Improving interference alignment of Gaussian MIMO X Channel and the Gaussian MIMO Z channel

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Abstract
We study about the Gaussian MIMO X channel and the Gaussian MIMO Z channel. In Gaussian MIMO X channel we have two multiple antennas in a transmitter and a receiver. One of the cross-links in the MIMO X channel is eliminated to obtain the MIMO Z channel. The sum-rate upper bound is derived for the MIMO Z channel and it is examined with an other existing bound. From the sum-rate upper bound of the MIMO Z channel we can obtain the sum-rate upper bound of the MIMO X channel. Again, the sum-rate upper bound of the MIMO X channel is obtained from the derived worst noise covariance matrix for two user multiple access channel. The sum-rate of MIMO X channel is further increased by reducing the detection of unknown signals using some detection algorithms. The derived sum-rate upper bounds for the MIMO X channel are compared with the Maddah Ali Motahari khandani (MMK) scheme. The above results are considered with MIMO interference channel and their results are examined. On further examining we derive some numerical values, which shows that the obtained sum-rate upper bounds capacity is much more tighter than the previous ones.

Keywords: component; The MIMO X channel, The MIMO Z channel, Sum-rate upper bound capacity, Worst noise covariance, The MIMO interference channel.

Inspiration
Use of multiple antennas at the terminals has become the most modern feature of the wireless communication. Increase in transmit power or increase in data rates with out increase in bandwidth like features are possible with MIMO technology, which is being an important part of wireless communication. We study about three important MIMO channels, namely, the Gaussian MIMO Z channel, the Gaussian MIMO X channel and the Gaussian MIMO interference channel, which are comprised of single antenna channels. We will be considering the sum-rate upper bounds which are valid for all SNRs.

MIMO X channel:

MIMO Z channel:

Notation:
Matrices are denoted as boldface uppercase letters, Vectors are denoted as boldface lowercase letters. Transpose operation is denoted as $[.]^T$, Hermitian operation is denoted as $[.]^H$. Trace operation is denoted as $\text{Tr}(.)$, Expectation matrix is denoted as $E[.]$. Determinant of a matrix $Z$ is denoted as $|Z|$ and $I$ is the identity matrix.
System Model
In the Gaussian MIMO XC system, there are two transmitters and two receivers. There are Mt antennas in transmitter and Nr antennas in receiver, r = t = 1, 2. Independent messages are sent from each transmitter to each receiver. So that, there will be four independent messages sent from transmitter ‘j’ to receiver ‘i’, messages would be \( W_{ij} \), i = j = 1, 2. While transmitting messages from transmitter to receiver, We will get a channel gain matrix \( H_{ij} = [h_{ij}] \), where \( h_{ij} \) is the channel gain, i = 1, 2,..,M and j = 1, 2,...,N.

\[
\begin{align*}
\gamma_i &= H_{11}x_1 + H_{12}x_2 + n_1 \\
\gamma_j &= H_{21}x_1 + H_{22}x_2 + n_2
\end{align*}
\]

Where, \( x_i \) is the input vector at transmitter ‘i’, t = 1, 2, \( y_i \) is the output vector at receiver ‘r’, r = 1, 2, and \( n_i \) is the circularly symmetric complex Gaussian noise at receiver ‘r’. There should be a power constrain for transmitter ‘t’, i.e., \( P_t \) and \( \text{Tr}(S_t) < P_t \). \( S_t \) is the set of input covariance matrices, which is equal to \( E[x_i x_i^H] \), where t = 1, 2. \( P_t \) is the total power transmitted by two transmitters, \( P_t = P_1 + P_2 \).

In the MIMO XC system, the message \( W_{2t} \) is not sent and made it as null, then the channel gain \( H_{21} \) will become zero. So that, we obtain the MIMO ZC from the MIMO XC. The sum-rate upper bounds is found for the MIMO XC, its found from the sum-rate of the MIMO ZC and from the worst covariance noise in MAC. First we go with the sum-rate upper bound of the MIMO ZC.

The Sum-Rate Upper Bound for the MIMO XC:
The noise is to be reduced at receiver 1, which is the main concept of upper bound. A new sum-rate upper bound is to be derived, so that all the three messages in MIMO ZC system are to be easily decoded in the channel.

**Theorem ZC:** If \( N_r > M_t \) , r = t = 1, 2, then the sum-rate capacity of the MIMO ZC will be bounded by the sum-rate of the MAC formed due to both transmitters and one receiver, the additive Gaussian noise at receiver 1, \( A = [H_{11}^2, H_{12}^2, H_{21}^2, H_{22}^2] \).

Here, \( W \) is a positive semi definite matrix, which satisfies \( W < (H_{12}^2, H_{12}^2) \) and \( W < (H_{21}^2, H_{22}^2) \). The sum-rate capacity bounded for MIMO ZC is

\[
S_{Z} < S_{Z_{\text{out}}} = \max_{\text{Tr}(S_t) = P_t} \log_{\text{Tr}(S_t) = P_t}^{[H_{11}^2, H_{12}^2, H_{21}^2, H_{22}^2] + A} \]

**The Sum-Rate Upper Bound for MIMO XC based on Sum-Rate Upper Bound of MIMO ZC:**
The sum-rate capacity of the MIMO XC be \( S_x \). The rate of message \( W_{ij} \) be \( R_{ij} \). The link which is removed from the MIMO XC to obtain the MIMO ZC, that associated message forms an upper bound on the sum-rate of the three remaining messages. So that, wondedly we are considering Z(12) channel which is obtained by eliminating message \( W_{12} \) and channel \( H_{12} = 0 \) from the MIMO XC. So the sum-rate capacity of Z(12) channel is bounded on the three messages, i.e., \( R_{11} + R_{21} + R_{22} \). The sum-rate capacity of Z(12) channel is unknown, so we are using the sum-rate upper bound from Theorem ZC and it is denoted by \( S_{Z(12)_{\text{out}}} \). A new sum-rate upper bound of the MIMO XC, \( S_{\text{out}-1} \) is obtained by utilizing the sum-rate upper bounds of the four MIMO Z channels associated with the MIMO X channel. A set of rate inequalities are given by considering four to three combinations of the following rate vectors \( (R_{11}, R_{12}, R_{21}, R_{22}) \) for each of the four Z channels in association with X channel.

**Theorem XC:** When, \( \min(N_t, N_r) > \max(M_t, M_r) \), this condition is required to satisfy the upper bound conditions of the MIMO ZC.

\[
R_{12} + R_{21} + R_{22} < S_{Z(11)_{\text{out}}} \\
R_{11} + R_{22} < S_{Z(22)_{\text{out}}} \\
R_{12} + R_{22} < S_{Z(12)_{\text{out}}} \\
R_{11} + R_{21} < S_{Z(112)_{\text{out}}}
\]

Using these sum-rate upper bounds we can calculate a new sum-rate upper bound of the MIMO XC, where the each rate inequality variable will be repeated thrice

\[
S_X < S_{\text{out}-1} = 1/4[S_{Z(11)_{\text{out}}} + S_{Z(12)_{\text{out}}} + S_{Z(22)_{\text{out}}} + S_{Z(112)_{\text{out}}}]
\]

**The Sum-Rate Upper Bound for the MIMO XC using the Detection Algorithms:**
There is a problem which is to be considered that involves the detection of many signals, and some unknown shapes all are part of noise, this is a very common problem. This can be reduced and for the required signal detection, the following detection algorithms are given below, for, i = 1, 2, j = 1, 2.

\[
\begin{align*}
\gamma_i &= a_i + \sum \theta_i (a_i - a_i) + \frac{\sigma^2}{d_{ij}} \\
y &= T(1, a_1) + T(2, a_2) + w
\end{align*}
\]

**The Upper Bound for MIMO XC from MIMO MAC with Worst Noise Covariance**
In here, MIMO MAC is formed when both receivers cooperate with the same individual power constraint at the transmitter, in this condition MIMO XC is considered. The sum-rate capacity of this MIMO MAC is considered as \( S_{MAC} \). As the \( S_X < S_{MAC} \), the upper bound is further tightened by imagining noise correlation at the both receivers,This affects the MIMO MACs sum-rate capacity region. The noise vector correlation at the receivers does not show any affect on the capacity region of the MIMO XC, as the MIMO XCs capacity region does not depend on the joint distribution but it is depended on the marginal transition probabilities. Any how, the MAC is the upper bound of the MIMO XC.

Let ‘a’ be the noise vector and ‘A’ be the noise covariance matrix in the MIMO MAC system, we have \( a = [n_1^T, n_2^T]^T \) and

\[
A = E[z^2]
\]

Let \( Y_{Ni} = E[n_i n_i^H] \), where i = 1, 2, and \( X = E[n_1 n_2]^T \).

Let S is set of all the positive semidefinite noise covariance matrices which satisfies the MAC upper bound conditions,

\[
S = \{ A > 0, A = [Y_{N1}, X, Y_{N2}] \}
\]

By minimizing the \( S_{MAC} \) about all the noise covariance matrices A, the upper bound of \( S_X \), i.e., the sum-rate capacity of MIMO XC is further tightened. We get, \( S_X < S_{\text{out}-2} = \inf S_{MAC} \).

The above MAC upper bound can be written as a min-max problem,

\[
S_{out-2} = \max_{S_{MAC}} \log_{S_{MAC}}^{[H_{11}^2, H_{12}^2, H_{21}^2, H_{22}^2]} + A
\]

**MIMO Interference Channel:**

**The Upper Bound based on the MIMO ZC:**
The MIMO ICs sum-rate upper bounds be derived from the sum-rate upper bounds for the MIMO ZC, the cross links constitutes the major part of the interference and by removing
one of the cross links the capacity can be increased. The sum-rate of the MIMO ZC is the upper bound on the MIMO IC’s sum-rate, by removing one of the cross links we obtain the sum-rate of MIMO Z-IC and the MIMO Z-IC is an upper bound for the sum-rate of MIMO IC and the sum-rate of MIMO ZC is upper bound of sum-rate of MIMO Z-IC, thus the MIMO ZC is an upper bound on the sum-rate of the MIMO IC. So that, the minimum of the upper bounds of the MIMO ZCs are to be considered so that it could match with the sum-rate upper bound for the MIMO IC.

**Theorem IC using ZC:**
When, \( \min(N_1, N_2) > \max(M_1, M_2) \), this condition is to satisfy the upper bound in Theorem ZC. The sum-rate of the MIMO IC is bounded as,
\[
S_i^{\text{out}} = \min \left[ S_{Z(12)}^{\text{out}}, S_{Z(21)}^{\text{out}} \right]
\]

**The upper bound of MIMO IC from the MIMO MAC with Worst Noise Covariance:**
When the Sum-rate of the MIMO XC is derived by considering the two receivers cooperation, by imagining the noise correlation for further tightening of that upper bound and the worst noise covariance matrix is derived. The sum-rate of the MIMO IC is bounded by the sum-rate of the MIMO XC, from min-max problem we derive, \( S_i^{\text{out}} = S_i^{\text{out}-1} \), the sum-rate upper bounds are found out with the help of mentioned input covariance matrices, \( S_1 \) and \( S_2 \). On comparing both the upper bounds \( S_i^{\text{out}-1} \) and \( S_i^{\text{out}-1} \), we consider the input covariance constrains as: \( S_i^{\text{out}-1} = S_{i^*}^{\text{out}-1} \), \( i = 1, 2 \), we do not consider trace constraint \( \text{Tr}(S_i^{\text{out}-1}) < P_t \). We formulate \( S_i^{\text{out}-1} \) in terms of the sum-rate upper bound of the MIMO ZC. Therefore, the sum-rate upper bound of the MIMO IC is,
\[
S_Z^{\text{out}} < \log |H_{11}S_1^{\text{out}-1}H_{11}^H + H_{12}S_2^{\text{out}-1}H_{12}^H + A|/|A|
\]

By considering the min-max problem the second upper bound of the MIMO IC is written by using similar arguments as above, for \( S_2^{\text{out}-1} = S_2^{\text{out}-1} + \Delta S_2^{\text{out}} \) with covariance constrains the inner maximization of the min-max problem will be maximized.
\[
S_i^{\text{out}-2} = \min_{A,\delta} \log \left| H_1S_1H_1^H + H_2S_2H_2^H + A_2^H A \right|
\]

**Obtained Results**
From all the proposed models, the obtained upper bounds for the MIMO XC, MIMO ZC and the MIMO IC are compared and evaluated with others.

**MIMO ZC:**
The sum-rate upper bound of the MIMO ZC from Theorem ZC is considered, we also consider a channel which is assumed to be used for \( n \) times, and the transmitter vector satisfies the power constrain \( P_t \) from Fano’s inequality we get the MIMO ZC as, \( H(W_j^T)^{r_0}A < n\epsilon_n \), we are taking three antenna at transmitter and at receiver, channel matrix is generated for 5000 realizations with a channel gain of \( h_{ij} \), \( i = 1, 2, \ldots, M_1 \), \( j = 1, 2, \ldots, N_1 \), we are considering \( M_1 = N_1 = 2 \), then the average sum-rate is calculated for required upper bounds. The total power transmitted is divided equally between the two transmitters, \( P_1 = P_2 = P_t/2 \), \( \text{SNR} = P_t/\sigma_n^2 \). Sum-rate upper bounds are further compared with sum-rate of MAC at receiver 1 and with worst noise covariance and found that Theorem ZCs upper bound is much more tighter than the compared ones for almost all the realizations among \( W \) and \( u \) there were no observations of positive definite ordering.

![Graph](image_url)

A plot of average sum-rate upper bounds in MIMO XC of CSCG matrices for 5000 realizations.

**MIMO XC:**
The sum-rate upper bounds of the MIMO XC, \( S_i^{\text{out}-1} \) and \( S_i^{\text{out}-2} \) found previously in the above theorem are evaluated and compared for 5000 random realizations, the number of antennas in transmitter and receiver are \( 3 \), we are comparing the obtained results with MMK scheme, when we reach high SNR, there will be a rapid change in the differences between the bounds, \( S_i^{\text{out}-1} \) is loose at higher SNR region and \( S_i^{\text{out}-1} \) is loose at low SNR region and is good at high SNR region. Results are shown in below graph.

![Graph](image_url)

A plot of average sum-rate upper bounds in MIMO XC found and CSCG matrices for 5000 realizations.

**MIMO XC using Detection Algorithms:**
The sum-rate upper bound of MIMO XC is much more tighter than the previous sum-rate upper bounds by using the detection algorithms mention previously, very helpful for
detection of the signal and reducing the noise, we compare both the sum-rate upper bounds, from the Theorem XC and the new MIMO XC using detection algorithms.

**MIMO IC:**

In the MIMO IC, we consider two antennas at both the transmitters and four antennas at both the receivers, $S_{I_{out-1}}^{*}$ and $S_{I_{out-2}}^{*}$, the obtained sum-rate upper bounds of the MIMO IC model are evaluated and compared for 5000 realizations and the below covariance constraints are used for evaluation,

\[
S_{1}^{*} = \frac{P}{2}I, \quad S_{2}^{*} = \frac{P}{2} \begin{bmatrix} 1 & 0.2 + 0.2i \\ 0.2 - 0.2i & 4 \end{bmatrix}
\]

The average sum-rate is evaluated for different input power values, the observations on comparing all the sum-rate upper bounds obtained by treating interference at both the receivers say that $S_{I_{out-1}}^{*}$ is much tighter and is closer to the achievable sum-rate evaluated with the above covariance constraints, $S_{1}^{*}$, $S_{2}^{*}$

**Conclusion**

We have evaluated and compared the sum-rate upper bound capacities of the MIMO XC, MIMO ZC, MIMO IC. The new obtained sum-rate upper bounds are tighter than the existing previous upper bound, this was shown in the above results. The proposed results are helpful for achieving these following conditions: Without increase in the bandwidth, we can significantly increase the data rates, The signal quality and the transmit power are increased, and the worst noise covariance is avoided successfully.

**References**


