. The nano $\alpha$-closed sets are $\{U, \varphi, \{c\}, \{a, c\}, \{b, c, d\}\}$. The Ngrα-closed sets are $\{U, \varphi, \{b\}, \{c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$.

**Remark 3.6**

From the above discussion, we have the following implications (Figure 1).
Theorem 3.7

The union of two \( N\alpha \)-closed sets is also a \( N\alpha \)-closed set.

Proof

Let \( A \) and \( B \) be two \( N\alpha \)-closed sets in \((U, \tau_\alpha(X))\). Let \( Ncl(Nint(A)) \subseteq V, Ncl(Nint(B)) \subseteq V, A \subseteq V, B \subseteq V \) and \( V \) be \( Nr_\alpha \)-open. Then we have, \( A \cup B \subseteq V \). Now \( Ncl(Nint(A \cup B)) = Ncl(Nint(A)) \cup Ncl(Nint(B)) \subseteq V \). Therefore \( A \cup B \) is a \( N\alpha \)-closed set in \( U \).

Remark 3.8

The intersection of two \( N\alpha \)-closed sets is not \( N\alpha \)-closed set as shown from the following example.

Example 3.9

Let \( U = \{a, b, c, d\} \), \( X = \{a, b\} \), \( \tau_\alpha(X) = \{U, \varnothing, \{a\}, \{b, d\}, \{a, b, d\}\} \). \( N\alpha \)-closed are \( \{U, \varnothing, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\} \). Here \( \{a, c\} \cap \{a, b, d\} = \{a\} \) is not \( N\alpha \)-closed.

Theorem 3.10

If A is a \( N\alpha \)-closed subset of \((U, \tau_\alpha(X))\), then \( Ncl(Nint(A)) \) – \( A \) contains no non-empty \( Nr_\alpha \)-open.

Proof

Suppose that A is \( N\alpha \)-closed in \( U \). We prove the result by contradiction. Let \( F \) be a \( Nr_\alpha \)-open such that \( F \subseteq Ncl(Nint(A)) \) – \( A \) and \( F = \varnothing \). Then \( A \supseteq U - F \). Since A is \( N\alpha \)-closed set, \( F \) is \( Nr_\alpha \)-open and \( U - F \) is nano is also \( Nr_\alpha \)-open, we have \( Ncl(Nint(A)) \subseteq U - F \). So \( F \subseteq U - Ncl(Nint(A)) \). Therefore \( F \cap (Ncl(Nint(A)) \cap U - Ncl(Nint(A))) = \varnothing \), this is a contradiction. Hence \( Ncl(Nint(A)) \) – \( A \) does not contain any non-empty \( Nr_\alpha \)-open set.

Theorem 3.11

If a subset A of \((U, \tau_\alpha(X))\) is \( N\alpha \)-closed, then \( Ncl(Nint(A)) \) – \( A \) does not contain any nano regular open set in \( U \).

Proof

Proof follows from the fact that every nano regular open set is \( Nr_\alpha \)-open.

Theorem 3.12

Let A be a \( N\alpha \)-closed subset of \((U, \tau_\alpha(X))\). Then A is nano regular closed if and only if \( Ncl(Nint(A)) \) – \( A \) is a \( Nr_\alpha \)-open.

Proof

Suppose \( A \) is a nano regular closed. Then \( Ncl(Nint(A)) = A \) and \( Ncl(Nint(A)) - A = \varnothing \), which is \( Nr_\alpha \)-open. Conversely, let \( Ncl(Nint(A)) - A \) be a \( Nr_\alpha \)-open in \( U \). Since \( A \) is \( N\alpha \)-closed, by theorem 3.10, \( Ncl(Nint(A)) - A \) does not contain any non-empty \( Nr_\alpha \)-open set. Then \( Ncl(Nint(A)) - A = \varnothing \), hence \( A \) is nano regular closed.

Theorem 3.13

If \( A \) is \( N\alpha \)-closed subset of \((U, \tau_\alpha(X))\) such that \( A \subset B \subseteq Ncl(Nint(A)) \), then \( B \) is \( N\alpha \)-closed set in \((U, \tau_\alpha(X))\).

Proof

Let \( A \) be a \( N\alpha \)-closed subset of \((U, \tau_\alpha(X))\) such that \( A \subset B \subseteq Ncl(Nint(A)) \). Let \( V \) be \( Nr_\alpha \)-open set such that \( B \subseteq V \), then \( A \subseteq V \). Since \( A \) is \( N\alpha \)-closed, \( Ncl(Nint(A)) \subseteq V \). Now we have \( Ncl(Nint(B)) \subseteq Ncl(Nint(Ncl(Nint(A)))) = Ncl(Nint(A)) \subseteq V \). This implies that \( B \) is \( N\alpha \)-closed.

Theorem 3.14

Let \( A \) be an \( N\alpha \)-closed. Then \( A \) is \( Nr_\alpha \)-closed if and only if \( Ncl(Nint(A)) \) – \( A \) is a \( Nr_\alpha \)-open.

Proof

Suppose \( A \) is a nano regular closed. Then \( cl(Nint(A)) = A \) and \( Ncl(Nint(A)) - A = \varnothing \), which is a \( Nr_\alpha \)-open in \( U \). Conversely, let \( Ncl(Nint(A)) - A \) be a \( Nr_\alpha \)-open set in \( U \). Since \( A \) is \( N\alpha \)-closed, by theorem 3.10, \( Ncl(int(A)) \) – \( A \) does not contain any non-empty \( Nr_\alpha \)-open in \( U \). Then \( Ncl(Nint(A)) - A = \varnothing \), hence \( A \) is nano regular closed.

Theorem 3.15

If \( A \) is \( N\alpha \)-closed and nano regular open, then \( A \) is nano rg-closed, nano pre closed and nanoclopen.

Proof

Since every nano regular open set is nano open and \( Nr_\alpha \)-open.
Let \( A \subseteq A \) be \( N\alpha \)-closed and nano regular open , \( Ncl(Nint(A)) \subseteq A \) and \( Ncl(A) \subseteq Ncl(A) \subseteq Ncl(Nint(A)) \subseteq A \). Therefore \( A \) is nanoclopen.

Definition 3.16

A space \((U, \tau_\alpha(X))\) is called \( T_{N\alpha} \) –space if every \( N\alpha \)-closed set is nano closed.
Definition 3.17
A space \((U, \tau_R(X))\) is called Ngrα- \(T_{1/2}\) space if every Ngrα-closed is nano \(\alpha\)-closed.

Theorem 3.18
For a Nano topological space \((U, \tau_R(X))\) the following are equivalent

(i) \(U\) is Ngrα-\(T_{1/2}\) space.

(ii) Every singleton of \(U\) is either Nr\(\alpha\)-closed (or) nano \(\alpha\)-open.

Proof

(i)\(\Rightarrow\) (ii)

Let \(x\in U\) and assume that \(\{x\}\) is not Nr\(\alpha\)-closed. Then clearly \(U-\{x\}\) is also not Nr\(\alpha\)-open and \(U-\{x\}\) is Ngrα-closed. By hypothesis, \(U-\{x\}\) is nano \(\alpha\)-closed and thus \(\{x\}\) is nano \(\alpha\)-open.

(ii)\(\Rightarrow\) (i)

Let \(A\subset U\) be Ngrα-closed. Let \(x\in \text{Ncl}(\text{Nint}(A))\). To show \(x\in A\).

Case(i) : If a set \(\{x\}\) is nano Nr\(\alpha\)-closed and \(x\in A\), then \(A\subset U-\{x\}\). Since \(A\) is Ngrα-closed and \(U-\{x\}\) is Nr\(\alpha\)-open, \(\text{Ncl}(\text{Nint}(A))\subset U-\{x\}\) and hence \(x\in \text{Ncl}(\text{Nint}(A))\), this is a contradiction. Therefore, \(x\in A\).

Case (ii)

The set \(\{x\}\) is nano \(\alpha\)-open. Since \(x\in \text{Ncl}(\text{Nint}(A))\), then \(\{x\}\cap A\neq \emptyset\) implies \(x\in A\). This shows that \(A\) is Nano \(\alpha\)-closed.

Theorem 3.19

(i) \(\text{NaO}(U, \tau_R(X))\subseteq \text{NGRαO}(U, \tau_R(X))\)

(ii) A space is Ngrα-\(T_{1/2}\) space if and only if \(\text{NaO}(U, \tau_R(X)) = \text{NGRαO}(U, \tau_R(X))\).

Proof

(i) Obvious.

(ii) Let \(U\) be a nano \(T_{1/2}\) space and \(A\in \text{NGRαO}(U, \tau_R(X))\). Thus, \(\text{NGRαO}(U, \tau_R(X)) = \text{NaO}(U, \tau_R(X))\). Conversely, let \(\text{NaO}(U, \tau_R(X)) = \text{NGRαO}(U, \tau_R(X))\). A is Ngrα-closed implies \(U-A\) is Ngrα-open. Hence A is nano \(\alpha\)-closed.

NGRA-CONTINUOUS AND NGRA-IRRESOLUTE

Definition 4.1
Let \((U, \tau_R(X))\) and \((V, \tau_R(Y))\) be two nano topological spaces. Then a mapping \(f: (U, \tau_R(X))\rightarrow (V, \tau_R(Y))\) is Ngrα-continuous if \(f^1(F)\) is Ngrα-closed in \(U\) if for every nano-closed set \(F\) of \(V\).

Definition 4.2
Let \((U, \tau_R(X))\) and \((V, \tau_R(Y))\) be two nano topological spaces. Then a mapping \(f: (U, \tau_R(X))\rightarrow (V, \tau_R(Y))\) is Ngrα-irresolute if \(f^1(F)\) is Ngrα-closed in \(U\) if for every Ngrα-closed set \(F\) of \(V\).

Theorem 4.3
Let \(f: (U, \tau_R(X))\rightarrow (V, \tau_R(Y))\) and \(g: (V, \tau_R(Y))\rightarrow (W, \tau_R(Z))\) be any two functions. Then

(i) \(g\circ f\) is Ngrα-continuous if \(g\) is nano-continuous and \(f\) is Ngrα-continuous.

(ii) \(g\circ f\) is Ngrα-irresolute if \(g\) is Ngrα-irresolute and \(f\) is Ngrα-irresolute.

(iii)\(g\circ f\) is Ngrα-continuous if \(g\) is Ngrα-continuous and \(f\) is Ngrα-irresolute.

(iv)\(g\circ f\) is nano \(\alpha\)-continuous if \(g\) is nano \(\alpha\)-irresolute and \(f\) is Ngrα-continuous and \(V\) is a Ngrα-\(T_{1/2}\) space.

Proof

(i) Let \(F\) be nano closed in \(W\). Then \(g^{-1}(F)\) is nano closed in \(V\). Since \(g\) is nano continuous. By hypothesis, \(f^1(g^{-1}(F))\) is Ngrα-closed in \(U\). Hence \(g\circ f\) is Ngrα-continuous.

(ii),(iii) and (iv). These proofs are obtained similarly as the proof of (i).

APPLICATIONS OF NANO TOPOLOGY

Example 5.1
Here we apply the nano topology to find the key factors of ‘Heart Attack’ using topological reduction of attributes in incomplete information system.

The following table gives information about patients those who are having High Blood Pressure, Alcohol and smoking, Stress and strain, Diabetics and Family history.
Here U = \{B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8\}, the set of patients and A = \{High Blood Pressure, Alchohol and smoking, Diabetics, Stress and strain\} and the family of equivalence classes, U/C corresponding to C is given by U/R(C) = \{\{B_1, B_4\}, \{B_2\}, \{B_3, B_6\}, \{B_5\}, \{B_7\}, \{B_8\}\}. The basis of \(\beta_c(X)\) is given by \(\beta_c(X) = \{U, \{B_1, B_2\}, \{B_3, B_4, B_5\}\}\).

Step 1: When the attribute ‘High Blood Pressure’ is removed from C, \(U/R(C-BP) = \{\{B_1, B_3, B_5\}, \{B_2\}, \{B_4, B_6\}, \{B_7\}, \{B_8\}\}\) and hence the lower and upper approximations of X corresponding to C-BP are given by \(L_{C-BP}(X) = \{\{B_2, B_7\}\}\) and the corresponding boundary region is \(B_{C-BP}(X) = \{B_1, B_3, B_5, B_6, B_7, B_8\}\).

Step 2: When the attribute ‘Alcohol and smoking’ is removed from C, \(U/R(C-AS) = \{\{B_1, B_4\}, \{B_2\}, \{B_3, B_6\}, \{B_5\}, \{B_7\}, \{B_8\}\}\) and hence \(\tau_{C-AS}(X) = \{U, \{B_2, B_7\}\}\) and \(\beta_{C-AS}(X) = \{U, \{B_2\}\}\).

Step 3: When the attribute ‘Diabetics’ is removed from C, \(U/R(C-DB) = \{\{B_1, B_4\}, \{B_2, B_6\}, \{B_3, B_5\}, \{B_7\}, \{B_8\}\}\) and hence \(\tau_{C-DB}(X) = \{U, \{B_2, B_7\}\}\) and \(\beta_{C-DB}(X) = \{U, \{B_2\}\}\).

Step 4: When the attribute ‘Stress and strain’ is removed from C, \(U/R(C-SS) = \{\{B_1, B_4\}, \{B_2\}, \{B_3, B_6\}, \{B_5\}, \{B_7\}, \{B_8\}\}\) and hence \(\tau_{C-SS}(X) = \{U, \{B_2, B_7\}\}\) and \(\beta_{C-SS}(X) = \{U, \{B_2\}\}\).

Step 5: When the attribute ‘Remove family History’ is removed from C, \(U/R(C-FH) = \{\{B_1, B_4\}, \{B_2\}, \{B_3, B_6\}, \{B_5\}, \{B_7\}\}\) and hence \(\tau_{C-FH}(X) = \{U, \{B_2, B_7\}\}\) and \(\beta_{C-FH}(X) = \{U, \{B_2\}\}\).

Table 1

<table>
<thead>
<tr>
<th>Patients</th>
<th>High Blood Pressure</th>
<th>Alcohol and smoking</th>
<th>Diabetics</th>
<th>Stress and strain</th>
<th>Family history</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>B_1</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>Nil</td>
<td>Nil</td>
<td>Yes</td>
</tr>
<tr>
<td>B_2</td>
<td>√</td>
<td>Nil</td>
<td>Nil</td>
<td>√</td>
<td>√</td>
<td>Yes</td>
</tr>
<tr>
<td>B_3</td>
<td>Nil</td>
<td>√</td>
<td>√</td>
<td>Nil</td>
<td>Nil</td>
<td>Yes</td>
</tr>
<tr>
<td>B_4</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>Nil</td>
<td>Nil</td>
<td>No</td>
</tr>
<tr>
<td>B_5</td>
<td>Nil</td>
<td>√</td>
<td>√</td>
<td>Nil</td>
<td>Nil</td>
<td>No</td>
</tr>
<tr>
<td>B_6</td>
<td>Nil</td>
<td>√</td>
<td>√</td>
<td>Nil</td>
<td>√</td>
<td>Yes</td>
</tr>
<tr>
<td>B_7</td>
<td>√</td>
<td>Nil</td>
<td>√</td>
<td>√</td>
<td>Nil</td>
<td>No</td>
</tr>
<tr>
<td>B_8</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Case 1 (Patients with Heart Attack) Let X = \{B_1, B_2, B_3, B_4\}, the set of patients with Heart Attack. Hence \(L_C(X) = \{B_1, B_2\}\), \(U_C(X) = \{B_1, B_2, B_3, B_4, B_5, B_6\}\) and \(B_C(X) = \{B_1, B_2, B_3, B_4, B_5, B_6, B_7\}\). The basis of \(\tau_C(X)\) is given by \(\beta_C(X) = \{U, \{B_2\}\}\).
Step 4: When the attribute ‘Stress and Strain’ is removed from C, U/R(C-SS) = {{B₁, B₄}, {B₂}, {B₃, B₆}, {B₅}, {B₇}, {B₈}} and hence τ_{C-(SS)}(X) = {U, φ, {B₅}, {B₈}} and hence β_{C-(SS)}(X) = {U, {B₅, B₈}} ≠ β_{C}(X).

Step 5: When the attribute ‘Remove family History’ is removed from C, U/R(C-FH) = {{B₁, B₄}, {B₂}, {B₃, B₅, B₆}, {B₇}, {B₈}} and hence τ_{C-(FH)}(X) = {U, φ, {B₅}, {B₈}, B₅, B₆, B₇, B₈} and β_{C-(FH)}(X) = {U, {B₅}} ≠ β_{C}(X). Therefore, CORE = {High Blood Pressure, Family history}.

**Observation:** From the CORE, we observed that ‘High Blood Pressure’ and ‘Family History’ are the key factors for heart attack. Proper medical care and change in the behavioral pattern can prevent the risk.

**REFERENCES**


