The Break-Even Point of the Utilities in the Real Estate Market of Bilateral Monopoly

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Abstract

In the real estate market, there are examples of bilateral monopoly in special situations related to the position and the configuration of the property. Typical cases are the trading involving: extra ordinary properties that are taken away from the normal interactions between supply and demand; landlocked funds and/or residual surfaces that get a certain value only if connected to neighboring properties. A model that summarizes the framework within which supply and demand move and interacts is defined, in order to identify a possible break-even point of the respective utilities.

Keywords: break-even point, utility theory, real estate market, bilateral monopoly.

Introduction

When the property for sale is an asset not reproducible or non-replaceable (for example with reference to the use of a complementary properties1), to determine the trading price it is possible to assume that supply will be able to exercise bargaining monopoly power. It is true, however, that once the property placed on the market – due to the constraints imposed2, its location3, or its physical characteristics4,5 – the bargaining power of supply has to deal with a single interlocutor interested in buying property. This is the hypothesis that defines the framework in which the model of interaction between supply and demand is developed for the definition of a likely trading price. This condition can be treated, with respect only to the phase of negotiation, as a scheme of bilateral monopoly.

In fact, in these conditions, the balance cannot be derived from an analysis conducted in terms of the marginal-productivity theory6,7, that is, building the curves of supply and demand, cost and marginal revenue. Since this is a property not replaceable or reproducible, within a quadrant that has the abscissa axis the quantity and on the ordinates the price, these curves would be infinitely rigid in quantity. They would be as vertical lines with opposite directions (upwards the supply, starting from the minimum price for the sale, down the demand, starting from the maximum price for the purchase).

The solution appears at first sight indeterminate; the only obvious statement concerns the possibility that the exchange takes place or not. That is, given the level of the maximum price that demand is willing to pay and the minimum price level that supply is willing to accept, an indispensable condition for the start of trading is that the first is greater than the second8. The equilibrium, i.e. the price fixed for the exchange, is then determined by non-economic factors such as the bargaining power, strategies, etc.

From these premises and based on assumptions about the behavior of supply and demand a model that defines a probable equilibrium has developed. The balance is sought by resorting to measures of the utilities arising from exchanges and identifying the likely break-even point.

With reference to the Italian traditional theory of appraisal10, it should be noted that in this case the estimate of the equilibrium price is not generally valid. It is rather the expression of a specifically judgment formulated in relation to: 1) the specific nature of the property, 2) the subjectivity of the actors, 3) the relationship between them and the asset, 4) and the different motivations that cause the exchange.

The Model

Balance in the exchange:

The interest of the study is aimed at defining a probable equilibrium in a two-bargaining which is between a private entity interested in investing to enhance a particular property and another party willing to alienate it.

The exchange will be the result of negotiation between the parties, if they reach an agreement on the equivalence between the resource and a quantity of money. It is evident that this equivalence should be referred to two different utility functions that have different measurement scales. The one and the other party measures the utility derived from the exchange using the monetary parameter, but each gives to this parameter a different marginal utility in relation to his economic capacity.

The unknowns of the problem are initially three: the maximum price that demand is willing to pay (P_{MaxD}), the minimum price that supply is willing to accept (P_{MinD}) and the equilibrium price.

It is possible to define the first unknown building a financial analysis of the cash flows associated with the investment. In other words, you need to determine the value that can ensure the profit expected by the investor with regard to the intervention of exploitation, processing or union with other property. Placing NPV = 0, and finally setting the discount rate equivalent to the "opportunity cost" of the investment, the only unknown is the P_{MaxD} (initial outlay for the purchase of property).

\[
P_{\text{MaxD}} = \sum_{i=0}^{n} \left( \frac{R_i}{(1+r_d)^i} - \frac{C_i}{(1+r_d)^i} \right)
\]

(1)
For the solution, namely to estimate the other two unknowns - the minimum price supply is willing to accept and the equilibrium price - has built a linear system involves two equations and two variables. The two equations both represent the condition that is the basis of the model: the equilibrium is defined by the break-even point of the weighted utilities of the two parties. This condition, however, is described with different approaches: one analytical the other graph. For each of the approaches are valid, the following hypothesis, but it should be noted, there is no interaction or relationship between them:

1. The first hypothesis involves a simplification of the mechanism that describes the real interaction between supply and demand. It requires not consider any non-economic factors that determine financial exchange; i.e. it is not brought into account an eventual thrust psycho-sociological that can induce a greater or lesser propensity to the exchange.

2. The second assumes the perfect symmetry of the information. Each of the two parties know the economic capacity of the other and is therefore able to measure its bargaining power, or more realistically, this information is known to the expert called upon to estimate the likely equilibrium price or to mediate between the parties.

Analytical approach to break-even point of the utilities:

The two utilities are expressed - on different scales - in terms of financial benefits from the exchange. The respective financial benefits of the contractors are therefore measured in the event that the transfer of property is realized at a price

\[ P \]

such income using the discount rate \( r^* \) (defined as capitalization rate):

\[ \frac{R}{r^*} \]

b) In the second scenario, the property under current conditions is not able to generate an income. The financial sacrifice that the owner is willing to support is partly commensurate with the loss of revenue, resulting in the draft agreement between the two parties. The loss is at least equal to the minimum price that supply is willing to accept. On the other hand, this amount should also be added to the financial cost of "conservation" of the property. The owner will still have to support a minimum annual maintenance cost \( (C) \) necessary for its "conservation". This cost is related to the willingness to pay of the property over time in order to avoid further deterioration of the property and to ensure the possibility of future transformation. Therefore it is a cost that can become a major item of expenditure, to which the possession taxes must also added, if due. The decision on whether to make the minimal maintenance so that some transformation of the property is possible in the future, depends on its cost and on the value that the owner envisions the property will reach once transformed.

Then it is clear that the financial sacrifice endured by the owner who does not sell the property is the sum of the minimum price accepted by the supply \( (P_{MinO}) \) and accumulation of the annual costs of the conservation \( \frac{C}{r^*} \).

This is the assumption on which the subsequent development of the analysis is built. Regarding the supply, the benefit from the sale, in financial terms, is therefore commensurate to the selling price (considered net of taxes, which for simplicity are not brought into account) less the financial sacrifice that the owner is willing to support to preserve the availability of the property:

\[ \overline{P} - \left( P_{MinO} + \frac{C}{r^*} \right) \]

In order to make commensurable the utilities of the two parties - measured in terms of financial benefits resulting from the exchange - the respective values of these benefits are multiplied by appropriate weighting coefficients. The coefficients, with respect to the hypothesis according to which the equilibrium price is determined solely by economic factors, should therefore express the different bargaining power of the two parties. This should be proportionate to the economic horizon and risk tolerance of the two economic subjects, and it is therefore equal to the rate corresponding to the different and subjective opportunity costs of supply and demand.
For the supply, the opportunity cost should be corresponding to sacrifice following the non-revenues deriving from the sale \((r_o)\). Therefore, there is a direct proportional relationship between this rate and the willingness to pay for conservation of the property \((C)\).

For the demand, the opportunity cost is instead defined by the minimum expected return on investment \((r_d)\), already used for the calculation of the maximum purchase price \((P_{MaxD})\). Economic logic indicates as a weighting factor the reciprocal value of the parameter that reflects the true size of the utility and it is therefore set equal to the inverse of this rate \((1/r)\).

The assumptions listed and the considerations which there may result lead to the following equation:

\[
\frac{P - P_{MinO} - C}{r_o} = \frac{P_{MaxD} - \bar{P}}{r_d}.
\]

This equation is then solved as a function of the unknown \(\bar{P}\):

\[
\bar{P} = \frac{r_o \cdot P_{MaxD} + \frac{r_d}{r} \cdot C + r_d \cdot P_{MinO}}{r_o + r_d}.
\]

At this point, it should be emphasized differentiation between the capitalization rate \(r^*\) and rates defining the opportunity cost, in other words the economic capacity of supply and demand \((r_{od})\). The first is to be understood as a coefficient (divider) which applied to a number that represents an average annual flow determines an equivalent value of capital (stock) reported at the time of the estimate. Compared to the other two it is reference to a time frame definitely more wide; The greatest uncertainties related to predictions of longer term may lead to assume the following relation \(r^* < r_{od}\).

On the other hand, the certainties (few) of the estimate of the expected flow (annual and medium) formed by income and / or the costs of conservation, compared to the level of risk as measured by the opportunity cost, lead to assume the opposite relationship: \(r^* > r_{od}\). In particular, the rate \(r^*\) can be set equal to the remuneration of financial investments without risk.

**Graphical approach to break-even point of the utilities**

Given that money is the yardstick for the utility, the two utility functions have linear trends, growing the supply utility \((Uo)\) and diminishing the demand utility \((Ud)\), with zero value respectively at the minimum price that supply is willing to accept \((P_{MinO})\) and the maximum price that demand is willing to pay \((P_{MaxD})\).

The assumption of linearity of the utility functions assumes that the corresponding marginal utility of money is constant in the range considered \([P_{MinO}; P_{MaxD}]\).

The Figure 1 shows, as an example, the chart pattern on the assumption that supply and demand have equal bargaining power, or the same economic and financial capacity, and a similar propensity to transfer from side to buy on the other side, generated by different motives psycho-sociological. In this case the two functions utilities have an identical but opposite inclination, so that equilibrium is logically defined by the midpoint of the interval \([P_{MinO}; P_{MaxD}]\). It is clear however that, in practice, it is far from such a condition. In fact, note that the two parties, supply and demand, have utility functions not commensurable as measured on different scales. The diagram in Figure 1 should be amended by assigning a different angle to the utility functions that, within the framework outlined by the assumptions of the model, is able to reflect the economic and financial capacity (and therefore the different bargaining power) of the contractors (Figure 2).

**Figure 1: Chart pattern break-even point of the utilities**

**Figure 2: Graphical approach to break-even point of the utilities with the explanation of the constraints**

In particular the two utility functions are constructed by fixing them to the following constraints that respond to the economic logic of the model:
- If supply and demand have the same economic capacity, the exchange price is equal to the maximum price that demand is willing to pay:

\[ (\text{if } r_o = r_d \text{ then } P = P_{\text{MaxO}}) \]

- If supply has no bargaining power the sale price coincides with the minimum price that supply is willing to accept:

\[ (\text{if } r_o \to 0 \text{ then } P = P_{\text{MinO}}) \]

- If \( r_d \) is much larger than \( r_o \), then \( P \) comes close to \( P_{\text{MinO}} \).

The above conditions are met if utility functions are described, respectively, for supply and demand, by the following equations:

\[ U_O = \frac{1}{r_o} \cdot (P - P_{\text{MinO}}) \quad (7) \]

\[ U_D = \frac{1}{r_d - r_o} \cdot (P_{\text{MaxD}} - P) \quad (8) \]

Set equal the two utility functions, at the price at which the exchange takes place, it is:

\[ \frac{1}{r_o} \cdot (P - P_{\text{MinO}}) = \frac{1}{r_d - r_o} \cdot (P_{\text{MaxD}} - P) \quad (9) \]

From equation (9), in turn, you get

\[ P = \frac{r_o \cdot P_{\text{MaxD}} + P_{\text{MinO}} \cdot (r_d - r_o)}{r_d} \quad (10) \]

In this way it is built a system of two equations, (6) and (10), in the two unknowns, \( P \) and \( P_{\text{MinO}} \), that once solved provides the following expressions of the unknowns:

\[
\begin{align*}
P &= P_{\text{MaxD}} + \frac{r_d}{r_o} \left( \frac{r_d}{r_o} \right)^2 \cdot \frac{C}{r^*} \\

P_{\text{MinO}} &= P_{\text{MaxD}} - \left( \frac{r_d}{r_o} \right)^2 \cdot \frac{C}{r^*} 
\end{align*}
\]

Finally it should be imposed as a binding condition for a plausible solution:

\[ P_{\text{MinO}} > 0. \quad (12) \]

**Appropriateness of the model:**

When \( r_o = r_d \), that is, when supply and demand have equal bargaining power or the same economic capacity, the most likely trading value is:

\[ P = P_{\text{MaxD}} \]

with

\[ P_{\text{MinO}} = P_{\text{MaxD}} - \frac{C}{r^*}. \]

The latter equation, in relation to the constraint imposed by (12), indicates the impossibility of an agreement between the contracting parties when the total financial cost of conservation exceeds the maximum purchase price. 

On the basis of this statement, the initial accumulation of the annual costs of conservation (\( C \)) of the property, can be construed as the production cost of the same property. In practice it is stated that the costs involved in the "passive protection" of the property coincide with its cost of production. It is then logical and coherent that, in the event \( r_o = r_d \), this amount represents the minimum price that supply is willing to accept (\( P_{\text{MinO}} \)).

When, instead, the economic capacity of demand (\( r_d \)) is well above the capacity of supply (\( r_o \)), so that the minimum price that the property is willing to accept is close to zero (\( P_{\text{MinO}} \approx 0 \)), the most likely trading value is:

\[ P = \sqrt{P_{\text{MaxD}} \cdot \frac{C}{r^*}}. \quad (13) \]

This formulation is obtained by imposing the condition \( P_{\text{MinO}} = 0 \) to the second equation of the system (11), thus obtaining the ratio \( \frac{r_d}{r_o} \), and finally by replacing the latter in the first equation of the same system. The equation (13), with reference to the hypotheses from which derives, suggests defining the relationship between \( P \) \( C/r^* \), that is, between the selling price and what has been referred to as cost of production of the property. This is achieved through the construction of the function which relates the percentage ratio \( \frac{P}{P_{\text{MaxD}}} \% \) and the analogous ratio \( \frac{C}{P_{\text{MaxD}}} \% \), where \( P \) remember - is measured by taking the value of the ratio \( \frac{r_d}{r_o} \), such that \( P_{\text{MinO}} \) is null (Figure 3).

The algebraic expression of the function is as follows:

\[ \frac{P}{P_{\text{MaxD}}} \cdot 100 = 100 \cdot \sqrt{\frac{C}{P_{\text{MaxD}}} \cdot \frac{r^*}{r^*}} \cdot 100. \quad (14) \]

Reading the graph some final remarks are useful to validate the proposed model: a) the trading price is always higher than the cost of conservation of the property; b) the trading price increases with the cost of conservation, but less than proportionately, until the two amounts coincide when the trading price equals the maximum price that demand is willing to pay.
Conclusions
The model defines the balance likely in a pattern of bilateral monopoly able to approximate the actual conditions characterizing the exchange of properties in the real estate market. It considers supply unable to achieve any profits by the property that in turn forces to support a cost of conservation. However, given its physical and qualitative characteristics, or the location of the property, supply and demand are both able to exercise bargaining monopoly power. A number of assumptions that simplify the mechanism of interaction between supply and demand, the definition of the basic parameters used to represent the different bargaining power of the players involved in the exchange and an original formulation allow to give solution to a problem apparently indefinitely.

First step of the model is to determine the maximum limit of the range within which the price will be fixed eventually (maximum amount that demand is willing to pay). The lower limit of the same range (minimum amount that supply is willing to accept) together with the equilibrium price are the unknowns of a system of two linear equations. The writing of these equations is derived respectively from two types of approaches to the same problem (one analytical, the other graph): the breakeven point of the weighted utilities of the two actors involved (supply and demand).

References