Neonatal Seizure Detection using Time-Frequency Renyi Entropy of HRV signals

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Abstract
The Time-Frequency Renyi Entropy (TFRE) uses a time-frequency distribution (TFD) of a signal to provide a measure of the signal information content and complexity in the time-frequency (TF) plane. The concept is applied to the problem of newborn seizure detection using the HRV signal. This paper provides an experimental comparison of the performance of the TFRE obtained from selected TFDs, including the Wigner-Ville distribution (WVD), the spectrogram (SPEC), the Choi-William Distribution (CWD), the Born-Jordan Distribution (BJD), and the Modified B Distribution (MBD). The test signals include a multi-component Gabor logon and a three component linear FM signals, as well as real-life signals. Such a choice of synthetic test signals has been motivated by the fact that Gabor logons are essentially the building blocks in the TF plane of all signals, while the LFM is a good model for many real-life signals. The comparison results provided in this paper, illustrate the effects of the signals TF parameters (namely, the components time and frequency separation, their amplitude modulation, the changes in the components time duration, as well as the bandwidth variations and noise effects) on the TFRE evaluated from the considered TFDs. The results are shown to benefit practical applications of the TFRE in general. For the specific application considered in this paper, it has been shown that the MBD based TFRE of newborn heart rate variability (HRV) can be successfully used as a critical feature in neonatal seizure detection.

Keywords: Time-Frequency Renyi Entropy, quadratic time-frequency distribution, non-stationary, Instantaneous Frequency

INTRODUCTION
The concept of Renyi entropy, RE, originates from information theory and it is used to characterize the complexity of a time series [1]. The smaller the value of RE is the less random the time series. Information measures from probability theory were applied to the TF plane by treating TFDs as density functions [2], thus defining the TFRE. The properties of RE for deterministic signals [3] have indicated that in the TF domain the TFRE intimately relates to the number of components present in the signal. Intuitively, a signal that is constructed from a large number of components must have higher information content. The study in [3] has focused on the WVD and the results were extended in [4] for random white Gaussian signals while the performance of the TFRE for other TFD has not been reported so far. The study in [4] concluded that the TFRE of a white Gaussian noise is bounded, and the bounds depend only on the type of TFD kernel as well as the number of time and frequency samples of the distribution.

Previous studies [3] [4] described findings that show the TFRE changes depending on the type of TFD. Therefore, when measuring the complexity of a signal using TFRE, i.e. detecting the number of its components, different TFDs of non-stationary signals need to be considered. Our goal, in this paper, is to determine which of the selected TFDs is the most accurate in providing such information, and hence to be used when analyzing the complexity of HRV signals. So for practical applications, the aspect of knowing which TFD is the most appropriate for the TFRE measurement is far more immediate and important than the mathematical properties of TFRE, which have been a primary focus of the previous related works in the literature.

The paper provides an experimental comparison of the performance of TFRE evaluated using several commonly used quadratic TFDs: the Wigner-Ville distribution (WVD) as a key TFD, the Spectrogram (SP) as one of the most popular TFD in practical applications, the Choi-William Distribution (CWD) and the Born-Jordan Distribution (BJD) as two well-known and studied reduced interference TFDs, and the Modified B Distribution (MBD) which was shown to perform exceptionally well for various signals in practice[5]. The performances of those TFDs are tested on multi component signals composed of Gabor logons and LFM components.
The main contributions of the paper include:

**The effect of the choice of TFDs on the TFRE values**: As the choice of TFD is important in order to obtain an accurate TFRE result [2, 3, 4], this paper provides a comparison of TFRE obtained from five different TFDs. While in [4], a comparison between WVD, BJD and the Minimum Entropy Distribution (MED) for a two-component Gabor logon signal is given, in this paper in addition to Gabor logons we also study multi component signals composed of LFM, which are far more models for many real-life signals, including the signals considered in the main application described in this paper, namely newborn EEG seizures[5]. Other real-life signals for which LFM are good models include radar, sonar and telecommunications signals.

**The effect of signal parameters on the TFRE**: A comparison of nonstationary signal parameters on the TFRE has only been done in the previous studies to limited extends. For example, in [4] only the time separation between the Gabor logons and the specific case of a signal in noise for the signal to noise ratio (SNR) of 8dB were considered. This paper investigates the relationship between the following factors and the TFRE for each of the considered test signals as stated in Table 1.

**Table 1**: The factors and their effect on the TFRE to be studied in this paper.

| Time and frequency separation between signal components – both to be considered as different TFDs create different interference patterns depending on the components location |
| Amplitude variations of the signal components |
| Time duration changes in the signal components |
| Bandwidth variation in the signal components |
| Instantaneous frequency variations in LFM signal components |
| Noise performance for SNR in the range -16dB to 6 dB |

Finally, the paper investigates the complexity of the selected TFDs for the case of a real-life signal that can be well modeled as a combination of piece-wise linear FM components. We measure the TFRE of newborn heart rate variability (HRV) in order to discern differences between non-seizure and seizure newborns.

**RENYI ENTROPY OF QUADRATIC TFDs**

**A. Renyi Entropy of Quadratic TFDs**

**A Brief Review of Quadratic TFDs**: The general expression for the quadratic class of TFD is [5].

$$\rho_s(t, f) = \rho(t, f)^* \cdot \gamma W_s(t, f)$$  \hspace{1cm} (1)

where $W_s(t, f)$ is the Wigner-Ville distribution of the analytic signal $z(t)$, associated with the real signal at hand, defined as:

$$W_s(t, f) = \int_{-\infty}^{\infty} z(t+\tau/2) z^*(t-\tau/2) e^{-j2\pi f \tau} d\tau$$  \hspace{1cm} (2)

and $\gamma(t, f)$ is the time-frequency kernel filter, which defined the TFD and its properties [5]. In addition, Equation (1) can be also re-written as:

$$\rho_s(t, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(v-r, \tau) \cdot g(v, r) \cdot z(v+r/2) z^*(u-r/2) dvdu$$  \hspace{1cm} (3)

The function $g(v, r)$, defined in the Doppler-lag domain $(v, r)$, is the 2-dimensional Fourier transform equivalent of $\gamma(t, f)$ [5]. From (3), the Wigner-Ville distribution is obtained if the kernel $g(v, r)$ is equal to one [5], while the spectrogram with the kernel filter, performs a local or “short-time” spectral analysis of the signal by using a sliding analysis window $w(u)$. The SPEC is positive and cross-terms free, but it generally has poor TF concentration when compared to other TFDs. This is due to the inherent trade-off between the time resolution, which requires a short analysis window, and the frequency resolution, which requires a long analysis window [5]. While the WVD provides better TF resolution than the SPEC for monocomponent signals, its main drawback is the existence of spurious cross-terms which appear in between the true signal components in the case of multicomponent signals [5].

$$g(v, r) = \int_{-\infty}^{\infty} w(u) e^{-jfM \omega \rho} du$$  \hspace{1cm} (4)

To alleviate the problem of cross-terms, different kernels have been designed by convolving the WVD with a 2D smoothing filter. The CWD, BJD and the MBD are the smoothed versions of the WVD which can be used as an alternative to the WVD and SP. The CWD and BJD are two widely used TFDs. The MBD has been shown to yield promising results in attaining high resolution of signal components while significantly reducing the effect of cross-terms [6]. Table 2 defines the kernels for TFDs considered in this paper. The CWD has the parameter $\alpha$, which allows the user to select the amount of filtering in the Doppler-lag $(\alpha)$ domain [5]. The BJD, also called the “since distribution”, has a cone shaped kernel in the $(\alpha)$ domain that acts as a low pass filter, preserving the auto-terms that are close to the origin while eliminating the cross-terms that are located away from the origin. The amount of smoothing is controlled by the parameter $\alpha$. The function $(\alpha)$ in the MBD kernel stands for the gamma function and is a real, positive number that controls the trade-off between components’ resolution and cross-terms suppression [6]. Note that MBD is a member of a special subclass of quadratic TFDs known as the time-lag independent kernel TFDs [5].

**The Renyi Entropy**: Entropy is a measure of information for a given probability density function. By analogy, we can extend the measures of information from probability theory to the TF plane by treating TFDs as pseudo-density functions [1]. Two
important issues that arise when entropy functions are adapted to the TF plane are normalization and the non-positivity of the TFDs [3].

The normalized TFD assures that the TFD behaves like a PDF [5] as presented in Equation (5):

$$\rho_{\text{norm}}(t, f) = \frac{\rho(t, f)}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(t, f) dt df}$$

The Shannon entropy [2] can only be employed as a TF information measure when the TFD is non-negative (for example, the SPEC or the smoothed SPEC) [3]. To overcome this limitation, the Renyi entropy has been used instead, and it is defined in Equation (6). The order Renyi entropy of a normalized TFD is [3]:

$$R_{\alpha} = \frac{1}{1-\alpha} \log \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho_{\text{norm}}\alpha(t, f) dt df$$

where $\alpha > 0$, and $\alpha \neq 1$. As $\alpha$ approaches 1, TFRE approaches the Shannon entropy [3]. Note that $\alpha = 1$ is not recommended in case of time-frequency distributions with negative values because of the logarithm in the integrand in the definition of the Shannon entropy.

Table 2: TFDs and their kernels (see the complete list in [5]).

<table>
<thead>
<tr>
<th>TFD</th>
<th>Kernel filter $g(v, \tau)$ in its discrete form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wigner-Ville</td>
<td>$\delta[n]$</td>
</tr>
<tr>
<td>Spectrogram (window $w$)</td>
<td>$w[n + m]w[n - m]$</td>
</tr>
<tr>
<td>Choi-William ($\sigma$ - scaling factor)</td>
<td>$\sqrt{\sigma} \exp\left(-\frac{\sigma}{4m^2}\right)\text{sinc}(n)\text{sinc}(m)$</td>
</tr>
<tr>
<td>Born-Jordan ($\alpha$ - scaling factor)</td>
<td>$\frac{1}{4</td>
</tr>
<tr>
<td>Modified B ($\beta$ - smoothing factor)</td>
<td>$\sum_n \cosh^{-2}\beta_n \text{sinc}^{-2}\beta_n$</td>
</tr>
</tbody>
</table>

Even values of $\alpha$ are recommended only for interference-free TFDs since the cross-terms oscillatory structure cancels under the integration operation with odd powers [3]. Thus, in the general case of TFRs with cross-terms, odd integers values for $\alpha$ are recommended, as discussed in [3]. Indeed, the TFRE for $\alpha = 3$ has been found to have interesting and useful properties for a large class of signals [3], and it has been used in [3,4,8].

The main properties of the third order Renyi entropy are [3]:
- It can provide an estimate of the number of components in a multicomponent signal (the counting property)
- For odd values of $\alpha > 1$ it is asymptotically invariant to TFD cross-components
- It exhibits extreme sensitivity to phase differences between closely spaced components
- Its lower bound is attained for the WVD by a single Gaussian pulse
- It is time and frequency shift invariant.

In Fig. 1, it is shown how $\alpha = 3$ gives the most accurate estimate of the signal component number (the increment in the TFRE) for a signal composed of two Gabor logons centered at frequencies 0.1 Hz and 0.25 Hz, respectively, each with the time duration of 15 s, for the varying time separation between the logons. The TFRE has been calculated using the WVD.

**EXPERIMENTAL RESULT**

B. *Calibration of the TFRE method using simulated data*

In this section, we systematically compare the TFRE results obtained from the WVD, SP, CWD, BJD and MBD for a set of multi-component test signals. The test signal components used are the Gabor logon [5] and the LFM [5] signals. These test signals are selected with the aim of illustrating the performance of TFRE when obtained using different TFDs and taking into consideration the effects of the factors listed in Table 1 on the TFRE.

![Figure 1: TFRE gain for various $\alpha$ (obtained using the WVD) for the signal composed of two Gabor logons placed at 0.1Hz and 0.25Hz, respectively, and separated in time by $\Delta t$.](image-url)
Experimental setup: For all the test signals, the time duration is 128 samples and the sampling rate, $F_s = 1\text{Hz}$. The optimal parameters are found for SPEC, CWD, and MBD, and they result in a compromise between TF resolution and cross-terms suppression for each test signals. The method used to obtain these optimal parameters is described in following subsections.

Test signal 1: Two Gabor logons

Two Gabor logons of 15s of time duration each were first used. The optimal parameters for SP, CWD and MBD are tabulated in Table 3. The optimal parameters are obtained by visually comparing the TF plots of the signal for different values of parameters. The parameters that resulted in the “cleanest” TF plots (best energy concentration and least interference present, as discussed in Chapter 7 in [5]) were selected as optimal. Figure 2 shows the TF plot obtained for each of the TFD considered in Table 3 with its optimal parameters.

**Table 3: Optimal parameters for TFDs**

<table>
<thead>
<tr>
<th>TFDs</th>
<th>Optimal Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP</td>
<td>Hamming with length of 31</td>
</tr>
<tr>
<td>CWD</td>
<td>$\sigma = 10$</td>
</tr>
<tr>
<td>MBD</td>
<td>$\beta = 0.5$</td>
</tr>
</tbody>
</table>

Let us next consider how different signal non-stationary parameters affect the TFDs’ TFRE values.

Time separation between components: The performance of the 5 TFDs for separating two Gabor logons for the case when the components are closely spaced and for the case when they are well separated in frequency are presented in Figure 3 and 4 respectively. We have plotted the variations in TFRE when the time separations, $\Delta T$, between the components varied.

From the Figures 3 and 4, it can be concluded that for closely spaced components in frequency, the TFRE value oscillates and overshoots for almost all the TFD studied. The SP exhibited a reliable measure by reaching a stable level well before the other distributions.

![Figure 2: TF plots of the TFDs in Table 3.](image-url)
Figure 3: TFRE gain w.r.t. $\Delta T$ between the two Gabor logons placed at 0.1Hz and 0.15Hz respectively (i.e. closely spaced in frequency).

Figure 4: TFRE gain w.r.t. $\Delta T$ between the two Gabor logons placed at 0.1Hz and 0.25Hz respectively (i.e. well separated in frequency).

However, TFRE obtained from SP is higher than the saturation value. This is due to the smoothing introduced by the SP that had added “extra information” to the components. This is evident from the energy concentration of the components in TF plots when compared to the other TFDs’ plots. The TFRE obtained from MBD and WVD reaches the 1 bit at $\Delta T$=35s. Meanwhile, $\Delta T$=25s is needed for the TFRE of the CWD and BJD to reach the 1 bit. The oscillations in TFRE for all the distributions are reduced when the components are more separated in frequency. It is evident that the TFRE oscillates the most and takes the longest for WVD to reach the 1-bit saturation level for both closely spaced and well-separated components in frequency. A similar observation is reported in [4]. MBD performs the best for fairly disjoint components in frequency, followed by CWD and BJC.

Frequency separation between components: The performance of the 5 TFDs for separating two Gabor logons for the case when the components are closely spaced and for the case when they are well separated in time are presented in Figure 5 and 6 respectively. We have plotted the variations in TFRE when the frequency separations, $\Delta F$, between the components varied. From Figures 5 and 6, it can be concluded that for closely spaced components in time, the TFRE value oscillates and overshoots for almost all the TFD studied except for SP. The SP exhibits a frequency invariant measure by reaching close to 1 bit before the other distributions without oscillations. However, as explained before, its value is slightly higher than the saturation value. The TFRE obtained from CWD and BJD, MBD, and WVD reaches the 1 bit once the $\Delta F$ is 0.18Hz, 0.23Hz and 0.27Hz respectively. The measure from all the distributions is almost frequency invariant when the components are well separated in time. This is consistent with the finding in [3]. The measure oscillates the most for TFRE gained through WVD, followed by MBD, CWD and BJD. It is worth noting that, like in the case of SP, the measure attained from BJD also does not reach the 1-bit level.

Figure 5: TFRE gain w.r.t. $\Delta F$ between the two Gabor logons placed at 20s and 40s respectively (closely spaced in time).

Figure 6: TFRE gain w.r.t. $\Delta F$ between the two Gabor logons placed at 20s and 70s respectively (well separated in time).

Amplitude variation of the components: The performance of the TFDs for two Gabor logons with different amplitude ratios is illustrated in Figure 7. We have considered the amplitude variation for closely spaced and for well-separated components in frequency. The test is repeated for the case when the logons had different separations in time. The difference in TFRE when decreasing the amplitude of one of the Gabor logon is plotted. For all the cases investigated, as
the ratios of the amplitudes between the two components were increasing, TFRE was decreasing, gradually approaching 0 bit (that is only 1 component present in the signal). Decreasing the amplitude of the signal reduces the signal energy level and eventually the component can no longer be identified. Thus, the amplitude ratio between the components in a signal contributes to significant changes in the complexity measure values.

**Figure 7:** TFRE w.r.t. decreasing amplitude of one of the Gabor logons. The Gabor logons are placed at 20s and 60s, and 0.1Hz and 0.2 Hz, respectively.

**Noise performance:** The differences in TFRE of the five TFDs for the two Gabor logons in additive white Gaussian noise for the signal-to-noise ratio (SNR) ranging from -16dB to 6dB were investigated. Figures 8 and 9 show some representative results.

Overall, the presence of noise is, as expected, adding to the complexity of the signal, hence increasing the TFRE above the 1-bit saturation level. For the case of lower SNR, Figure 6 shows that WVD performs the best, having the minimum increase in RE, followed by MBD, CWD, BJD and SP. For higher SNR, the values from all the distributions rapidly oscillate and exhibit an increase in TFRE regardless of ΔT and ΔF between the two logons.

**Figure 8:** TFRE of two Gabor logons placed at 0.1Hz and 0.25Hz in additive white Gaussian noise at SNR=-16dB

**Test signal 2: Two component LFMs**

Two LFMs, each of 48s of time duration and 0.05Hz of bandwidth, were first used. The optimal parameters for SP, CW and MBD are tabulated in Table 4. The optimal parameters are obtained by visually comparing the TF plots of the signal for different values of the parameters; in terms of TF resolution. Figure 10 shows the TF plot obtained for each of the TFD considered in Table 4 with its optimal parameters.

**Table 4:** Optimal parameters for TFDs

<table>
<thead>
<tr>
<th>TFDs</th>
<th>Optimal Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP</td>
<td>Hamming with length of 61</td>
</tr>
<tr>
<td>CW</td>
<td>σ = 5</td>
</tr>
<tr>
<td>MBD</td>
<td>β = 0.1</td>
</tr>
</tbody>
</table>

**Figure 9:** TFRE of two Gabor logons placed at 0.1Hz and 0.25Hz in additive white Gaussian noise at SNR=6dB
**Time separation between components:** The performance of the 5 TFDs for separating two LFMs for the case when the components are closely spaced and for the case when they are well separated in frequency are presented in Figure 11 and 12 respectively. We have plotted the variations in TFRE when the time separations, $\Delta T$, between the components varied.

Figures 11 and 12 show that for closely spaced components in frequency, MBD performs the best by attaining the TFRE values closest to the 1-bit saturation level with the least oscillations. The TFRE obtained from the WVD performs the worst by exhibiting unreliable TFRE. However, WVD performs the best for fairly disjoint components in frequency followed by MBD, CWD and BJC. The SP exhibits almost similar result as MBD for both cases but, as explained earlier, its TFRE is higher than the saturation value. The gradual increase/decrease in TFRE for SP and MBD, as shown in Figure 11, is a result of different amounts of overlap of the components’ time supports.

**Figure 10:** TF plots of the TFDs in Table 4.

**Figure 11:** TFRE gain w.r.t. $\Delta T$ between the two LFMs placed at 0.1-0.15Hz and 0.15-0.2Hz respectively (closely spaced in frequency).

**Figure 12:** TFRE gain w.r.t. $\Delta T$ between the LFMs placed at 0.1-0.15Hz and 0.3-0.35Hz respectively (well separated in frequency).
Frequency separation between components: The performance of the 5 TFDs for separating two LFMs for the case when the components are closely spaced and for the case when they are well separated in time are presented in Figure 13 and 14 respectively. We have plotted the variations in TFRE when the frequency separations, ∆F, between the components varied.

Figure 13: TFRE gain w.r.t. ∆F between the two LFM each placed at 0-48s (closely spaced in time).

Figure 14: TFRE w.r.t. ∆F between the two LFMs placed at 0-48s and 80-128s respectively (well separated in time).

Figure 15: TFRE w.r.t. decreasing amplitude of one of the LFM component. The two LFMs are placed at 0-48s and 80-128s, and at 0.1-0.15Hz and 0.15-0.2 Hz, respectively.

Noise performance for SNR from -16dB to 6 dB: The difference in TFRE gained from the five TFDs for the two LFMs in additive white Gaussian noise for the signal-to-noise ratio (SNR) of -16dB and 6dB were investigated and are shown in Figures 16 and 17. Overall, the presence of noise increases the TFRE more than the 1-bit saturation level adding to the complexity of the signal. The obtained results are identical to those for two Gabor logons (Figures 8 and 9).

Figure 16: TFRE of two LFMs placed at 0.1-0.15Hz and 0.3-0.35Hz in additive white Gaussian noise at SNR= -16dB.

For the case of lower SNR, Figure 16 shows that WVD performs the best, having the minimum increase in TFRE, followed by BJD, CWD, MBD and SP. For higher SNR, the
values from all the distributions oscillate and exhibit an increase in TFRE regardless of $\Delta T$ and $\Delta F$.

Figure 17: TFRE of two LFM placed at 0.1-0.15Hz and 0.3-0.35Hz in additive white Gaussian noise at SNR=6dB.

**Bandwidth variation in LFM components:** Next, the experiments presented in parts A to D are repeated for the case when the LFM have different bandwidths. For all the cases considered, the TFRE obtained from CWD, BJD, SP and MBD increases as the bandwidth of the LFM components increases adding to the complexity of the signals. The WVD is a special case, as TFRE obtained from the WVD is insensitive to the bandwidth changes. Figure 18 shows the results obtained when the test in Figure 14 is repeated with each component bandwidth being now twice as big, i.e. 0.1Hz.

**Time duration changes in LFM components:** The performance of the 5 TFDs for two LFM with different time durations is investigated. The time duration of one of the LFM component is 48s and of the other is ranging from 127s to 0s. The difference in the TFRE for decreasing the time duration of one of the LFM is shown in Figure 19. The TFRE from all the TFDs is higher than 1-bit saturation level when the time duration of the second component is approximately longer than 50s adding to the information content of the test signal, except for WVD for which the TFRE reduces and raises again at around 80s of time duration of the component

As the duration of the component reduces, the TFRE decreases gradually approaching 0 bit for all the TFDs. This indicates only 1 component present in the signal. In this example, the LFM is considered as a component, adding to the overall signal complexity, only when its minimum time duration is approximately 50s. Overall, TFRE of WVD seem to be least effected by time duration changes and TFRE of SP seem to be affected the most. This test shows that the time duration of signal components contributes to considerable changes in the complexity measure.

**Instantaneous frequency variation in LFM components:** The tests in parts A to F are repeated for the case when the LFM have different instantaneous frequency laws, that is one component has increasing frequency and another has decreasing frequency with time. It was observed that such a change in the components IF laws results in no differences in the TFRE for all the cases considered. Hence, the instantaneous frequency law of LFM does not influence the TFRE gained from the five TFDs investigated. Figures 20 and 21 below show the results obtained when the tests in Figures 12 and 14 are repeated when one component has decreasing instantaneous frequency.

Figure 18: TFRE w.r.t $\Delta T$ between the two LFM placed at 0-48s and 80-128s with bandwidth of 0.1Hz each.

Figure 19: TFRE w.r.t. decreasing time duration of one of the LFM. The LFMs are placed at 0.1-0.2Hz and 0.3-0.4 Hz.

Figure 20: TFRE w.r.t. $\Delta T$ between the LFMs placed at 0.15-0.1Hz and 0.35-0.3Hz respectively (well separated in frequency).
Test signal 3: Three LFM

The experiment in Section B is now extended to a signal with three LFM components, each component having a time duration of 88s, an instantaneous bandwidth of 0.05 Hz, and an IF that increases with time. The values of TFRE obtained using the five TFDs of the signal are summarized in Table 6. Four different cases, as defined in Table 5, for the three components (C1, C2, and C3) of the LFM signal are investigated, taking into consideration the following factors:

- Ratio of frequency separations (FSR) between the components C1, C2 and C3
- Ratio of amplitudes (AR) between C1, C2 and C3
- Noise (SNR).

Table 5: The characteristics of three-component LFM signal for different cases investigated.

<table>
<thead>
<tr>
<th>Case</th>
<th>Characteristics of the 3 Component (C1, C2, C3) LFM signal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FSR (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>0.1:0.1</td>
</tr>
<tr>
<td>2</td>
<td>0.05:0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.1:0.1</td>
</tr>
<tr>
<td>4</td>
<td>0.1:0.1</td>
</tr>
</tbody>
</table>

Table 6: TFRE obtained using different TFDs for different cases involving three-component LFM signal.

<table>
<thead>
<tr>
<th>Case</th>
<th>TFRE from different TFDs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WVD</td>
</tr>
<tr>
<td>1</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>1.46</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
</tr>
<tr>
<td>4(SNR=6)</td>
<td>1.22</td>
</tr>
<tr>
<td>4(SNR=-16)</td>
<td>2.45</td>
</tr>
</tbody>
</table>

Table 6 shows that TFRE obtained from WVD exhibits unreliable measure of complexity for the cases 1 and 2. The TFRE obtained from BJD and MBD shows an excellent performance by exhibiting frequency invariant TFRE for the cases 1 and 2, followed by SP and CWD. TFRE obtained from MBD is the closest to 2 bits, which is an indication of three components being present in the test signal [3]. However, MBD shows a poor performance in noise. The TFRE obtained from MBD increases by 25.9%, whereas the TFRE obtained from SPEC increases by 20.75% for SNR = 6dB. This shows that SPEC performs better when the signal is contaminated by noise.

C. Real-life signal

The pattern of changes in the TFRE measure that result of the changes in TF parameters of a signal can be used as a potential feature for signal classification. In this paper, we propose to use the TFRE of the newborn HRV as a feature to discriminate between the seizure and non-seizure states. In newborns, sympathetic and parasympathetic activities manifest themselves in the low frequency (LF) and the high frequency (HF) components of the HRV respectively. The mid frequency (MF) is both parasymptomatically and sympathetically mediated [6].

The newborn HRV is initially mapped to the TF domain using the MBD. This is because the HRV components is similar to LFM and we have shown in the previous sections that the MBD results into a consistent TFRE measures when it is almost noise free for LFM signals. Furthermore, the sensitivities of this complexity measure towards the signal’s TF parameter is a good candidate to be used as feature for signal classification.

Figure 22(a) and (b) show the result of applying the MBD to HRV related to non-seizure and seizure epochs respectively. Figures 22(a) and (b) represents the time series (left plots), the
joint TFDs by using MBD (centre plots), and the spectra (bottom plots) of the HRV related to non-seizure and seizure newborns respectively.

The plots indicate that the characteristics of the HRV components are almost similar to LFM with the frequency content varying slowly with time. In addition, the MBD has high resolution of signal components and significant cross-terms reduction. The TFRE for 19(a) and 19(b) is 10.49 and 9.90 respectively. The Renyi entropy for HRV related to non-seizure is higher than the HRV related to newborn seizure. Our result is consistent with results in [8] which reported that the complexity of HRV is reduced in infants with brain injuries compared to the healthy ones. As can be seen from the Figure 19(b), the reduction in TFRE is due to the shorter time durations of the components and greater energy ratio difference among the components during seizure. It is worth noting that in Figure 19(b), the energy in LF is significantly greater compared to MF and HF that makes the components unnoticeable.

The method is tested on HRV obtained from the ECG recordings of newborns admitted to the Royal Brisbane and Womens Hospital, Brisbane, Australia. The one channel newborn ECG was recorded simultaneously along with 20 channels of Electroencephalogram (EEG). The EEG seizure were identified and annotated by a neurologist. The ECG was sampled at 256 Hz. In the present study, we analyzed 28 seizure and 12 non-seizure epochs of 64 seconds each from 5 newborns. The HRV were preprocessed and resampled at 2 Hz [7, 12].

For all the data considered, we found consistent and significant difference in the values of TFRE between HRV of non-seizure and seizure newborn obtained. Table 7 shows the mean TFRE for HRV corresponding to non-seizure and seizure newborns computed in our study. A receiver operating characteristics (ROC) is used to acquire appropriate sensitivity (Sn), specificity (Sp), the area under the ROC curve (AUC) and the 95% confidence interval (CI) of AUC. If the lower limit of the CI for the AUC is > 0.5, the feature tested is considered to have discriminatory potential. The Sn and Sp are defined as presented in Equation (7) [13, 14]:

$$\text{Sn} = \frac{TP}{TP + FN}; \quad \text{Sp} = 1 - \frac{FP}{TN + FP}$$

where TP, TN, FN, and FP respectively represent the number of true positive, true negative, false negative and false positive. The Sn is the proportion of seizure events correctly recognized by the test, while Sp is the proportion of non-seizure events correctly recognized by the test.

<table>
<thead>
<tr>
<th></th>
<th>Non-Seizure</th>
<th>Seizure</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFRE(Mean ± SD)</td>
<td>10.31 ± 0.43</td>
<td>9.63 ± 0.71</td>
</tr>
<tr>
<td>Sn</td>
<td>70.27 %</td>
<td>75.00 %</td>
</tr>
<tr>
<td>Sp</td>
<td></td>
<td>75.00 %</td>
</tr>
<tr>
<td>AUC(CI)</td>
<td>0.7950 (0.6740 - 0.9161)</td>
<td>0.7950</td>
</tr>
<tr>
<td>Threshold</td>
<td></td>
<td>9.98</td>
</tr>
</tbody>
</table>

From the table, we get a clear statistical separation of Renyi entropy between seizure and non-seizure ones. It is also...
evident from the AUC (CI) that the Renyi entropy could reliably discriminate HRV related to newborn with and without seizure with 70.27% of sensitivity and 75.00% of specificity. The optimal threshold was found to be 9.98. Note that these results can be further improved on by filtering the MBD of HRV which allows the TF locations of true HRV components to go unaltered and blocks the remaining TF locations not associated with the true HRV components before the Renyi entropy is computed. This post-process will ensure that the complexity measure is free from spurious and insignificant HRV peaks.

DISCUSSION AND INTERPRETATION OF RESULT

An experimental comparison of the performance of TFRE obtained from Wigner-Ville Distribution, Spectrogram, Choi-William Distribution, Born-Jordan Distribution and Modified B Distribution is presented. The performances are tested on two component Gabor logon and LFM signals, extended to three component LFM signals. The main aim was to investigate the effects of signal’s non-stationary parameters on TFRE which was obtained using above TFDs. The results obtained indicate that the value of TFRE vary depending on the time separation between signals components, frequency separation between the components, ratio of amplitudes between the components, time duration of signal component, bandwidth of components and the presence of noise. For the case of Gabor logons, it can be concluded that Modified B Distribution outperforms other popular TFDs when the components are sufficiently disjoint in time and frequency and are almost free from noise. For the LFM signals, Modified B Distribution outperforms other TFDs when the signal is almost free from noise (SNR=6dB and higher). Generally, the TFRE obtained from Spectrogram is always higher than the saturation value for the considered signals due to its poorer TF resolution that adds extra smoothing/information. The TFRE obtained using WVD shows a good performance when the signals are contaminated by noise.

The results presented in this paper show that the choice of TFD is crucial in order to obtain accurate Renyi entropy measurements. The TFD should be chosen according to the characteristics of the signal components in TF plane. The TFD that has high resolution of signal components and significant cross-terms reduction, such as the MBD, ensures more consistent TFRE values towards many signal’s TF parameters. Furthermore, the sensitivities of this complexity measure towards the time duration and bandwidth of signal components make it a good candidate to be used as a feature for biological signal classification. The TFRE was shown to be a reliable feature capable of discriminating between the HRV related to seizure from the one related to non-seizure.

The extensive analysis presented in this paper shows that for non-stationary signals the parameters and performance limitations of TFDs affect significantly the TFRE properties, and it is important to take this into account for a better use in practical applications that involve representations in the TF plane. This study is especially useful for classification of real signals and the correct interpretation of TFDs complexity of signals based on TFRE.

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REFERENCES


