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A Review of the Fundamentals on Process Capability, Process Performance, and Process Sigma, and an Introduction to Process Sigma Split

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Abstract

There are several indices for quality control, like Capability and Process Performance indices, within the scope of design and process optimization. Given the amount of existing literature produced in the last decades, and its level of complexity reached so far, this article aims to work as an essential digest of Process Capability and Process Performance indices, in relation also to Process Sigma (or Sigma Level), introduced within Lean Six Sigma. Particularly, the article provides a clear review of the fundamental concepts and applications of the above-mentioned indices to calculate and evaluate process performance and process yield. Moreover, this article offers an overview of the features of each metric, specifying the main differences and defining hypotheses of their applicability in process performance evaluation. The article further clarifies the different methods to calculate Process Sigma (for centered and off-center processes with and without 1.5σ shift). Besides outlining the essentials, reaching out also to a more operative audience in manufacturing and service industry, the novelty of this paper is also to suggest a rapid method to standardize statistical language and to confront either similar or competing processes. In fact, the authors have devised a new index named “Capability Difference”, to obtain a new method to calculate Process Sigma through “Process Sigma Split”, which allows to isolate the current performance from future performance, for a clearer evaluation of the process. This research, its results and insights, might further studies, and lead scientists to develop new, improved, and practical performance dashboards.

Keywords: Design, Optimization, Quality Control, Lean Six Sigma

INTRODUCTION

Over the years, a series of methods, like Lean Six Sigma [1], World Class Manufacturing [2], and TRIZ [3] have been devised for Quality Improvement, with activities that any organization should undertake in order to verify that the characteristics of their processes match the target. This can also be helpful in terms of reliability requirements, as far as production and product development are concerned, and it is widely acknowledged that all manufacturing and service companies need to analyse process defectiveness and yield. Several metrics have been used to detect this information, i.e. Process Capability Indices (PCI), Process Performance Indices (PPI), and Process Sigma.

The statistical techniques [4] to attribute a value to Process Capability, Process Performance and Process Sigma (or Sigma Level) are necessary for the whole cycle of production of a product/service, for a correct evaluation of customer/market requests (VOC, “Voice of Customer”) and process performance (VOP, “Voice of Process”) [5].

In view of the amount of existing literature on the topic [6] [7], and of the increasing complexity of the indices, this article aims to offer a review of the fundamentals of Process Capability, Process Performance and Process Sigma, to isolate the crucial concepts and applications needed to start evaluating/calculating process performance and yield. Process Capability Analysis frequently entails characterizing or assessing process or products based on more than one engineering specification or quality characteristics [8]. Interpolation methods are used in engineering problems characterised by iterative calculation procedures, since they create an effective tool to model and optimise in the multi-objective field [9]. Multivariate Analysis and Response Optimization are not covered by this paper, as they are not directly related to the Process Sigma calculation and to the quantification of Process Performance.

In order to retrieve the essentials, this article offers an overview of each metric, highlighting differences and hypotheses of applicability, to clarify the different methods to calculate Process Sigma, and to suggest a rapid method to standardize statistical language and confront either similar or competing processes.

Section 2 covers the main Process Capability Indices (PCI), C_p and C_{pk}, discussing the centering index C_a, and the index k. Section 3 debates Process Performance Indices, specifically P_p e P_{pk}. Section 4 looks into Process Sigma (or Sigma Level), specifying how to calculate Process Sigma for centered processes, and for off-center processes. Section 5 introduces a new index (Capability Difference Index) for a better understanding of how to compare and contrast different processes. Capability Difference and Process Sigma Split are applied to a specific case study on a mechanical component of hydraulic pumps to suggest a possible application. Section 6 offers final remarks and further perspectives of research.

PROCESS CAPABILITY INDICES (PCI)

When dealing with Continuous Improvement of quality, and for Lean Six Sigma Programs [10], Process Capability Indices (PCI) are considered some of the fundamentals quality measurement tools needed. This because they evaluate related process performance, and compare one material or supplier with others, when applied properly to improve process control performance.

In 1939, Walter Shewhart [11] first developed Statistical Process Control (SPC) to measure quality and variability during a manufacturing process. According to this methodology, every process is subject to two types of variation: variation from an ordinary cause that is naturally present in any process in nature, is innate in the process, and it is expected to be present; variation from special causes is the variation which comes from the fact that an extraordinary event has occurred [12]. SPC regime is implemented in two phases. Phase I aims at a better understanding of the process, and at assessing process stability. The aim of Phase 2 is to find assignable cause of variation to bring the process in control [13].

Capability studies generally apply to the so-called “stable processes”, i.e. processes showing uniformity of behavior. Variability measures the uniformity of the characteristic feature of the outgoing product, and it can be of two types: natural variability, pertaining to a particular moment (also instant variability), and variability versus time [12]. This is why a Six Sigma range of the distribution of the quality features of the product is used to measure Process Capability (Limits of Natural Tolerance or Control Limits).

As previously anticipated, it is now more evident that Process Capability analysis is vital for any program aiming at improving quality [14].

Data collected from such analysis are useful to:

- predict how the process will comply with the tolerances;
- assist the research and development of the process changes;
- establish the sampling frequency for the evaluation procedures;
- establish the performance requirements of new equipment;
- select suppliers;
- plan production even in presence of interaction of the process on the tolerances;
- reduce process variability.

It is therefore easy to understand why Process Capability analysis, and statistical analysis in general, are employed in several areas on a wide range of applications [12] [15], which happens to create a proper scientific basis in design area as well [16].

To calculate process performance, Process Capability Indices (PCI), Process Performance Indices (PPI), and Process Sigma are mainly used [17] [18].

Capability analysis can rely either on historical data, or on data specifically collected for the purpose of the analysis itself. In this paper, the theoretical analysis has been developed with Normal data, that is to say the Normality hypothesis for the distribution of the process output, because, when applied to Non-normal data, statistical indices could turn out to be unreliable [12].

Process Capability Analysis helps to quantify, monitor and, eventually, reduce process variability. By using Capability Indices, Process Capability Analysis compares the output of a stable process to the specification limits, and it is a measurement with respect to inherent precision of a manufacturing process. Process Capability can be calculated via several Process Capability ratios and indices. Process Capability Indices provide single number assessment of the ability of the process to meet specification limits for the quality characteristics of interest. Therefore, it identifies opportunities to improve quality and operational performance [19].

Process Capability Index (PCI) indicates the extent to which the process can produce the output that conforms to certain requirements. It is determined by comparing the natural variability of a process to the customer or engineering specifications [20].

The relationship between the current process performance and the specification limits may be quantified by the use of appropriate indices [21] [22].

$C_{p,}$ also called precision index [23] is the first Process Capability Index appearing in literature**,** and it was defined as the ratio of specification width (USL–LSL) over the process spread (6σ), with USL and LSL stand for Upper and Lower Specification Limits. The specification width represents customer and/or product requirements [24] [11]. Process variation is represented by the specification width:

$$C_p = \frac{VOC}{VOP} = \frac{(USL - LSL)}{6\sigma_{S.T.}}$$

where $6\sigma_{ST}$ is the natural tolerances and σ_{ST} is the Short-Term standard deviation of the process [25] [26]. The value of C_p is small, when process variation is large, and it represents a low Process Capability [27] [18] [28] [29].

The value of C_p helps to better understand process performance. For example, if it is greater than 1.33, which corresponds to the percentage of non-conforming items of 63 parts per million (ppm), process performance is satisfactory for a centered process. The quality conditions and the corresponding C_p values are reported in Table 1 [30] [31].

Table 1: Quality Conditions and C_p values for centered process.

Quality Condition	**C_p**
Super Excellent	$C_p \geq 2.00$
Excellent	$1.67 \leq C_p < 2.00$
Satisfactory	$1.33 \leq C_p < 1.67$
Capable	$1.00 \leq C_p < 1.33$
Inadequate	$0.67 \leq C_p < 1.00$
Poor	$C_p < 0.67$

Short-Term standard deviation (or Sigma) is indicated with $\sigma_{S.T.}$, while Long-Term standard deviation with $\sigma_{L.T.}$. Short-Term Sigma is generally obtained through the formula:

$$\sigma_{S.T.} = \frac{\bar{R}}{d_2}$$

where $\bar{R}$ represents the average Range of m samples analyzed and d_2 represents a constant, depending on the sample size (n) of m samples, as shown in Table 2 [32] [33].

Table 2: Values of d_2 as a function of sample size.

Sample Size (n)	**d_2**
2	1.128
3	1.693
4	2.058
5	2.325
6	2.536
7	2.706
8	2.844

C_p provides information just on the theoretical capacity of the process, because it does not take into account where the process mean is located, relative to the specifications. C_p measures only the spread of the specification relative to the six sigma spread in the process, and thus gives no indication of the actual process performance. Kane [23] (1986) introduced C_{pk} index to respond to this problem [34].

C_{pk} index is used to relate process variability, by showing how a process conforms to its specifications. C_{pk} is generally used to relate the "Natural Tolerances ($\pm 3\sigma_{S.T.}$)" to the specification limits. C_{pk} describes how well the process is comprised within the specification limits, with reference to the the process mean [28] [12] [35].

$$C_{pk} = min\{C_{pku}; C_{pkl}\}$$

where [36] [37]:

$$C_{pku} = \frac{(USL - \mu)}{3\sigma_{S.T.}}$$

$$C_{pkl} = \frac{(\mu - LSL)}{3\sigma_{S.T.}}$$

C_{pk} provides information on the actual process capability.

Further developments in the studies on Capability resulted in new PCIs to be introduced and discussed in the related literature, like the C_a index, which concentrates on the location of the process mean:

$$C_a = \frac{C_{pk}}{C_p}$$

C_a is also defined as [32] [35] [31]:

$$C_a = 1 - k$$

where:

$$k = \frac{|\mu - m|}{d}$$

k describes the Process Capability in terms of distance of the process mean μ from midpoint m and provides a measure of an off-centered process. m=(USL+LSL)/2 is the midpoint between the Upper and Lower Specification Limits and d=(USL-LSL)/2 is the half specification width related to the manufacturing tolerance [28] [38] [39].

therefore:

$$C_a = 1 - \frac{|\mu - m|}{d}$$

Table 3 displays various C_a values and their corresponding ranges of μ.

Table 3: The relationship between C_a values and ranges of μ.

C_a values	Ranges of μ
$C_a = 1.00$	$\mu = m$
$0.75 < C_a < 1.00$	$0 < (\mu\text{-}m) < d/4$
$0.50 < C_a < 0.75$	$d/4 < (\mu\text{-}m) < d/2$
$0.25 < C_a < 0.50$	$d/2 < (\mu\text{-}m) <$ ¾ d
$0.00 < C_a < 0.25$	¾ d $< (\mu\text{-}m) < d$

A new modality to calculate the C_{pk} index can be devised, starting from the formulae used to calculate C_a and through a series of mathematical steps:

$$C_{pk} = \frac{d - |\mu - m|}{3\sigma}$$

PROCESS PERFORMANCE INDICES (PPI)

The Automotive Industry Action Group (AIAG) recommends to use the Process Capability Indices C_p and C_{pk} only when the analyzed process is under statistical control, with data collected on an assigned Short-Term Sigma, calculated with the Range average method. A process can be defined under statistical control if it is stable, that is when all its points fall within the stability range, and if all the sensitivity rules of the SPC techniques have been met [40].

When a process, on the other hand, is not under statistical control, the AIAG advises to use the Process Performance Indices P_p e P_{pk}, with:

$$P_p = \frac{VOC}{VOP} = \frac{(USL - LSL)}{6\sigma_{L.T.}}$$

where the standard deviation is called Long-Term Sigma and is calculated with the traditional formula that analyzes quadratic divergences of any point compared to the mean, on the number of degrees of freedom [12].

$$\sigma_{L.T.} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

$$P_{pk} = min\{P_{pku}; P_{pkl}\}$$

where:

$$P_{pku} = \frac{(USL - \mu)}{3\sigma_{L.T.}}$$

$$P_{pkl} = \frac{(\mu - LSL)}{3\sigma_{L.T.}}$$

In 1996, the American National Standard Institute has also adopted in the ANSI Z1 Standard a position on the analysis of Capability studies, confirming that the P_p and P_{pk} indices can be used only when the process is not under control. It is shown, indeed, that when the process has a Normal distribution and is under statistical control, P_p is essentially close to C_p and P_{pk} to C_{pk}, because the Short-Term and Long-Term standard deviations are more or less similar [32] [12] [41].

Some literature, however, argues on the exclusive use of P_p and P_{pk}, explaining how these indicators can work at cost, but cannot, instead, act as a preventive therapy [42][35].

Montgomery [12] partly agrees with this opinion and declares that in some sectors, e.g. the automotive, the performance indicators P_p e P_{pk} can represent a wasted managing effort, but do not add any information more than those indicated by the C_p and C_{pk} indexes.

Process performances are visually summarized in terms of Process Capability and Process Performance with Control Charts, together with the histograms and the related diagrams of the density of probability of monitored distributions [12] [43] [44].

Both the Control Charts for attributes and those for variables data can be used in the Capability Analysis. The X Bar Chart and R Chart, for example, which are the most used ones due to their power and quality of offered output, give information on the Process Capability with the R Chart and on Process Performance with the X Bar Chart [45] [46] [47].

Very recent studies have confirmed that the use of the Capability Indices is better in order to analyze existing processes and those under statistical control. The Performance Indices, instead, are used to analyze new processes or to exclusively have a quick and qualitative estimate on processes that are not under statistical control and subject to high variability [32] [41] [31].

PROCESS SIGMA OR SIGMA LEVEL

In order to create one common metric to quantify Process Performance on all sort of data, the Six Sigma methodology has advanced Process Sigma or Sigma Level (the Z value in the Z-Table [48]) as an alternative method to calculate PCI and PPI, that is, as a measurement of the ability to satisfy customer requirements: the higher the value of Process Sigma, the higher will be the Capability of the process itself [49].

The first evident advantage of this new metric is allowing processes to be benchmarked across different industries, data worlds (continuous, count, attribute), technologies, etc.

Process Sigma represents a measurement of process yield, that is a measurement of success in achieving an output of a product with no defects and of the quality of the process itself [50].

The Six Sigma metric extends the concept of yield, since it is able to evaluate differences in terms of performance corresponding to minimal percentage changes of the reference index. Actually, if the following processes were characterized by a yield of 99%, Harry [51] specifically outlined that there could be:

- 20,000 lost articles of mail per hour
- Unsafe drinking water almost 15 minutes per day
- 5,000 incorrect surgical operations per week
- 2 short or long landing at most major airports each day
- 200,000 wrong drug prescriptions each year
- No electricity for almost 7 hours per month

Obviously, this level of quality is not satisfactory; the evaluation of Sigma Level can therefore offer a refined measurement of the quality of a product/process. The relation between Sigma Level and Defect rate is not linear [1], as it results from Figure 1. Therefore, another advantage of Process Sigma, being on a non-linear scale, is to increase resolution at a low defect rate, grasping even the slightest differences, as shown in Table 4.

Table 4: Process Sigma (centered: without sigma shift) and defects (centered process).

Process Sigma	Defects (ppm)
1	317,300
2	45,500
3	2,700
4	63
5	0.57
6	0.002

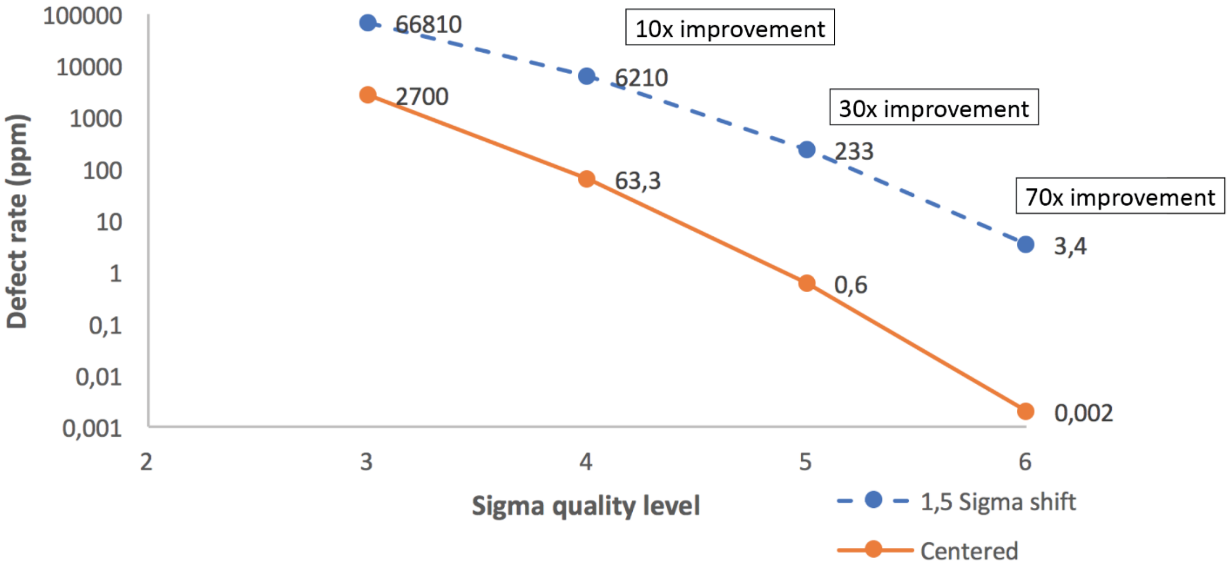


Figure 1: Relationship between Sigma Quality Level (with 1.5σ shift) and process defects.

Process Sigma can be obtained according to the type of data analyzed, and through various methods [52]. For continuous variables [53], Process Sigma is calculated through the standardized Normal curve; for discrete variables Process Sigma is calculated through calculation of the anomalies.

In case of normal data, according to process characteristics, there are different metrics to calculate Process Sigma. The different metrics are influenced by:

- process centering according to customer or engineering specifications;

- the use of a Sigma Level with or without 1.5σ shift.

As per centering, processes can either be centered or off-center. In a centered process, the mean matches exactly with the midpoint of the interval between the specifications. When using Sigma Level with or without shift, it can be useful to recap the concept of shift as introduced by Motorola. According to normal distribution, only 2 ppb (parts per billion) fall outside a 6σ distance from the target value, which is different from the 3.4 ppm (parts per million) indicated by the Six Sigma method. Such difference is due to the fact the Motorola has always considered a Process Sigma with 1.5σ mean shift. The 1.5σ shift is added to include the long-term mean shift. Table 5 features the relation among Sigma Level, yield, and ppm, in both cases, i.e. with 1.5σ shift and without 1.5σ shift.

Table 5: The relationship among several process perfomance metrics.

Without σ shift (centered)				With 1.5 σ shift			
Sigma Level	**Percentage in specification**	**Percentage defective**	**PPM**	**Sigma Level**	**Percentage in specification**	**Percentage defective**	**PPM**
1	68.2689	31.7311	317311	1	30.2328	69.76721	6976722
1.5	86.6386	13.3614	133614	1.5	49.865	50.13499	501350
2	95.4500	4.5500	45500	2	69.123	30.87702	308770
2,5	98.7581	1.2419	12419	2.5	84.1313	15.86869	158687
3	99.7300	0.2700	2700	3	93.3189	6.68106	66811
3,5	99.9535	0.0465	465	3.5	97.725	2.27504	22750
4	99.9937	0.0063	63.3	4	99.379	0.62097	6210
4,5	99.9993	0.0007	6.8	4.5	99.865	0.13499	1350
5	99.99994	0.00006	0.6	5	99.9767	0.02326	233
5,5	99.999996	0.000004	0.04	5.5	99.9968	0.00317	31.7
6	99.9999998	0.0000002	0.002	6	99.9997	0.00034	3.4

Introducing 1.5σ shift has often been criticized, also because it is often applied incorrectly. When the Six Sigma program started, no long-term data were available, therefore, to have an estimate of future performance, the only possibility was to estimate a long-term variability that was function of the short-term varibility.

Process Sigma for Centered Processes

With Normal distribution of continuous variables, Process Sigma is calculated by evaluating how many times the standard deviation (σ) is measured in the semi-width of the process tolerance, imposed by the customer [51]. When the process is centered, Process Sigma is equal to 3 [51], and the standard deviation is contained 3 times in the interval between the mean μ and the USL specification limit, and between the mean μ and the LSL lower specification limit. Using the same values listed in Table 4, Table 6 explains the further relation between C_p value and Process Sigma for a centered process.

Table 6: Process Sigma, C_p value and defects (centered process).

Defects (ppm)	**C_p**	**Process Sigma**
317,300	0.33	1
45,500	0.67	2
2,700	1	3
63	1.33	4
0.57	1.67	5
0.002	2	6

Table 6 is based on process yield (“Defects”, expressed in ppm). What reported so far can be analytically summed up with:

$$Process\ Sigma_{without\ sigma\ shift} = 3 * C_p$$

This formula is valid for processes that could be approximated to Gaussian distribution, and perfectly centered.

Another method to calculate Process Sigma (taking into account 1.5σ shift) is to calculate ppm through the normal distribution curve, then estimating the related Process Sigma using the Schmidt formula [54], as displayed in table 7.

$$Process\ Sigma_{with\ 1.5\ sigma\ shift} = 0.8406 + \sqrt{29.37 - 2.221\ln(ppm)}$$

Table 7: Calculation of Process Sigma with Schmidt formula.

Defects (ppm)	Process Sigma	Process Sigma (Schimdt)
697,700	1	0.999998764
308,700	2	1.989143766
66,810	3	3.017524208
6,210	4	3.988442962
233	5	4.995505967
3.4	6	6.003156999

Figure 2 exemplifies a centered process, where Process Sigma is calculated with both formulae (with and without 1.5σ shift)

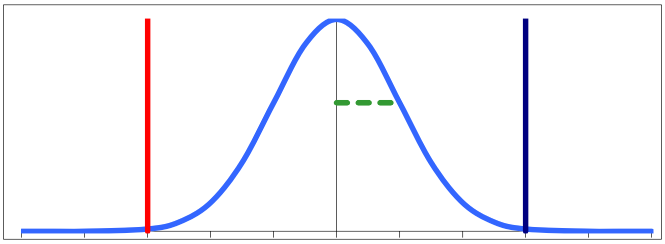


Figure 2: Normal distribution of a centered process with μ=0; σ=1; USL=3; LSL=-3.

μ	**0**
σ	**1**
USL	**3**
LSL	**-3**

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{6}{6} = 1$$

$$C_{pk} = min\left\{\frac{USL-\mu}{3\sigma}; \frac{\mu - LSL}{3\sigma}\right\} = min\left\{\frac{3}{3}; \frac{3}{3}\right\} = 1$$

$$Process\ Sigma_{without\ sigma\ shift} = 3 * C_p = 3 * 1 = 3$$

Estimating the ppm defect rate generated by the process:

$$Z_{USL} = 3$$

$$Z_{LSL} = -3$$

$$P(X_i \geq USL) \cong 0.00135$$

$$P(X_i \leq LSL) \cong 0.00135$$

$$P(X_i \geq USL) \cup P(X_i \leq LSL) \cong 0.27\%$$

0.27% is the percentage related to the probability of failing to meet the specification target, equalling 2700 ppm defect rate.

Including the ppm value inside Schmidt formula:

$$Process\ Sigma_{with\ 1.5\ sigma\ shift} = 0.8406 + \sqrt{29.37 - 2.221\ln(ppm)} \cong 4.28$$

Estimating Process Sigma with the two formulae leads to two different numerical results, but with the same ppm defect rate. Both formulae provide a correct result because it is only a matter of different notation being used.

Process Sigma for off-center processes

As already mentioned for PCI and PPI, it is important to point out that the yield for a specific variable does not describe the whole performance of the process. Where the specification limits are not symmetrical compared with the average, the minor semi-width of the interval is taken into account [55].

Since

$$Process\ Sigma_{without\ sigma\ shift} = \frac{\min\{USL - \mu, \mu - LSL\}}{\sigma}$$

and

$$C_{pk} = min\left\{\frac{USL-\mu}{3\sigma}; \frac{\mu - LSL}{3\sigma}\right\}$$

therefore, $Sigma\ Level$ can also be calculated as

$$Process\ Sigma_{without\ sigma\ shift} = 3 * C_{pk}$$

It is worth remembering that Process Sigma is an index of process yield, tightly connected with ppm. Given two processes A and B, A is considered better (with a better yield) than B if the ppm defect rate of A is minor than the one in B. Estimating Sigma Level with C_{pk} can lead to wrong conclusions, because there is no one-to-one correspondence between C_{pk} and ppm value. C_{pk} is indeed function of the minimum of two specification intervals, namely $C_{pk} = f\left(min\{C_{pku}; C_{pkl}\}\right)$ and not of both intervals. Calculating Process Sigma based on C_{pk}, a relevant part of process information is lost, i.e. ppm related to the larger specification interval. Different processes can exist with same C_{pk} and Sigma Level, and completely different ppm. This can be exemplified by a theoretical process with different specific limits, arranged on three examples, A, B, and C. The process has μ=8 and σ=1; example A features USL=10 and LSL=2; example B features USL=10 and LSL=5; example C features USL=10 and LSL=6. Example A outlined all relevant calculations, while data relating to Examples B and C are reported in Table 8, in comparison with data from Example A.

Example A

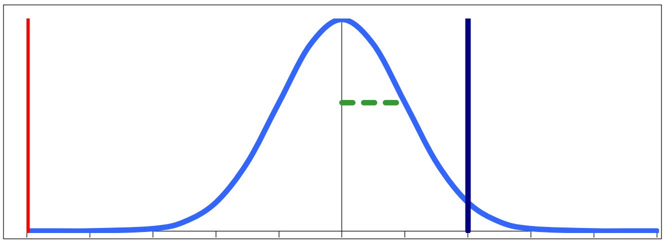


Figure 3: Normal distribution of a process with μ=8; σ=1; USL=10; LSL=2.

μ	8
σ	1
USL	10
LSL	2

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{8}{6} \cong 1.33$$

$$C_{pk} = min\left\{\frac{USL - \mu}{3\sigma}; \frac{\mu - LSL}{3\sigma}\right\} = min\left\{\frac{2}{3}; \frac{6}{3}\right\} \cong 0.67$$

$$Process\ Sigma_{without\ sigma\ shift} = 3 * C_{pk} = 3 * 0.67 \cong 2$$

Estimating the ppm defect rate generated by the process:

$$Z_{USL} = \frac{10 - 8}{1} = 2$$

$$Z_{LSL} = \frac{2 - 8}{1} = -6$$

$$P(X_i \geq USL) = 0.02275$$

$$P(X_i \leq LSL) \cong 0$$

$$P(X_i \geq USL) \cup P(X_i \leq LSL) \cong 2.275\%$$

2.275% is the percentage related to the probability of failing to meet the specification target, equalling 22750 ppm defect rate.

Table 8: Calculating PCI and Process Sigma with and without 1.5σ shift of the process A, B, and C.

	μ	σ	USL	LSL	C_p	C_{pk}	ppm	Process Sigma without shift	Process Sigma with shift
Example A	8	1	10	2	1.333	0.667	22750	2	3.5
Example B	8	1	10	5	0.833	0.667	24100	2	3.48
Example C	8	1	10	6	0.667	0.667	45500	2	3.19

Process Sigma is a performance indicator. In all examples A, B, and C, the same Process Sigma without shift has been obtained, therefore a first conclusion would entail that the three processes have the same yield/performance; however, ppm defect rates are very different, as the yield of the three processes is. In these cases, calculating Process Sigma with $Process\ Sigma_{without\ sigma\ shift} = 3 * C_{pk}$ could not isolate the differences in yield among the three processes. Process Sigma calculated with Schmidt formula (and based on ppm) highlights process different yields $Process\ Sigma_A = 3.5; Process\ Sigma_B = 3.48; Process\ Sigma_C = 3.19$

INTRODUCING A NEW INDEX: PROCESS SIGMA SPLIT

On the basis of the above-mentioned review, the authors have introduced a new index, the Capability Difference, expressing the difference between theoretical capability and actual capability of a process. On the basis of Capability Difference, Process Sigma Split can be calculated.

The Process Sigma Split has the advantage of keeping separating Process Sigma (related to C_{pk}) which provides information on actual process performance, and Capability Difference, (related to C_p), i.e. the increase of Process Sigma that would happen if the process were centered. The authors maintain that, by separating the two, it is possible to have better information on the actions to undertake after measuring performances, in order to improve process yield and capability. To increase process performance, and with a relatively big Capability Difference, process centering ought to be improved; while if Capability Difference is relatively small, and Process Sigma Split is not satisfying, to reduce variability, the process ought to be deeply modified.

$$Capability\ Difference = (C_p - C_{pk}) * 3$$

$$Process\ Sigma\ Split = 3 * C_{pk} + Capability\ Difference$$

In order to clarify the usage of Process Sigma Split, it has been applied to the three examples introduced in Section 4.2. Example A features a process with actual performance of Process Sigma =2, but, if process was to be centered, a Capability Difference =2 could be obtained, therefore a final Process Sigma Split =4 could be attained. Accordingly, Example B shows and actual performance of Process Sigma =2, but, if process can be centered, a Capability Difference =0,5 could be obtained, therefore a final Process Sigma Split =2,5 could be attained. In Example C, where the process is centered, Capability Difference =0, therefore the Process Sigma Split equals the Process Sigma without shift. Table 9 outlines the results.

Table 9. Calculating PCI and Process Sigma with and without 1.5σ shift of the process A, B, and C, applying Process Sigma Split.

	μ	σ	USL	LSL	C_p	C_{pk}	$C_p - C_{pk}$	ppm	Capability Difference	Process Sigma Split
Example A	8	1	10	2	1.333	0.667	0.667	22750	2	4
Example B	8	1	10	5	0.833	0.667	0.167	24100	0.5	2.5
Example C	8	1	10	6	0.667	0.667	0	45500	0	2

Extensions and applications: Case study

This section introduces a case study whereby effective application of Process Capability Indices, Process Performance Indices and Process Sigma (with its new Process Sigma Split Index) are applied.

The case study is taken from a Continuous Improvement project made through the application of the Lean Six Sigma methodology, developed by an International Corporation, producing hydraulic pumps.

The purpose here is to compare, through Minitab17® software, the mathematical relation between the Process Capability, Process Performance and Process Sigma.

The object of the analysis is the process pertaining the data collection of the defectiveness of the internal diameter of a ball bearing inside a hydraulic pump. The tolerance interval is defined by USL=14.05 mm and LSL=13.85 mm.

The sampling has been made by measuring the diameter of 9 ball bearings every day for 125 days, thus collecting 1125 data. The distribution of this sampling can be approximated with a Gaussian model (the Normality P-value is 0.425), with a mean of 14.00 mm and standard deviation of 0.051 mm as in the Graphical Summary of Figure 4.

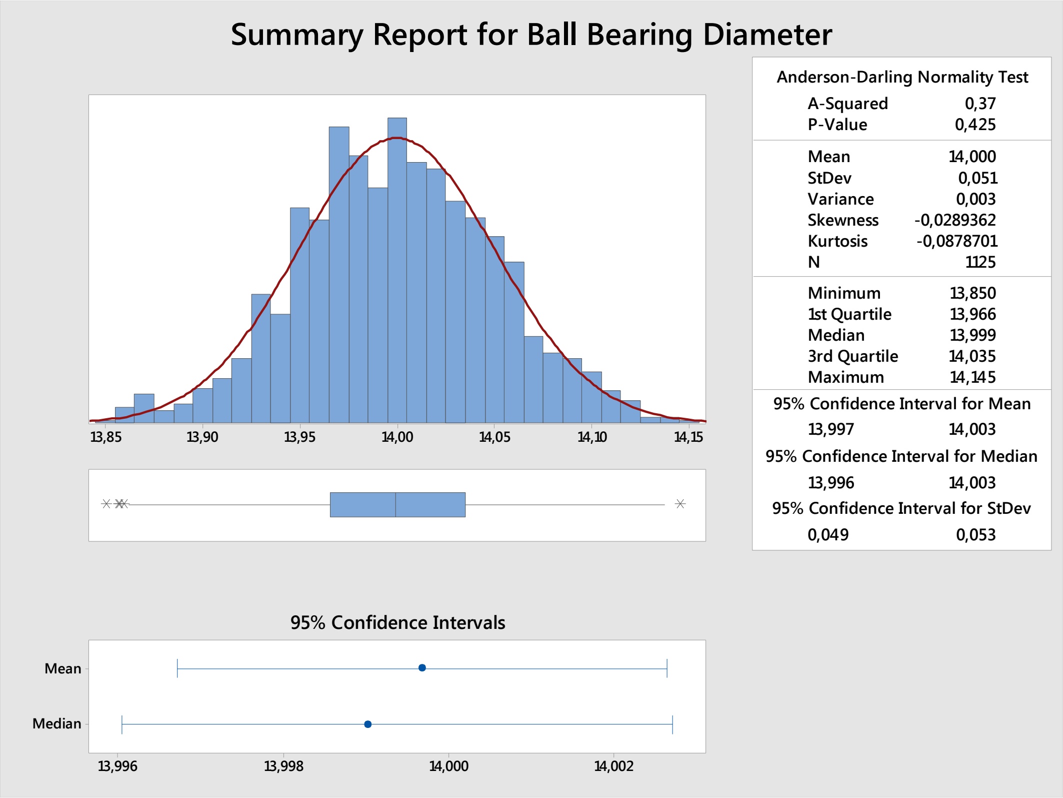


Figure 4: Graphical Summary for Ball Bearing Diameter

As described in Montgomery [12], the process of establishing control may be iterative and the control limits are usually viewed as trial limits. The goal is to make sure that a process is operating at or near acceptable target(s) under some natural (common) causes of variation and that no special causes or concerns are present [13].

In order to calculate the Process Capability, the necessary hypothesis is the stability of the analyzed process, which is evaluated through a Control Chart, as in Figure 5 [56] [57].

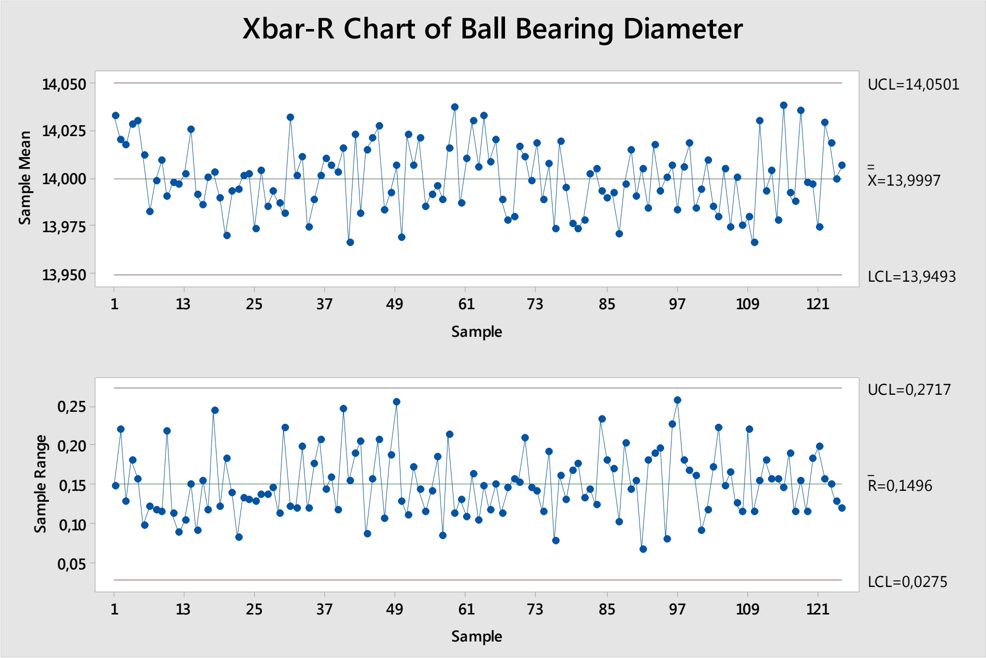


Figure 5: Control Chart Xbar-R for Ball Bearing Diameter.

By analyzing the time trend of the sampling distribution, the process is stable, since all the points are included in the VOP (Voice of Process), calculated as a mathematical difference between UCL and LCL.

PCI Calculation:

$$C_p = \frac{VOC}{VOP} = \frac{(USL - LSL)}{6\ \sigma_{S.T.}}$$

$$C_{pk} = min\{C_{pku}; C_{pkl}\} = min\left\{\frac{USL - \mu}{3\ \sigma_{S.T.}}; \frac{\mu - LSL}{3\ \sigma_{S.T.}}\right\}$$

Dealing with subgroup sampling, to estimate $\sigma_{S.T.}$ the formula below is used:

$$\sigma_{S.T.} = \frac{\bar{R}}{d_2}$$

where d_2 is an unbiasing costant to make estimators more unbiased, therefore not dependent on sample dimensions.

The value of d_2 constant is equal to 2.97, since the dimension of the analyzed subgroup is equal to 9.

From R Chart, in Figure 5, that plots the daily Range it can be noted that the average range is equal to 0.1496 mm.

Therefore:

$$\sigma_{S.T.} = \frac{0.1496\ mm}{2.97} = 0.0504\ mm$$

$$C_p = \frac{14.05\ mm - 13.85\ mm}{6 * 0.0504\ mm} = 0.66$$

$$C_{pku} = \frac{14{,}05\ mm - 14\ mm}{3 * 0.0504\ mm} = 0.33$$

$$C_{pkl} = \frac{14\ mm - 13.85\ mm}{3 * 0.0504\ mm} = 0.99$$

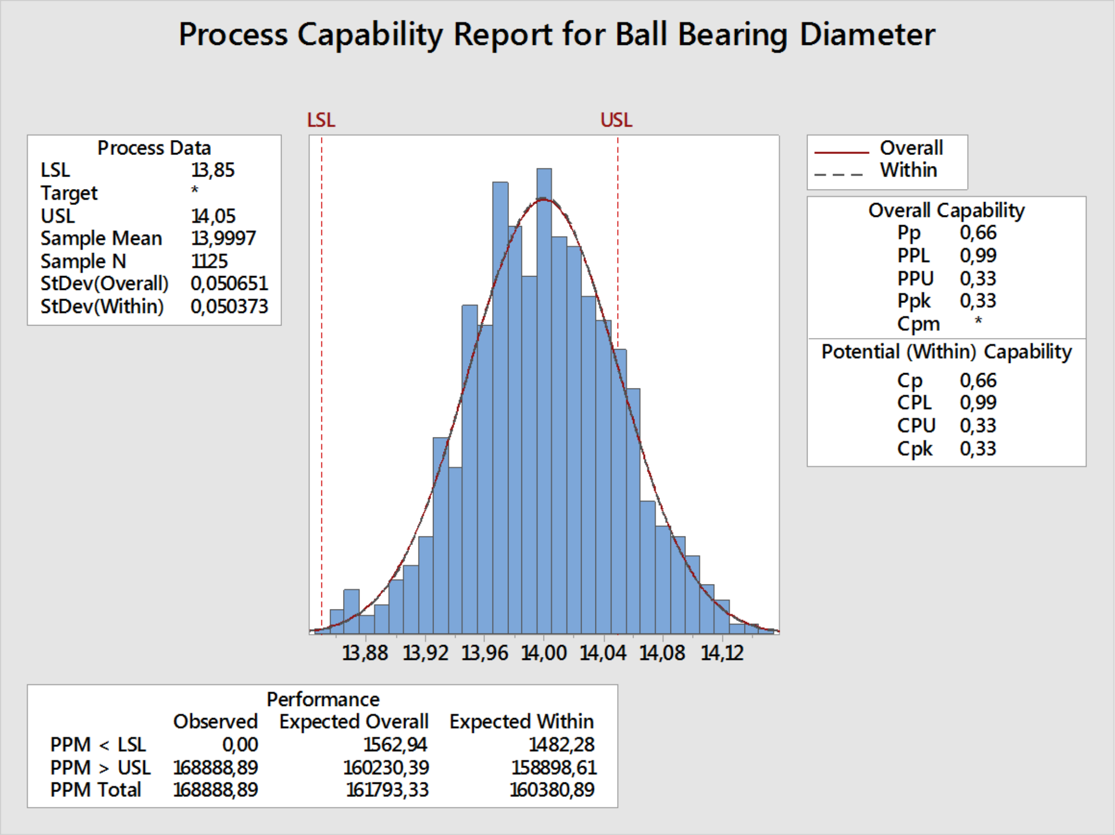
$$C_{pk} = \min\left(C_{pku}; C_{pkl}\right) = 0.33$$

$$C_{pk} = \frac{d - |\mu - m|}{3\sigma} = \frac{0{,}1 - |14\ mm - 13.95\ mm|}{3 * 0.0504} = 0.33$$

where:

$$d = \frac{USL - LSL}{2} = \frac{14.05\ mm - 13.85\ mm}{2} = 0.1\ mm$$

$$m = \frac{USL + LSL}{2} = 13.95\ mm$$



Process Data

LSL	13,85
Target	*
USL	14,05
Sample Mean	13,9997
Sample N	1125
StDev(Overall)	0,050651
StDev(Within)	0,050373

Overall Capability

Pp	0,66
PPL	0,99
PPU	0,33
Ppk	0,33
Cpm	*

Potential (Within) Capability

Cp	0,66
CPL	0,99
CPU	0,33
Cpk	0,33

Performance

	Observed	Expected Overall	Expected Within
PPM < LSL	0,00	1562,94	1482,28
PPM > USL	168888,89	160230,39	158898,61
PPM Total	168888,89	161793,33	160380,89

Figure 6. Process Capability Report for Ball Bearing Diameter

PPI Calculation:

$$\sigma_{L.T.} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} = 0.05065\ mm$$

The standard deviation ($\sigma_{L.T.}$) of the analyzed distribution is equal to 0.05065 mm.

$$P_p = \frac{VOC}{VOP} = \frac{USL - LSL}{6 * \sigma_{L.T.}} = \frac{14.05\ mm - 13.85\ mm}{6 * 0.05065\ mm} = 0.66$$

$$P_{pku} = \frac{14{,}05\ mm - 14\ mm}{3 * 0.05065\ mm} = 0.33$$

$$P_{pkl} = \frac{14\ mm - 13.85\ mm}{3 * 0.05065\ mm} = 0.99$$

$$P_{pk} = min\{P_{pku}; P_{pkl}\} = 0.33$$

Process Sigma Calculation:

The sampling distribution can be approximated to a Gaussian Distribution through Andersson-Darling Normality Test. To calculate the Sigma Level, the standardized Normal curve may be used, starting from:

- Average μ = 14.00 mm
- Standard deviation σ = 0.05065 mm
- Upper Specification Limit (USL) = 14.05 mm
- Lower Specification Limit (LSL) = 13.85 mm

$$Z_i = \frac{X_i - \mu}{\sigma}$$

Therefore:

$$Z_{USL} = \frac{14.05\ mm - 14.00\ mm}{0.05065\ mm} = 0.99$$

$$Z_{LSL} = \frac{13.85\ mm - 14.00\ mm}{0.05065\ mm} = -2.96$$

By using the Z-Table, the area (or probability) of total defectiveness results in:

Pr (X_i ≥USL) = 0.16109

Pr (X_i ≤LSL) = 0.00154

$$P(X_i \geq USL) \cup P(X_i \leq LSL) \cong 16.263\%$$

16,263% corresponds to the defectiveness of 162,630 ppm.

$$Sigma\ Level = 0.8406 + \sqrt{29.37 - 2.221\ln(ppm)} = 2.49$$

$$Process\ Sigma_{without\ sigma\ shift} = 3 * C_{pk} = 3 * 0{,}33 = 0.99$$

Referring to the new Process Sigma Split Index introduced by the authors earlier in this article:

$$Capability\ Difference = (C_p - C_{pk}) * 3 = (0.667 - 0.333) * 3 \cong 1$$

$$Process\ Sigma\ Split = 3 * C_{pk} + Capability\ Difference = 3 * 0.333 + 1 \cong 2$$

DISCUSSION AND FINAL REMARKS

The paper offered a comprehensive review of the main indicators that monitor processes (Process Capability, Process Performance and Process Sigma) aimed at determining the defect rates of a process, and, therefore, at quantifying defective parts per million (ppm). Such review is aimed at any organizations wanting to achieve high standards of quality and process yield. Indeed, by keeping process under statistical control, it is possible to have more robust information on the process, with more reliable indicators, which also allows to better satisfy customers' requirements.

Process Capability indices can be used to analyze existing processes, and only when the analyzed process in under statistical control. Process Performance indices are used to analyze new processes, or when the processes are not under statistical control. Process Sigma, instead, is likely to create one common metric that can be applicable to all data and environments. This review puts forward another crucial implicit assumption that is data distribution for all above-mentioned indices needs to be Normal. If the distribution happens not to be Normal, these indices can be calculated, but the information obtained is not reliable [58].

The originality of the present review lies in introducing a new index (Capability Difference) that, according to the authors, allows to understand clearly and more directly the effect on Process Sigma of an increase of process capability. In order to do so, Process Sigma Split has been devised. Process Sigma Split features two components; the first includes C_{pk} and provides information on actual process performance; the second represents the increase of Process Sigma once the process has been centered. A remarkable result of Process Sigma Split is to overcome some criticalities of Process Sigma itself, which often induces to make questionable evaluations. Due to its novelty, Process Sigma Split opens up for further contributions, where the index should be applied to other case studies, in order to refine and enrich its range of applicability.

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