International Journal of Applied Engineering Research ISSN 0973-4562 Volume 10, Number 23 (2015) pp 43705-43708
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Analysis Stability of Multi-Agents System With Disturbance Function

Sentot Achmadi*, Miswanto, Marjono*****

**Prodi TeknikInformatika, ITN, Malang, Indonesia E-mails:sentot_achmadi@yahoo. co. id*
***Department of Mathematics, Airlangga University, Surabaya, Indonesia E-mails: miswanto@fst. unair. ac. id*
****Department of Mathematics, Brawijaya University, Malang, Indonesia E-mails: marjono@ub. ac. id*

Abstract
In this paper, we utilize the swarm behavior in nature for stability of the multi agents system. We propose a mathematical model describing the movement of the agent of multi agents system with disturbance function. Different from the existed models which do not disturbance. This model is a dynamics system consisting of many agents with disturbance function. The first, we study stability of the multi-agent model with disturbance function, and we do some simulation. An estimate of the time when all agents start entering the annulus is presented. The second similar behavior will also be presented in this paper that the agents aggregate and enter a certainε-annulus. It is shown that the members of a swarm will aggregate and eventually form a cohesive cluster of finite size around the swarm center. In the last section will show the results of numericalsimulationof the swarm behavior with disturbance function.

Keywords: Multi Agents System, Disturbance Function, Aggregate, Numerical Simulation.

I. Introduction

First swarm phenomena modeled mathematically by Breder[1] which discussed the attractor force and repellent force between two members of swarm. Somescientistsalsodiscussedswarm, i. e., Gazi and Passino[3, 4]proposed a swarm model and studied its aggregation, cohesion and stabilityproperties. Wang et al [11] generalized their model. They introduced the coupling matrix and a real positive parameter and studied the properties of aggregation. The coupling matrix is symmetric, that is the interaction between two agent of the swarm are reciprocal. Chu et all [2] generalized Gazi and Passino model, where they introduced a coupling matrix. The coupling matrix is symmetric, that is theinteractions between two agents of the swarm are reciprocal. Shi et all [10]proposed a swarm model analogous with Chu et all model, but the couplingmatrix is asymmetric. Miswanto, et al [8], study about stationary of swarm center and stability analysis of themodel with Lyapunov method. This work is a modification of a modelproposed by Miswanto et al [7]. Gazi and Passino[5] study thestability of the collectivebehavior of social foraging swarms, i. e., swarms moving in a profile of nutrient/toxic substances (an attractant/repellent profile). Liu and Passino [6] study swarm cohesiveness as a stability property and use a Lyapunov approach to analysis cohesive social foraging even in the presence of "noise" characteristic. Parrish, Viscido and Grunbaum [9] studied some differences in behavior algorithms and aggregation statistics among existing schooling models. Warburton and Lazarus [12] also considered an individual-based swarm model and studied the effect on cohesion of a family of attraction/repulsion functions.

In this paper, we propose and study an extension of the Miswantoet all [7] where we introduced perturbation to the system. Analysis of the stationarity of the swarm center and behavior of swarm agents around the swarm center are reported.

In the next section, we present the swarm model with disturbance and behavior of swarm agent around the swarm center. In section 3, we showthe illustrate our resultsof numerical simulations for different attractant/repellent profile.

II. Swarm Model with Disturbance

Consider a swarm model of M agents with disturbance in an N dimensional Euclidean space. The model is given by

$$\dot{x}_i = \sum_{j=1}^{M} w_{ij} f(x_i - x_j) + \gamma_i \, , i = 1,2,3,\ldots,M \tag{1}$$

here$\dot{x}_i = \frac{dx_i}{dt}$, $x_i \in R^N$represents the position of the$i$-th individual, $G = [w_{ij}] \in R^{NxN}$is the coupling matrix with w_{ij}elements of nonnegative integer for all$i, j = 1, \ldots, M$. In this paper, matrix$G$is assumed to be symmetric, thatis$w_{ij} = w_{ji}$ for all$i$, j and$w_{ii} = 0$ for all $i = 1, \ldots M$. The symbol $f(.)$represents the term of attraction andrepulsion among members. The attraction/repulsion function is given by

$$f(y) = -y\left(a - \frac{r}{b + c\|y\|^2}\right), \tag{2}$$

where$a$, b, $c$and $r$are positive constant with$a \ll r$and$\|y\| = \sqrt{y^T y}$ is the Euclidean norm. The parameter $a$represents the attraction and the term$\frac{r}{b+c\|y\|^2}$represents the repulsion. The function is attractive for large distance and repulsive for small distance. The perturbation function satisfies $\gamma i(t) \to 0$for $t \to \infty$. The functions are assumed to have a decreasingproperties: $\gamma_i(t) \le Ke^{-\beta_1 t}$ where K and$\beta_1$is a positiveconstantand t is non-negative time variable. Next, the stationarity of the swarm center and behavior of swarmagents around a certain point called swarm center are stated.

The swarm center is given by

$$\bar{x} = \frac{1}{M} \sum_{i=1}^{M} x_i . \tag{3}$$

***Theorem1*:** The center of the swarm (x) described in (3) is stationary for$t \to \infty$.

Proof: This proof follows from the proof of theorem in [4]
Since

$$\bar{x} = \frac{1}{M}\sum_{i=1}^{M} x_i,$$

Then

$$\dot{\bar{x}} = \frac{1}{M}\sum_{i=1}^{M} \dot{x}_i.$$

Thus, we obtain

$$\dot{\bar{x}} = \frac{1}{M}\sum_{i=1}^{M}\left(\sum_{j=1}^{M} w_{ij} f(x_i - x_j) + \gamma_i\right).$$

From the definition of the function attraction andrepulsion $f(y), f(-y) = -f(y)$ for all $y \in R^n$. Hence

$$\dot{\bar{x}} = \frac{1}{M}\sum_{i=1}^{M} \gamma_i(t),$$

Therefore, $\dot{\bar{x}} \to 0$as $\to \infty$. This completes the proof.
The following theorem shows that all agents will aggregate and approach a bounded region around the swarm center.

***Theorem 2*:** Let the swarm model be (1) with an attraction and repulsion functions (2). Then, all agents will aggregate and approach a bounded region

$$\Omega = \{x: \textstyle\sum_{i=1}^{M} \|x_i - \bar{x}\|^2 \le \rho^2\},$$

Where

$$\rho = \frac{AB + 2K}{\sqrt{2}awM}, w = \min_{i \neq j, i, j = 1,..,M}(w_{ij})$$

and

$$A = \textstyle\sum_{j=1}^{M} w_{ij}.$$

Proof:
Let $e_i = x_i - \bar{x}$thus$\dot{e}_i = \dot{x}_i - \dot{\bar{x}}$. Choose$V = \sum_{i=1}^{M} V_i$as a Lyapunov function for the swarm where$V_i = \frac{1}{2} e_i^T e_i$. Then

$$\dot{V} = \sum_{i=1}^{M} e_i^T \left\{-\sum_{j=1}^{M} w_{ij}\,(x_i - x_j)\left(a - \frac{r}{b + c\|x_i - x_j\|^2}\right) + \gamma_i(t) - \frac{1}{M}\sum_{j=1}^{M} \gamma_j\,(t)\right\}$$

$$= -\sum_{i=1}^{M}\sum_{j=1}^{M} e_i^T\, w_{ij}(x_i - x_j)\left(a - \frac{r}{b + c\|x_i - x_j\|^2}\right) + \textstyle\sum_{i=1}^{M} e_i^T \left(\gamma_i(t) - \frac{1}{M}\sum_{j=1}^{M} \gamma_j(t)\right),$$

$$= -a\sum_{i=1}^{M}\sum_{j=1}^{M} e_i^T\, w_{ij}(x_i - x_j) + \sum_{i=1}^{M}\sum_{j=1}^{M} e_i^T w_{ij}(x_i - x_j)\frac{r}{b + c\|x_i - x_j\|^2} + \textstyle\sum_{i=1}^{M} e_i^T \left(\gamma_i(t) - \frac{1}{M}\sum_{j=1}^{M} \gamma_j(t)\right).$$

Thus

$$\dot{V} \le -2awMV + \sum_{i=1}^{M}\sum_{j=1}^{M} e_i^T w_{ij}(x_i - x_j)\frac{r}{b + c\|x_i - x_j\|^2} + \textstyle\sum_{i=1}^{M} e_i^T \left(\gamma_i(t) - \frac{1}{M}\sum_{j=1}^{M} \gamma_j(t)\right),$$

$$\le -2awMV + \sum_{i=1}^{M}\sum_{j=1}^{M} w_{ij}\|e_i^T\|\,\|x_i - x_j\|\frac{r}{b + c\|x_i - x_j\|^2} + \textstyle\sum_{i=1}^{M} e_i^T \left(\gamma_i(t) - \frac{1}{M}\sum_{j=1}^{M} \gamma_j(t)\right).$$

Since function $\|x_i - x_j\|\frac{r}{b + c\|x_i - x_j\|^2}$ is a bounded function whosemaximum is given by$\frac{1}{2}r\sqrt{\frac{1}{bc}}$, we have

$$\dot{V} \le -2awMV + \frac{1}{2}r\sqrt{\frac{1}{bc}}\sum_{i=1}^{M}\left(\|e_i^T\|\sum_{j=1}^{M} w_{ij}\right) + \textstyle\sum_{i=1}^{M} e_i^T \left(\gamma_i(t) - \frac{1}{M}\sum_{j=1}^{M} \gamma_j(t)\right),$$

$$\le -2awMV + \frac{1}{2}r\sqrt{\frac{1}{bc}}\sum_{i=1}^{M}\left(\|e_i^T\|\sum_{j=1}^{M} w_{ij}\right) + \textstyle\sum_{i=1}^{M}\|e_i^T\|\left(\|\gamma_i(t)\| + \frac{1}{M}\left\|\sum_{j=1}^{M} \gamma_j(t)\right\|\right).$$

$$\le -\sqrt{V}\left(2awM\sqrt{V} - \sqrt{2}\left(\frac{1}{2}r\sqrt{\frac{1}{bc}}A + 2K\right)\right).$$

$$\dot{V} < 0 \text{then} V > \left(\frac{\frac{1}{2}rA\sqrt{\frac{1}{bc}} + 2K}{\sqrt{2}awM}\right)^2$$

■.

III. Numerical Simulation

In this section, some numerical simulations to illustrate model (1) are reported. Fig. 1-4 show the simulation results, where $M = 10, a = 1, b = 1, c = 0.2$and $r = 20$. Thisconstantusing the constantGaziandPassino(2003). The coupling matrix $G$is generated randomly bynonnegative integer (0-10) and is symmetric. Fig. 1 shows the numerical simulations of the trajectoriesof the members of the swarm model without disturbance. It can be seen from Fig. 1 that the swarm members aggregate towards a bounded region. Then, they continuously move together in a spiral motion.
Fig. 2 shows the trajectories of the members of the swarm model with disturbance$(\gamma_i(t) = \gamma(t))$ for every$i = 1, \ldots M$ where$\gamma(t) = exp(-8t)\sin(0.5t)$. It can be seen that thesystem shifts to the same direction as y axes but the systemis stable although there are disturbances.
Fig. 3 shows the trajectories of the the swarm center withoutdisturbance. It can be seen, that the swarm center is stationary.

Fig. 4 shows the trajectories of the the swarm center with disturbance. It can be seen, that the swarm center is stillstationary although there are disturbances.

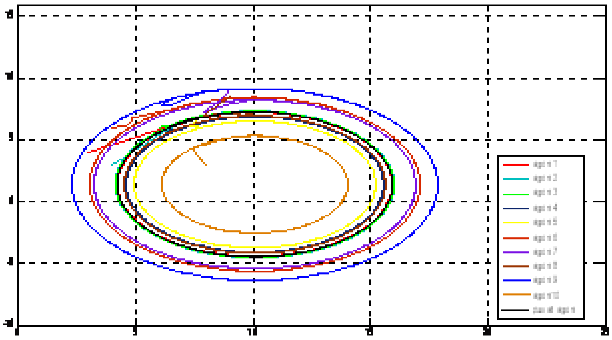

Fig. 1. The trajectories of the agent of the swarm model Withoutdisturbance. Number 1, 2, 3, etc denote agent of the swarm model.

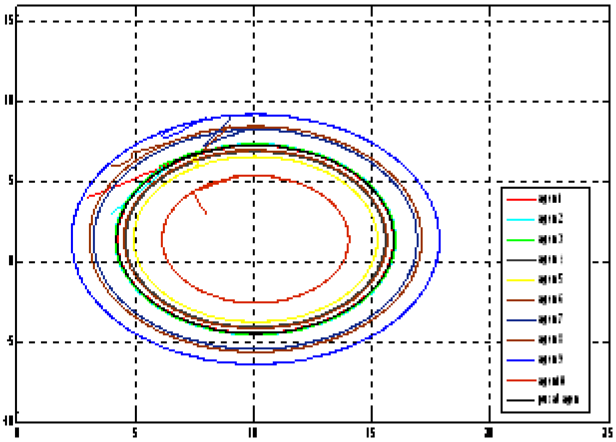

Fig. 2. The trajectories of the agent of the swarm model with disturbance. Number 1, 2, 3, etc denote agent of the swarm model.

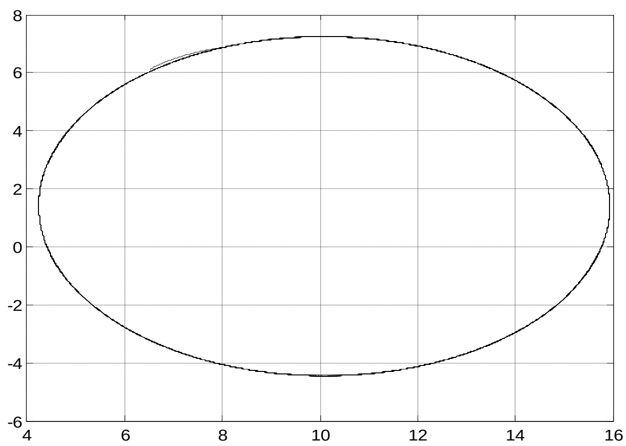

Fig. 3. The trajectories of the swarm center without disturbance.

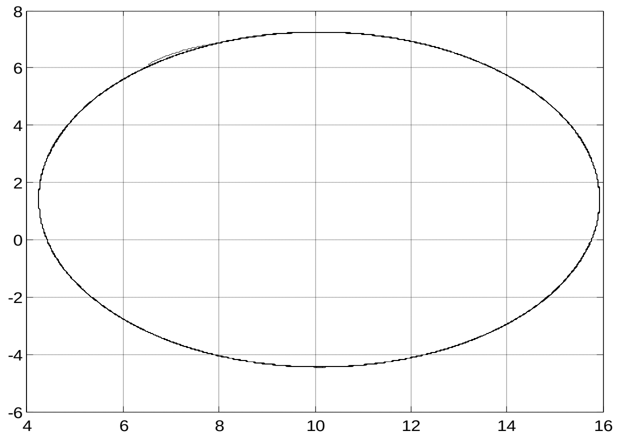

Fig. 4. The trajectories of the swarm center with disturbance

FUTURE WORKS

The analysis method for the swarming behavior of multi-agents with disturbance under certain condition on the coupling matrix have been presented in this paper. From the numerical simulations results, the swarm's center is still stationary and the swarm agents aggregate around the swarm's center. In the future works, it is interesting to apply this model in engineering field such as in flying inverted "V" formation of the airplane.

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