Restoration of Displaced Centre Frequency in Band-pass Distributed Filters using Predistortion Technique

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Abstract

This paper outlines a concise method for high frequency distributed filter design based on predistortion of the filter transfer function by appropriate loss/or gain factor. The approach enables the designer to realize all the filter specifications without resorting to the traditional circuit optimization and tuning (tweaking).

To appreciate the benefits derivable from predistorting filter circuits, this work begins by demonstrating the defects on filter responses due to lossy lumped circuit elements. The predistortion of the lossy circuit is then implemented to show the usefulness of the predistortion technique. As an extension of the concept, the predistortion of distributed bandpass filter is implemented to correct the shift in the centre frequency of a microstrip-line filter operating at 5GHz centre frequency.

Keywords: Predistortion, distributed elements, lumped elements, Transfer function, Substrate, Microstrip line

INTRODUCTION

The classic network synthesis procedures assume ideal and dissipationless elements. If a reactive network is built in the laboratory using practical dissipative elements, the resulting network response would have a transmittance which deviates from the prescribed response. In addition to a passband insertion loss the poles of the transfer function will be displaced. This is undesirable, especially in applications where precision is vital.

The traditional circuit optimization can be used to correct these defects, however it requires large computation time and memory size, particularly in distributed circuit design as it is not a systematic but random procedure and also does not always guarantee accurate results. On the other hand, circuit tweaking is tedious and not supported in fully integrated design systems. In this work predistortion technique has been employed to correct the defects. It is concise, systematic, and overcomes the disadvantages associated with the optimization and tweaking techniques.

THE PREDISTORTION TRANSFORMATION TECHNIQUE

The predistortion technique is based on the transformation of the complex frequency parameter, 's', into a new parameter, \( \lambda = \Sigma + j\Omega \) such that the new transfer function, \( H(\lambda) \), can be realized by a lossless \( L-C \) circuit. Transforming back to the frequency ‘s’ adds to each reactive circuit element a positive resistance. By so-doing a practical network can be realized to give a transmittance with the same poles and zeros as those of the prescribed transmittance, but only differing by a certain amount of flat loss.

Two main types of transformation can be performed:

(a) Equal (uniform) predistortion and
(b) Unequal predistortion transformations.

Equal Predistortion Transformation:

Equal predistortion assigns the same loss factor to every circuit element, hence requiring all the reactive elements in the final circuit to have the same quality factor, \( Q \).

Upon the addition of dissipation \( r \), to an ideal inductor, \( L \), the impedance becomes:

\[
 sL + r = L\left(s + \frac{r}{L}\right) = L(s + d) \quad \text{at unit } \omega .
\]

where \( d \) is a constant representing the value assumed by the equal resistance-reactance ratio \( (d = 1/Q) \) at unit \( \omega \). It follows that to remove the uniform dissipation from all the reactance elements of a prescribed network it is only necessary to design a network producing the predistorted function obtained by replacing \( s \) by \( \lambda - d \) in the prescribed transfer function.

| \( \lambda = s + d \) |
| \( \frac{1}{(s + d)C} \) |
| \( \frac{1}{(\lambda)C} \) |

\[
 F(s) \rightarrow \tilde{F}(\lambda) = F(\lambda - d)
\]

\[
 (s + d)L \quad \lambda L
\]

\[
 R
\]

Figure1: Equal Predistortion Transformation Procedure[1].
Using the transformation procedure in Figure 1, it is possible to synthesize a lossy practical network having the exact prescribed response required of an ideal lossless circuit, but with a flat pass-band loss.

**Synthesis of Equally Predistorted Network**

\[ \frac{\bar{S}_{21}(\lambda)}{S_{21}(s)} = \frac{1}{D(\lambda)} \]

Further, it is required that

\[ D(\lambda)D'(\lambda) = A(\lambda)A'(\lambda) \pm B^2(\lambda) \]

and

\[ \bar{S}_{21}(\lambda)\bar{S}_{21}(\lambda)^* \leq 1 \]

To satisfy Eqn.(9) while retaining the freedom of choosing \( R_s \) and \( R_L \), a constant of multiplication, \( K \) is included in \( H(\lambda) \), the value of \( K \) is then determined by (9). When \( \bar{S}_{21}(\lambda) \) is known Eqn.(8) is used to determine \( A(\lambda) \) and subsequently the input scattering parameter \( \bar{S}_{11}(\lambda) \), which in turn enables the input impedance \( \bar{Z}_{in}(\lambda) \) to be found as

\[ \bar{Z}_{in}(\lambda) = \frac{1 + \bar{S}_{11}(\lambda)}{1 - \bar{S}_{11}(\lambda)} \]

\( \bar{Z}_{in}(\lambda) \) is then expanded into a suitable continued fraction form to realize the predistorted network. After the circuit has been developed in \( \lambda \), Figure 1 is used to transform the elements to function of \( s \).

**Application of Equal Predistortion Concept**

In this section we demonstrate the application of equal predistortion using a 5th order low-pass Chebyshev filter with...
cut-off frequency, $f_c = 2.0 \text{GHz}$, and maximum passband ripple, $\alpha_{\text{max}} = 0.01 \text{dB}$.

The transfer function for the normalized lossless circuit is given as

$$H(s) = \frac{0.65}{s + 2.64s^2 + 4.74s^3 + 5.249s^4 + 3.71s^5 + 1.30174}$$  \hspace{1cm} (12)

Using standard synthesis procedure, the denormalised ideal filter circuit is designed as shown in Figure 3, which gives the simulated response in Figure 4, which is an ideal response.

From the response in Figure 6 it is clearly evident that, in addition to a passband insertion loss, the cutoff frequency of the practical (lossy) circuit ($S_{21}$) has shifted from the prescribed value of 2 GHz, and the band edge also severely rounded. Consequently the poles of the transfer function have been displaced.

**Figure 6:** Transmission coefficients of the ideal ($S_{21}$) and lossy ($S_{21}$) 5th order filter

Equal predistortion is then implemented using the transformation discussed in the previous section:

$$\lambda = s + d, \Rightarrow s = \lambda - 0.1$$

and therefore the transfer function of Equation (12) will take the following form:

$$\tilde{H}(\tilde{\lambda}) = \frac{\tilde{K}}{\tilde{\lambda}^5 + 2.144\tilde{\lambda}^4 + 3.779\tilde{\lambda}^3 + 3.974\tilde{\lambda}^2 + 2.793\tilde{\lambda} + 0.979}$$  \hspace{1cm} (13)

To determine $\tilde{K}$, we have

$$\max \tilde{S}_{21}(\lambda)\tilde{S}^*_2(\lambda) = 1$$  \hspace{1cm} (14)

$$4 \frac{R_s}{R_L} \tilde{K}^2 = 1$$

$$4 \frac{R_s}{R_L} \tilde{K}^2 = |D(\lambda)|^2$$

and

$$4 \frac{R_s}{R_L} \tilde{K}^2 = \min |D(\lambda)|^2_{\lambda = 0}$$

If $R_s = R_L = 1$, $\tilde{K} = 0.3759$

$$\tilde{S}_{21}(\lambda) = 2 - \tilde{K} \frac{D(\lambda)}{D^*(\lambda)} = \frac{0.751691}{D(\lambda)} = \frac{B(\lambda)}{D(\lambda)}$$

$$D(\lambda)D^*(\lambda) = A(\lambda)A^*(\lambda) + B^2(\lambda)$$

**Figure 3:** The ideal 5th order filter circuit

**Figure 4:** The transmission coefficient of the ideal 5th order filter

**Figure 5:** Lossy 5th order filter circuit
A(λ) = λ^3 + 1.54λ^4 + 2.679λ^3 + 2.55λ^2 + 1.79λ + 0.627

\[ \tilde{S}_{11}(λ) = \frac{A(λ)}{D(λ)} \]

From \( \tilde{S}_{11}(λ) \), the driving point impedance \( \tilde{Z}_m(λ) \) is calculated and is given as

\[ \tilde{Z}_m(λ) = \frac{1 + \tilde{S}_{11}(λ)}{1 - \tilde{S}_{11}(λ)} = \frac{D(λ) + A(λ)}{D(λ) - A(λ)} \]

\[ = \frac{2λ^3 + 3.687λ^2 + 6.467λ^3 + 6.524λ^2 + 4.586λ + 1.606}{0.6022λ^3 + 1.11λ^2 + 1.424λ^2 + 0.999λ + 0.352} \] (15)

The above equation generates an L-C circuit in \( \lambda \)-domain. The transformation in Figure 1 is then used to realize the predistorted circuit in \( s \)-domain, whose normalized transfer function is

\[ H(s) = \frac{0.82}{s^3 + 2.644s^4 + 4.746s^3 + 5.248s^2 + 3.7098s + 1.302} \] (16)

Using the classic method of deriving circuit prototype values, we will have the circuit shown in Figure 7.

**Figure 7:** The uniform predistorted 5th order low-pass filter

From Figure 8 the following observations are made:

i) The cutoff frequency of the lossy circuit (\( S_{21} \)) has shifted from the prescribed value of 2.0GHz, the bandedge severely rounded, and suffered an insertion loss of about 10.6dB.

ii) The consequence of (i) is that the poles of the lossy circuit have equally been displaced.

iii) The predistorted response (\( S_{21} \)) and the ideal response (\( \tilde{S}_{21} \)) have the same cutoff frequency, except for the 4.77dB insertion loss observed across the passband of the predistorted response.

iv) In addition to correcting the displaced cutoff frequency in the lossy circuit, the predistorted circuit also improved the insertion loss from -10.6dB to -4.77dB.

These defects clearly justify the implementation of predistortion, which has clearly restored all the prescribed specifications except for the insertion loss which can be compensated using active device.

**UNEQUAL PREDISTORTION TRANSFORMATION**

Similar steps are used to design circuits using unequal predistortion transformation, but here all inductors have one Q-factor and all capacitors have another Q-factor. The transformation technique is demonstrated in Figure 9, where \( s = jω \), \( d_L, d_C \) are the resistance-reactance ratios due to the inductors and capacitors respectively, and \( Q_L, Q_C \) are the quality factors of the inductors and capacitors, respective [2]. The application of this transformation to the design of distributed microwave filters is given as an example in section 4.1.

\[ \lambda = s + d \]

\[ d_L = d + \delta = \frac{1}{Q_L} \]

\[ d_C = d - \delta = \frac{1}{Q_C} \]

\[ Z(s) \rightarrow \tilde{Z}(\lambda) = \frac{\sqrt{\lambda - \delta}}{\sqrt{\lambda + \delta}} Z(\lambda - \delta) \]

\[ \frac{1}{(s + d_L)L} \rightarrow L\sqrt{\lambda^2 - \delta^2} \]

\[ \frac{1}{(s + d_C)C} \rightarrow \frac{1}{C\sqrt{\lambda^2 - \delta^2}} \]

\[ R \rightarrow R\sqrt{\lambda + \delta} \]
Synthesis of Predistorted Distributed Microwave Bandpass Filters

In order to effectively apply the predistortion technique to distributed circuits, appropriate mapping transformations are required to relate the distributed elements to their equivalent lumped elements. This permits the distributed transmission line functions to be treated as lumped LCR elements, not only for the purpose of realizability but also for synthesis and analysis. However, distributed elements are represented as infinite number of infinitesimal lumped-elements. In microstrip line the copper clad top conductor has a finite conductivity and can be modeled as a lossy inductor, while the substrate losses contribute to the losses associated with capacitance per unit length and hence we have a capacitor in shunt with a conductance. Figure10 therefore, accurately represents a microstrip distributed transmission line section whose effective Q-factor is determined by the combination of the losses in the individual elements. These losses are specified by the conductivity of the conductor material and the loss tangent of the substrate.

\[
\alpha_c = \frac{\beta}{2a_c} \\
\beta = \frac{2\mu\sqrt{\varepsilon_{eff}}}{c}
\]

where \(\beta\) is the phase constant, \(\alpha_c\) is the attenuation factor (nepers/mm) due to copper, \(c\) is the speed of light, and \(\varepsilon_{eff}\) is the effective dielectric constant which is related to the dimensions \((h, w)\) and the relative dielectric constant \((\varepsilon_r)\) of the substrate, as \([5,8]\)

\[
\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left(1 + \frac{12h}{w}\right)^{-0.5}
\]

Using standard tables, the transfer function for the standard filter is written as

\[
H(s) = \frac{K}{s^3 + 3.179s^2 + 5.802s + 520693} = \frac{k}{F(s)}
\]

To implement predistortion, we require the Q-factor \((Q_C)\) and \((Q_L)\), for the dielectric materials and conductor respectively.

\[
Q_c = \frac{\beta}{2\alpha_c}
\]

\[
\beta = \frac{2\mu\sqrt{\varepsilon_{eff}}}{c}
\]

An Example of unequal Predistortion

A 3rd order parallel-coupled bandpass microstrip filter, having a maximum passband attenuation of 0.01dB, is to be built on a Duroid (RT 5780) substrate, at 5GHz and bandwidth of 100MHz. The microstrip has the following specifications:

- Conductivity of the copper clad, \(\sigma = 5.813 \times 10^7 \text{ S/m}\)
- Loss tangent of the duroid substrate, \(\tan \delta = 0.0026\)
- Substrate dielectric constant, \(\varepsilon_r = 2.33\)
- Thickness of substrate, \(h = 0.795 \text{ mm}\)
- Thickness of copper clad, \(t = 0.03776 \text{ mm}\)

With the knowledge of the conductivity of the conducting material and the loss tangent of the substrate, their respective Q-factors are computed. Using standard design methods, the appropriate driving input impedance \(Z_m(s)\), for the standard filter is generated as a function of \(s\). The desired predistortion transformation is then implemented using the procedure in Figure9 to generate the predistorted input impedance \(Z_m(\lambda)\), from which the predistorted low-pass elements are obtained. Standard design method is again used to generate the dimensions of the parallel-coupled bandpass filter \([4,6,7]\). The procedure is demonstrated in the example below.
Using the transformation in Figure 9, the predistorted transfer function is obtained as follows:

\[
\delta = \frac{d_c - d_d}{2} = 2.35 \times 10^{-4}
\]

\[
d = \frac{d_c + d_d}{2} = 2.365 \times 10^{-3} \quad \lambda = s + d
\]

\[
\tilde{H}(\lambda) = \frac{K}{F(\lambda - d)} = \frac{K}{F(\lambda - d)}
\]

\[
\tilde{F}(\lambda) = \lambda^3 + 3.17164\lambda^2 + 5.78718\lambda + 5.19325
\]

Using the same procedure as in section 3.1.2, we determine \( \tilde{S}_{11} \) and subsequently \( \tilde{S}_{11} \), from which the expression for \( \tilde{Z}(\lambda) \) is found:

\[
\tilde{Z}(\lambda) = \frac{1 + \tilde{S}_{11}}{1 - \tilde{S}_{11}}
\]

\[
= \frac{2\lambda^3 + 3.18075\lambda^2 + 6.70152\lambda}{3.1625\lambda^2 + 4.87284\lambda + 5.18492}
\]

The effective input impedance of the predistorted filter becomes [6]

\[
Z_{in}(\lambda) = \sqrt{\frac{\lambda - \delta}{\lambda + \delta}} \tilde{Z}(\lambda)
\]

Expanding \( Z_{in}(\lambda) \) gives the element values for the predistorted normalized lowpass network, as given in Table 1, with its corresponding circuit shown in Figure 11.

**Table 1:** The element values of the predistorted normalized lowpass filter

<table>
<thead>
<tr>
<th>( g_0 )</th>
<th>( g_1 )</th>
<th>( g_2 )</th>
<th>( g_3 )</th>
<th>( g_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.6324</td>
<td>0.9243</td>
<td>0.6598</td>
<td>0.997</td>
</tr>
</tbody>
</table>

To compute the admittance inverters \( J_{j,j+1} \), the odd, and even characteristic impedances \( Z_{oo} \) and \( Z_{oe} \) respectively, equations 22 to 26 [5,9,10] are used and the results are shown in Table 2, while the admittance inverter action of the parallel-coupled filter is shown in Figure 13.

**Figure 12:** Impedance inverter

For the first coupling structure:

\[
J_{o1} = \frac{\pi\delta}{\sqrt{2g_0g_1}}
\]

For the intermediate coupling structures:

\[
J_{o,n+1} = \frac{\pi\delta}{2\sqrt{g_ng_{n+1}}} \quad (n \text{ odd})
\]

\[
J_{e,n+1} = \frac{\pi\delta}{2\sqrt{g_ng_{n+1}}} \quad (n \text{ even})
\]

where \( \delta = \text{fractional bandwidth} = \frac{f_2 - f_1}{f_0} \)

We require the odd and even mode coupled-line impedances \( Z_{oo} \) and \( Z_{oe} \) given by

\[
(Z_{oe})_{j,j+1} = Z_o(1 + aZ_o + a^2Z_o^2)
\]

\[
(Z_{oo})_{j,j+1} = Z_o(1 - aZ_o + a^2Z_o^2)
\]

where

\[
a = J_{j,j+1}
\]
Table 2: The admittance inverter element values and the corresponding odd and even characteristic impedances

| \( J_{o,j,l} \) | 3.99x10^3 | 6.58x10^3 | 6.44x10^3 | 3.91x10^3 |
| \((Z_{oo})_{o,j,l}/(\Omega)\) | 61.955 | 51.698 | 51.661 | 61.684 |
| \((Z_{oo})_{e,j,l}/(\Omega)\) | 42.01 | 48.41 | 48.443 | 42.137 |

Figure 13: The admittance inverter action of a parallel-coupled bandpass filter

Computation of Parallel-coupled filter Dimensions

The computation of the parallel-coupled filter dimensions proceeds from the knowledge of the odd and even mode characteristic impedances, \( Z_{oo} \) and \( Z_{oe} \), evaluated in section 4.2.

The length of the coupled region is that which ensures the maximum degree of coupling. This is \( \frac{\lambda_{gm}}{4} \), where \( \lambda_{gm} \) is the mid-band wavelength. These lengths are modified by the hypothetical electrical extensions \( \Delta L \), due to field fringing and radiation effects at the open-circuit ends of the line, given by [5]

\[
\frac{\Delta L}{h} = 0.412 \left( \varepsilon_r + 0.3 \right) \left( \frac{w}{h} + 0.263 \right) \left( \frac{w}{h} + 0.813 \right)
\]  (27)

where \( h \) is the thickness of the substrate, \( \varepsilon_r \) its dielectric constant (relative permittivity), and \( w \) the width of the conducting strip.

For narrow strips (when \( Z_o < (44 - 2\varepsilon_r)\Omega \)) [9,10]

\[
\left( \frac{w}{h} \right)_{sel,so} = \left( \frac{\exp H}{8} - \frac{1}{4\exp H} \right)^{-1}
\]  (28)

\[
H = \frac{Z_{sel,so}}{119.9} \sqrt{2(\varepsilon_r + 1) + 0.5 \left( \frac{\varepsilon_r - 1}{\varepsilon_r + 1} \right) \left( \ln \frac{\pi}{2} + \frac{1}{\varepsilon_r} \right)}
\]  (29)

\[
Z_{o,so} = \frac{1}{2} Z_{oe}
\]  (30)

\[
Z_{eso} = \frac{1}{2} Z_{oe}
\]  (31)

where \( so \) and \( se \) indicate the even-mode and odd-mode equivalent single lines respectively.

For wide strips (when \( Z_o > (44 - 2\varepsilon_r)\Omega \)) [5]

\[
\frac{w}{h} = \frac{2}{\pi} \left[ \frac{d_z - 1}{\ln(d_z - 1)} + \frac{\varepsilon_r - 1}{\varepsilon_r} \left( \ln(d_z - 1) + 0.293 - \frac{0.517}{\varepsilon_r} \right) \right]
\]  (32)

where

\[
d_z = \frac{59.95\pi^2}{\varepsilon_r Z_e}\n\]  (33)
\[
\left( \frac{w}{h} \right)_{se} = \frac{2}{\pi} \cosh^{-1}\left( \frac{2d - g + 1}{g + 1} \right) \quad (34)
\]

\[
\left( \frac{w}{h} \right)_{se} = \frac{2}{\pi} \cosh^{-1}\left( \frac{2d - g + 1}{g - 1} \right) + \frac{1}{\pi} \cosh^{-1}\left( 1 + \frac{2 \frac{w}{h}}{\frac{r}{h}} \right)_{r > 6} \quad (35)
\]

\[
\left( \frac{w}{h} \right)_{se} = \frac{2}{\pi} \cosh^{-1}\left( \frac{2d - g + 1}{g - 1} \right) + \frac{4}{\pi} \cosh^{-1}\left( 1 + \frac{2 \frac{w}{h}}{\frac{\varepsilon}{h}} \right)_{r = \varepsilon} \quad (36)
\]

where

\[
g = \cosh\left( \frac{\pi v}{2h} \right) \quad (37)
\]

\[
d = \cosh\left( \frac{\pi}{h} \left( w + \frac{s}{2} \right) \right) \quad (38)
\]

Equations (35) and (36) are solved simultaneously to obtain the shape and spacing ratios.

**Table 3:** The dimensions of the standard parallel-coupled filter

<table>
<thead>
<tr>
<th>W(mm)</th>
<th>S(mm)</th>
<th>L(mm)</th>
<th>( \varepsilon_{\text{eff}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.223</td>
<td>0.547</td>
<td>10.4091</td>
<td>1.9229</td>
</tr>
<tr>
<td>2.364</td>
<td>3.858</td>
<td>10.2883</td>
<td>1.974</td>
</tr>
<tr>
<td>2.223</td>
<td>0.547</td>
<td>10.4091</td>
<td>1.9229</td>
</tr>
</tbody>
</table>

**Table 4:** The dimensions of the predistorted parallel-coupled filter

<table>
<thead>
<tr>
<th>W(mm)</th>
<th>S(mm)</th>
<th>L(mm)</th>
<th>( \varepsilon_{\text{eff}} )</th>
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</tr>
</tbody>
</table>

**Figure 14:** The transmission coefficients of the standard \((S_{21})\) and the predistorted \((S_{43})\) 3rd order microstrip filter at 5GHz centre frequency
DISCUSSION OF SIMULATION RESULTS

The Agilent ADS simulation package was used in the design and simulation of the filters. The simulation responses of both the standard (unpredistorted) and the predistorted filter are shown in figures 14 and 15. In figure 14 the centre frequency of the predistorted filter is at the prescribed value of 5GHz, while that of the standard (unpredistorted) filter is at 5.10GHz, showing a shift of 100MHz. The return loss of both filters are shown in figure 15. It is seen that the standard filter has lost of its three poles while that of the predistorted filter shows all the thee poles expected from 3rd order filter.

CONCLUSION

In this work the classic theory of predistortion was demonstrated using a 5th order low-pass lumped-element filter whose response is shown in figure 4 (for the ideal filter), figure 6 (lossy and ideal filters.) and figure 8 (for ideal, lossy and predistorted filters). It is observed in figure 8 that the cutoff frequency of the lossy circuit (S_{43}) has shifted from the prescribed value of 2.0GHz, the bandedge severely rounded, and suffered an insertion loss of about 10.6dB. The predistorted filter restored the displaced frequency and improved the insertion loss to -4.77dB.

To demonstrate the potentials of the predistortion techniques, we have developed an application of the techniques by successfully implementing it in the design of microwave distributed filters using microstrip transmission line. As a practical example, a standard (unpredistorted) and a predistorted 3rd order parallel-coupled microstrip filters were designed. The simulation results in figures 14 and 15 have justified the usefulness of the predistortion technique.

With the predistortion technique we have avoided the usual the usual time-consuming optimization technique, whose rate of convergence largely depends on the starting values. Hence we established a systematic procedure which is devoid of trial-by-error.

REFERENCES


