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Energy Independent Estimation of the Equivalent Stationary Duration of Earthquake Accelerograms

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Abstract
The equivalent stationary duration of an earthquake time history is one of the most important parameter in the characterization of a seismic record. This parameter and his intensity value are strongly related to the non-linear behavior of both a structure and its foundation soil. The importance of this time quantity comes from the fact that its estimation implies the complete identification of the part of a seismic signal having higher energy content in terms of both time length (duration) and position along the time axis with the corresponding mean intensity value. Three alternative procedures for the estimation of the equivalent stationary duration parameter are here proposed. These definitions have the common character of being independent from any global or local value of the energy released during the strong ground motion. Two of them comes directly from commonly used signal analysis relations while the other is strictly related to the fundamental definition of stationary and amplitude modulated stochastic process; all the examined definitions share the same time positioning procedure. Particular attention is also devoted to investigating the effectiveness of the different formulations with regard to the linear elastic response of a single degree of freedom oscillator. In particular the advantage of each of the examined durations is pointed out in terms of the response spectra obtained for the natural record compared to the ones acquired using just the equivalent stationary part of the record extracted employing a common type of tapering window. The numerical example is based on two natural records that can be considered as representative samples extracted from two of the most significant seismic Italian events of the last forty years.

Keywords: Significant Duration, Stochastic Process, Recorded Accelerogram, Intensity Function, Response Spectra.

Introduction

The characterization of an earthquake for purposes related to structural engineering is essentially based on the definition of some quantities able to express in concise way the level of ground shaking, its frequency content and the duration of the shock in terms of structural response. On the other hand, seen from the point of view of a resistant structure, the equivalent duration of a strong ground motion refers to that part of the load history stressing the structure beyond some resistance level.

Since the beginning of earthquake engineering the main attention was focused on the definition and influence on a structural system of the intensity, the energy content and the spectral content combined with the influence of the magnitude, the distance from the source and soil conditions[1,2].

The determination of the part of a natural record outside of which the contribution of the seismic response of the structure can be reasonably considered to be of small entity (namely the significant or equivalent duration of the motion), was instead rather neglected compared to other characteristic quantities. Nevertheless different works [3-8] highlight the importance of this parameter especially for strongly non-linear problems [9] like liquefaction of saturated sands, overcoming of the elastic limit and resistance, damage level of structures and soil, linear systems with very low damping, generation of simulated accelerograms. Recent works [10, 11] give a new and improved form to the meaning of stationary duration of a strong ground motion pointing out some aspect about its frequency dependent evaluation via response analysis of S.D.o.F. structures. In scientific literature, almost all the different definitions proposed in past are more or less subjective: in most cases some threshold levels of different characterizing parameters are assumed from scratch for the inference of the claimed equivalent duration. Almost none of those methods can be considered as independent from a subjective statement and applicable to any earthquake record. Without any assumption of completeness, among the most known are cited the definitions given by Bolt [2] based on a threshold value of the acceleration record, Trifunac-Brady [3] similarly referring to a limiting value of the intensity function, Vanmarke and Lai derived from the theory of stochastic processes [6] and others [4-5,7-8]. All these definition are based on some predefined parametric value significantly affecting their outcome. In any case it is difficult to believe in the existence of a unique definition of equivalent stationary duration valid for any kind of structure, soil foundation and geology, and equally well applicable to all records and for any engineering problem, nevertheless it can be useful and effective to have a definition valid for the widest possible spectrum of applications.

This work take in exam three different methods for the estimation of the significant duration introduced by the author [12-14]. Two are essentially based to the definition of bandwidth used in signal analysis [15, 16], the other is strictly related to the fundamental definition of stationary and amplitude modulated stochastic process [12]. Although the examined idealizations are clearly independent from each other, the same time positioning procedure [12] is effectively used for all the proposals. Particular attention is devoted to investigating the pros and cons of the formulations, even considering that their use is here limited to the linear response analysis.

A comparison between the three different definitions is performed on the basis of the response spectra of the natural record and the ones calculated using the equivalent stationary process obtained by windowing procedure of the reference time histories on the basis of the considered equivalent duration. In particular the most significant portion of the record, the equivalent stationary process, is acquired as a segment of appropriate length and position extracted using a suitable window function of common use in signal analysis. The quality of the examined definitions is then compared employing two different records considered as representative samples of two of the most significant seismic Italian events of the last fifty years: Ancona (1972) and Friuli (1976) [17].

Basic Relations

The meaning of the term "significant duration" of a seismic record refers to that part of the accelerometric signal $r(t)$ in time t, with an higher shaking degree with respect to the remaining parts. It is also strictly connected to the concept of "duration of the locally stationary phase" of the shaking. The locally stationary portion can be in first approximation defined as the portion of a time history that, when recognized and extracted from the remaining part of the signal, behaves like a segment of finite duration d_0 of a Gaussian stationary random process with zero mean $s(t)$. In that framework an accelerometric record $r(t)$ of a seismic event can be idealized as an amplitude modulated stationary process having an equivalent stationary duration of length d_0:

$$r(t)=i(t)\ s(t) \quad , \qquad t=0, d \tag{1}$$

with d total duration of the signal and $i(t)$ the intensity function, a unit peak function giving the characteristic three-phase-shape observed in any seismic record with: 1) the starting of the strong ground motion or rising part of duration t_r, 2) a quasi-stationary portion of duration d_0, 3) a final part or decaying portion of the accelerogram, staring at $t_d=t_r+d_0$ till the end of the time history at $t=d$.

In that context the equivalent stationary duration also play a fundamental role in the definition of the power spectral density function $G_r(f)$ of $r(t)$ in the frequency domain f:

$$G_r(f) = A_r^2(f)/d_0 \tag{2}$$

Being $A_r(f)$ the one-sided amplitude spectrum of the under-lying amplitude modulated stationary stochastic process. Beside the power spectral density function $G_s(f)$ of $s(t)$ is similarly defined:

$$G_s(f) = A_s^2(f)/d \tag{3}$$

being $A_s(f)$ the one-sided amplitude spectrum of the stationary stochastic process $s(t)$.

Moreover equation (1) can be further clarified taking into account of the common definition of the intensity function $i(t)$ as the envelope function $e(t)$ of the record scaled to unit:

$$i(t) = e(t)/\max(e(t)) \tag{4}$$

with $\max(e(t))$ the peak value of the envelope function:

$$e(t) = \sqrt{r^2(t) + r_H^2(t)} \tag{5}$$

being $r_H(t)$ the Hilbert transform of the record [16].

While moving in the given context, the main target of the present paper is to examine different definitions of the "Equivalent Stationary Duration Parameter", from here on named also in brief ESDP, from the point of view of the signal, regardless of its intensity and peak value.

ESDP from amplitude modulation definition

The stationary random process $s(t)$ in equation (1) can be expressed using the very definition of "amplitude modulated process": it is a stationary signal amplitude modulated by a basic intensity function $i(t)$, the "unit boxcar function" $\Pi_{0,d}(t)$ mathematically defined as [18, 19]:

$$\Pi_{0,d}(t) = H_0(t) - H_d(t) \tag{6}$$

being $H_c(t) \equiv H(t-c)$ and $H(t)$ the Heaviside step function.

It is also immediate to state that the equivalent stationary duration d_0 of a time limited stationary process of duration d is coincident with the total duration of the signal itself or in mathematical notation:

$$\int_0^d \Pi_{0,d}(t)\, dt = d \equiv d_0 \tag{7}$$

By simple extension of equation (7) to the generic intensity function $i(t)$ substituting $\Pi_{0,d}(t)$, the following holds:

$$\int_0^d i(t)\, dt \equiv d_0 \tag{8}$$

that gives a clean interpretation of the meaning of the ESDP. The just formalized definition is only dependent on the shape of the intensity function but not sensitive to the peak value or to the total energy released by the accelerometric record $r(t)$.

ESDP from bandwidth definition

Considering the shape of the intensity function as an uncommon distribution of a random variable, the expectations of $i(t)$ can be of help in the evaluation of some basic parameters usually adopted as concise descriptors of the shape of the function. In particular recalling the definition of moment m_j of order j as:

$$m_j = \int_0^d t^j \, i(t) \, dt \quad , \qquad j = 0,1,2,.... \tag{9}$$

then the most used parameters for the shape characterization of the $i(t)$ distribution are the first three m_0, m_1 and m_3 from which the following quantities can be obtained :

$$c_t = \frac{m_1}{m_0} = \frac{\int_0^d t \, i(t) \, dt}{\int_0^d i(t) \, dt} \tag{10}$$

is the time coordinate of the centroid of $i(t)$ where the area is more concentrated. Similarly:

$$t_c = \sqrt{m_2/m_0} \tag{11}$$

is a time parameter usually named central time t_c of the intensity function. Moreover the following dimensionless shape factor can be defined:

$$q_t = \left(1 - \frac{m_1^2}{m_0 m_2}\right)^{1/2} , \qquad 0 \leq q \leq 1 \tag{12}$$

Small q values imply a narrow function: $q=0$ is an impulse at c_t, while $q=1$ means a flat constant value function. From the shape factor q the 'bandwidth measure' $B_w=q_t \; t_c \equiv d_0$ is obtained and used in the followings as one of the three statements here adopted for the ESDP evaluation.

A different bandwidth definition can also be introduced considering now the intensity function $i(t)$ as a signal delivering smooth transitions from the rising part to the peak and to the fall off, the following definition of "effective bandwidth" B_{we} holds [15]:

$$B_{we} = \left(\int_0^d i(t) \, dt\right)^2 \Big/ \int_0^d i^2(t) \, dt \equiv d_0 \tag{13}$$

Time position of the ESDP

The previously given definitions of the equivalent stationary duration come from different interpretation of the used modulating function within the assumption made in equation (1) for the throughout description of the generic seismic signal $r(t)$ idealized as an amplitude modulated stationary stochastic process with equivalent stationary duration d_0, but no hints were given to locate along the time axis the defined quantity. To accomplish this task the procedure previously proposed by the author in [14] is here adopted with a plain variation. In particular the cumulate $C_i(t)$ of $i(t)$ is at first introduced:

$$C_i(t) = \int_0^t i(\tau) \, d\tau \quad , \qquad t = [0, d] \tag{14}$$

This function can be interpreted as a measure of the energy released by the amplitude modulated signal at current time t. Being the ESDP also presented as the portion of signal with the highest energy variation, the position of d_0 is obtained by satisfying the following maximizing condition posed on the given energy function $C_i(t)$:

$$t_r = t(\max[C_i(t + d_0) - C_i(t)]) \; , t = [0, d - d_0] \tag{15}$$

Therefore for the generic amplitude modulated signal $r(t)$, t_r gives the beginning of quasi-stationary portion of duration d_0 with the record being ideally divided, as previously described, in three parts: the rising interval $[0, t_r]$, the stationary portion $[t_r, t_d]$ of duration $d_0 = t_d - t_r$, and the decaying part $[t_d, d]$.

Numerical example

The numerical application is here set up with the intention to show the effectiveness of the different definitions of the equivalent stationary duration parameter previously defined. In particular the comparison is here reported in terms of variations in the peak response of a linear S.D.o.F. oscillator with changing natural frequency, when forced by the selected seismic record of entire length d. In fact the response spectra of the natural time-history, at different damping ratios, are compared with the parallel ones obtained using as seismic inputs only the quasi-stationary portions of the record as defined by the different ESDP here examined.

The seismic records selected for the numerical example belong to the public Italian seismic database [17]. In particular the first record is the NS component of the acceleration signal reported during the Ancona event at the station of Ancona-Palombina in date 21.06.1972, starting time h.15:06:44, sampling period 0.01s, record length 14.08s and instrumentally corrected. The second record used is the NS component of the acceleration signal reported during the Friuli event at the station of Forgaria-Cornino in date 15.09.1976, starting time h.09:21:18, sampling period: 0.01s, record length: 24.59s and instrumentally corrected.

The time characterization procedure can be concisely described by performing the following steps:

- estimate of the intensity function $i(t)$ by Eq.(5) and Eq.(4)
- evaluation of d_0 by Eq.(8)
- evaluation of B_w using Eq.(11), Eq.(12) and $B_w=q_t \, t_c$
- estimate of B_{we} employing Eq.(13)
- time positioning of d_0, B_w and B_{we}, building at first the cumulate of the intensity function $C_i(t)$ in Eq.(14) and then applying the maximizing condition in Eq.(15).

The two reference records and the results obtained by implementation of the described time characterization procedure are summarized in Fig.1 (AN-Palombina) and Fig.2 (Forgaria).

The ESDP gives an indication of the most significant portion of the signal that can be easily extracted by a suitable windowing procedure. In particular the significant parts, identified by d_0, B_w and B_{we}, of the two used records are isolated employing a Tukey window [20], also known as "tapered cosine window". The resulting window-tapered records $r_t(t)$ are reported in Fig.3 (AN-Palombina) and Fig.4 (Forgaria). As previously stated the comparison between the three different ESDPs are driven by the response spectra of the corresponding tapered records evaluated for three reference values of the damping parameter: 0%, 2% and 5% compared with the source record.

The results are plotted in Fig.5 (AN-Palombina) and Fig.6 (Forgaria) in terms of the pseudo-velocity response spectra *V(f)* estimated in the range *f*=[0.04; 25] Hz.

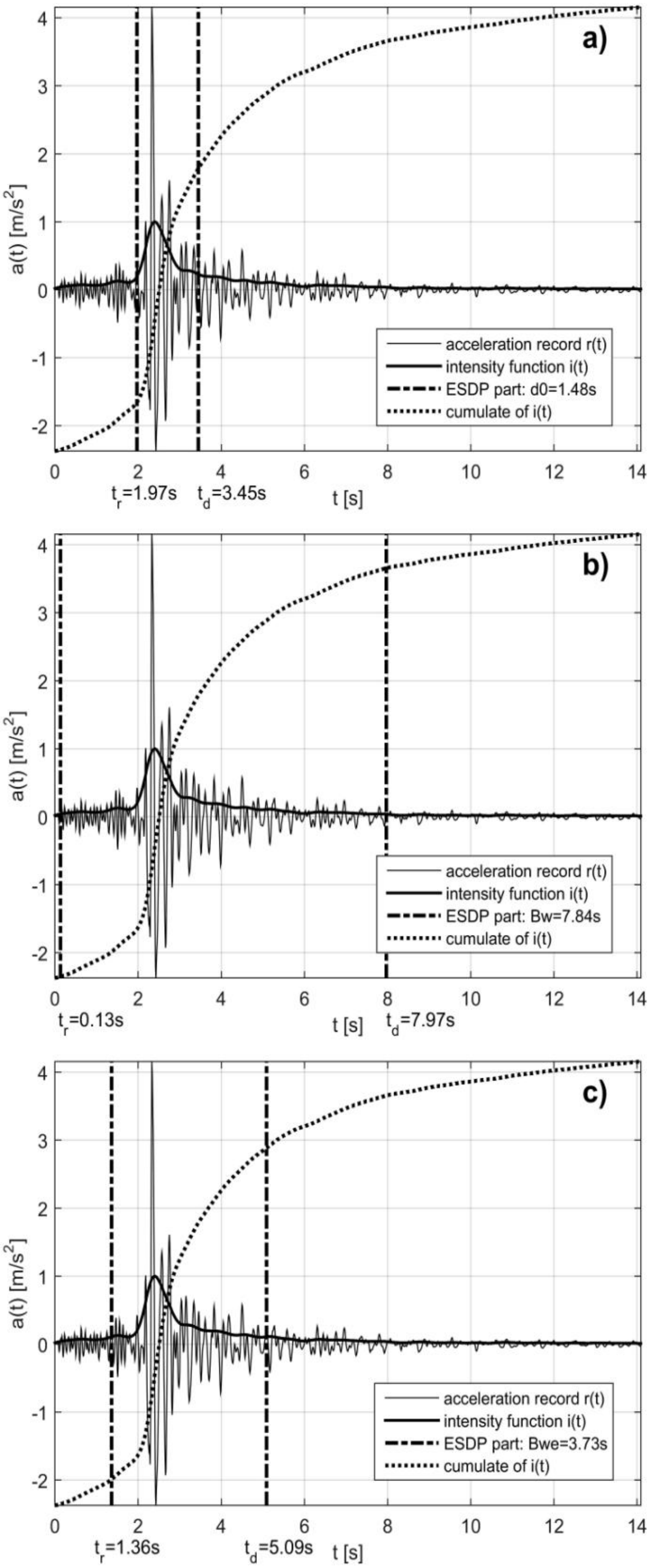

Fig. 1 - The Ancona-Palombina record *r(t)*, its intensity function *i(t)* with the cumulate $C_i(t)$ and the location of the ESDP evaluated with the definitions here considered.

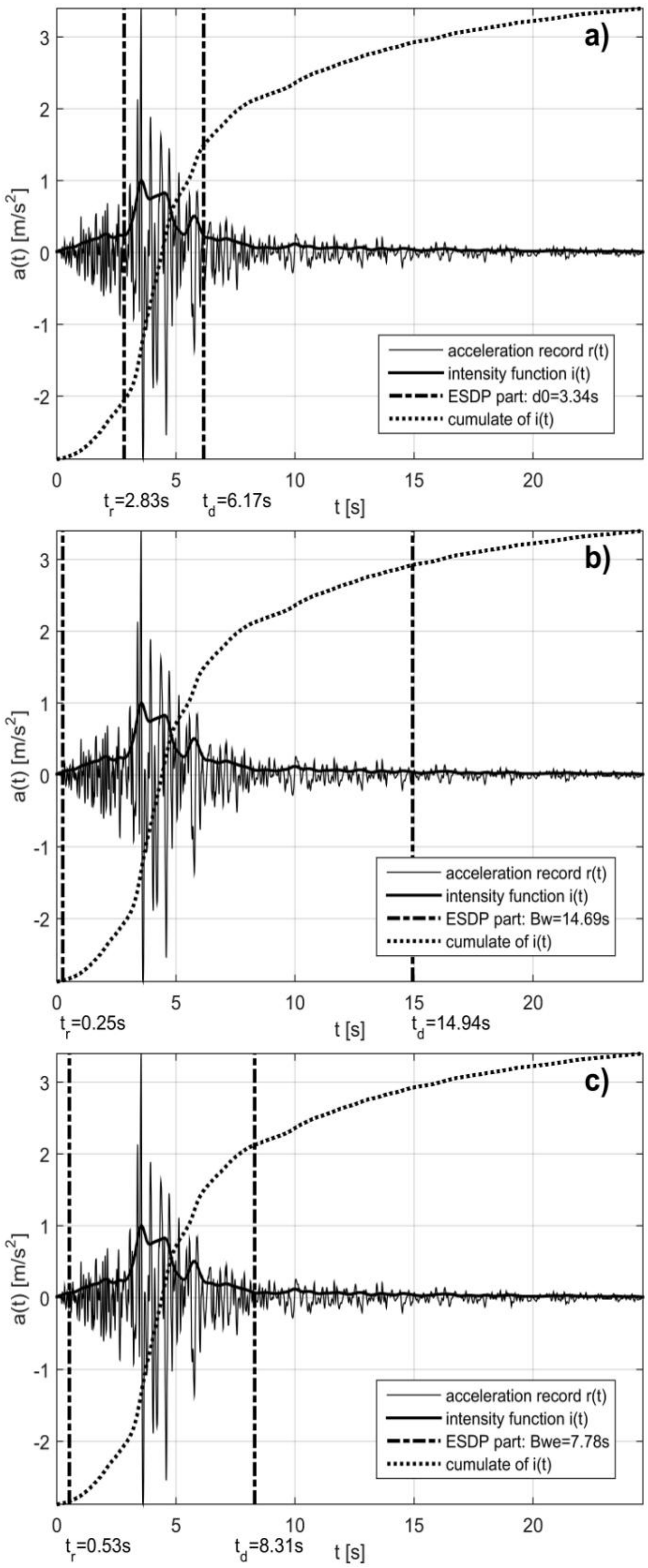

Fig. 2 - The Forgaria-Cornino record *r(t)*, its intensity function *i(t)* with the cumulate $C_i(t)$ and the location of the ESDP evaluated with the definitions here considered.

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Conclusions

The three different definition of equivalent stationary duration studied in the present paper give very different duration values: d_0 gives the smallest values, B_w by far the largest while B_{we} stays in the middle. Therefore the resulting window-tapered records $r_t(t)$ are extremely different for both the time-length and even more, the energy content. Nevertheless the results obtained in terms of the pseudo-velocity spectral ordinates for such different signal show very small variations. In particular both the records tapered accordingly to d_0 exhibit few relevant gaps with respect to the full record. These gaps are significant only in case of null damping ratio and exclusively for second order peaks located in the low and high frequency range. A surprising result for tapered signals of so limited duration.

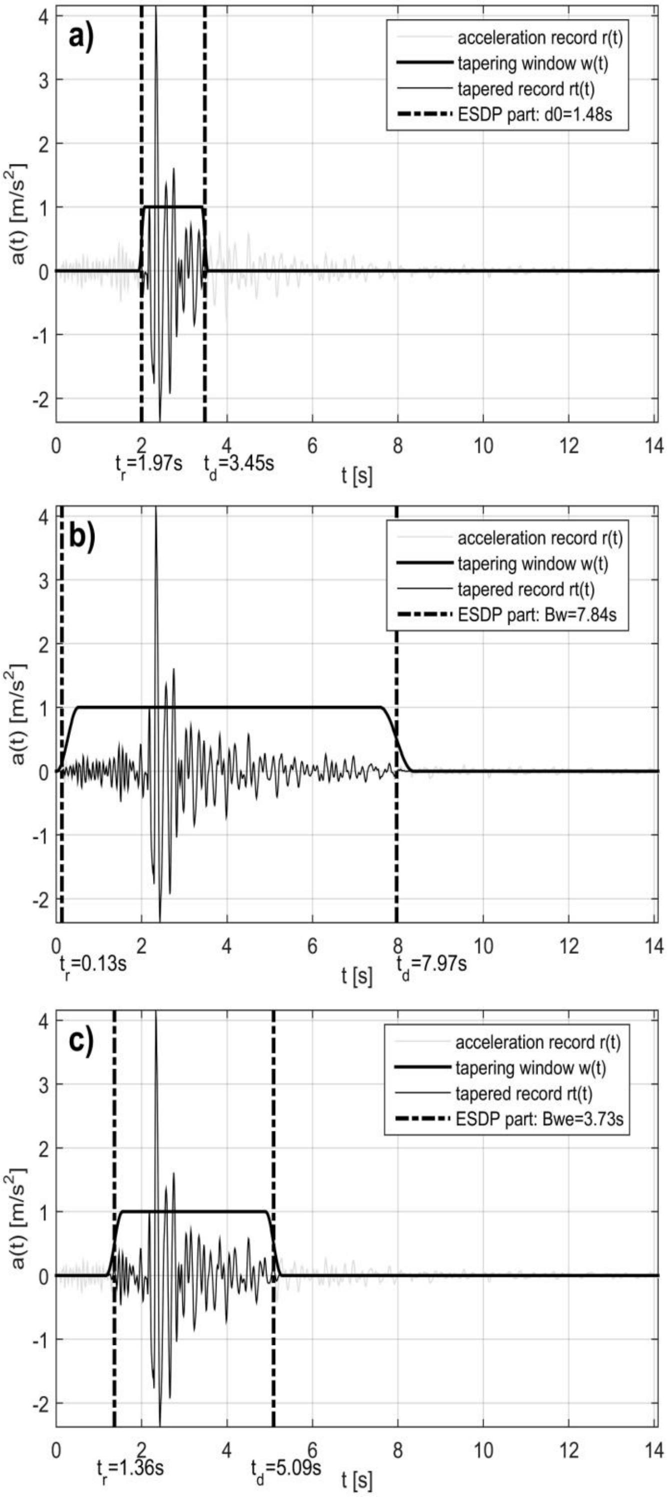

Fig. 3 - Ancona-Palombina record *r(t)*, the window *w(t)*, the resulting tapered record $r_t(t)$ with the related ESDP.

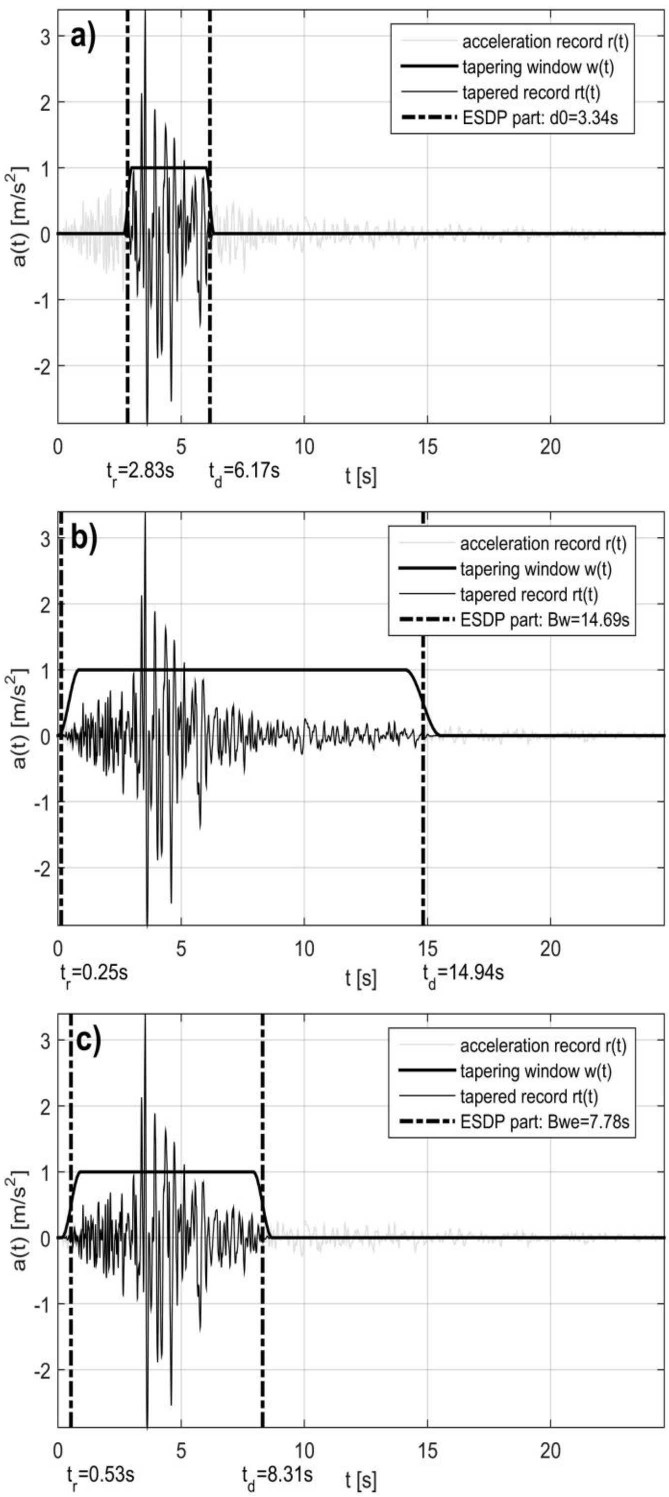

Fig. 4 - Forgaria-Cornino record *r(t)*, the window *w(t)*, the resulting tapered record $r_t(t)$ with the related ESDP.

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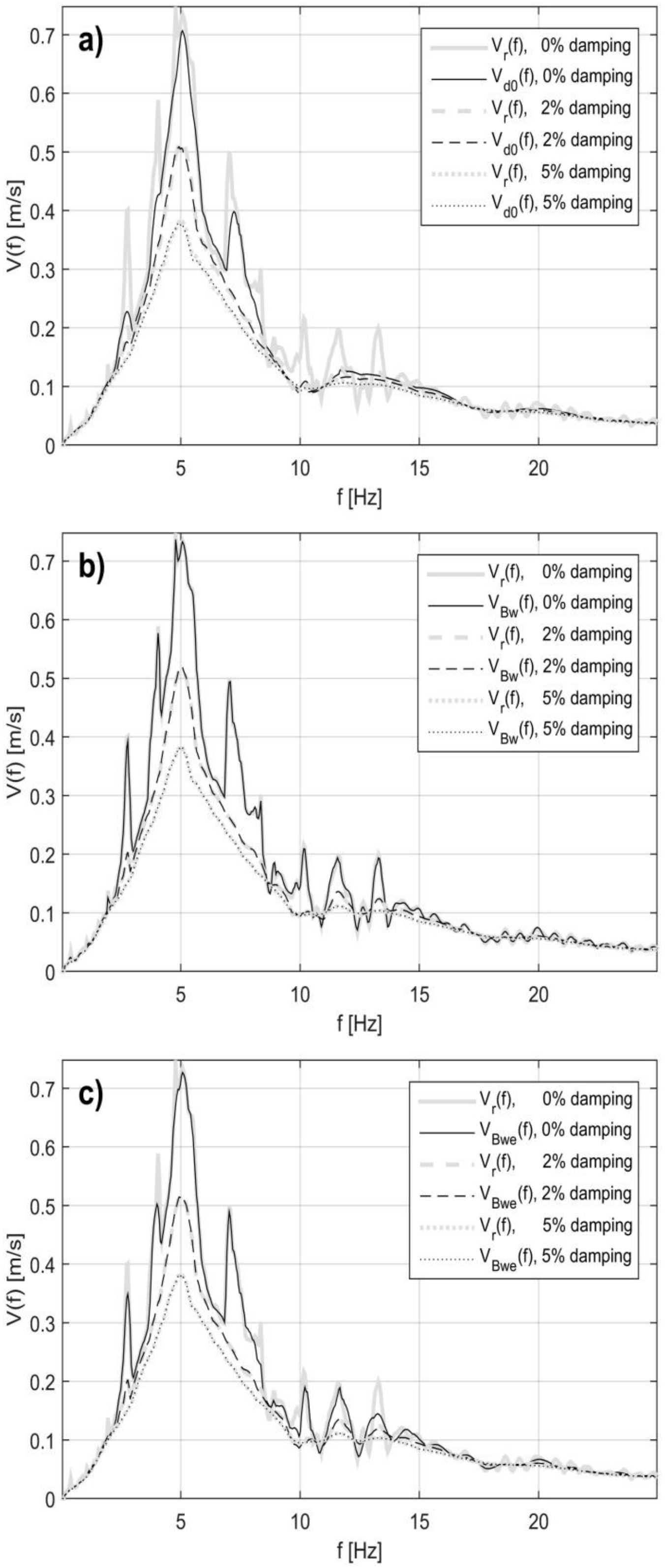

Fig. 5 - The Ancona-Palombina pseudo-velocity spectra at different damping ratio compared with the correspondent spectra of the tapered records $r_t(t)$. The gaps in a), for the shortest equivalent stationary duration d_0, are significant only in case of null damping ratio and only for second order peaks located in the low and high frequency range.

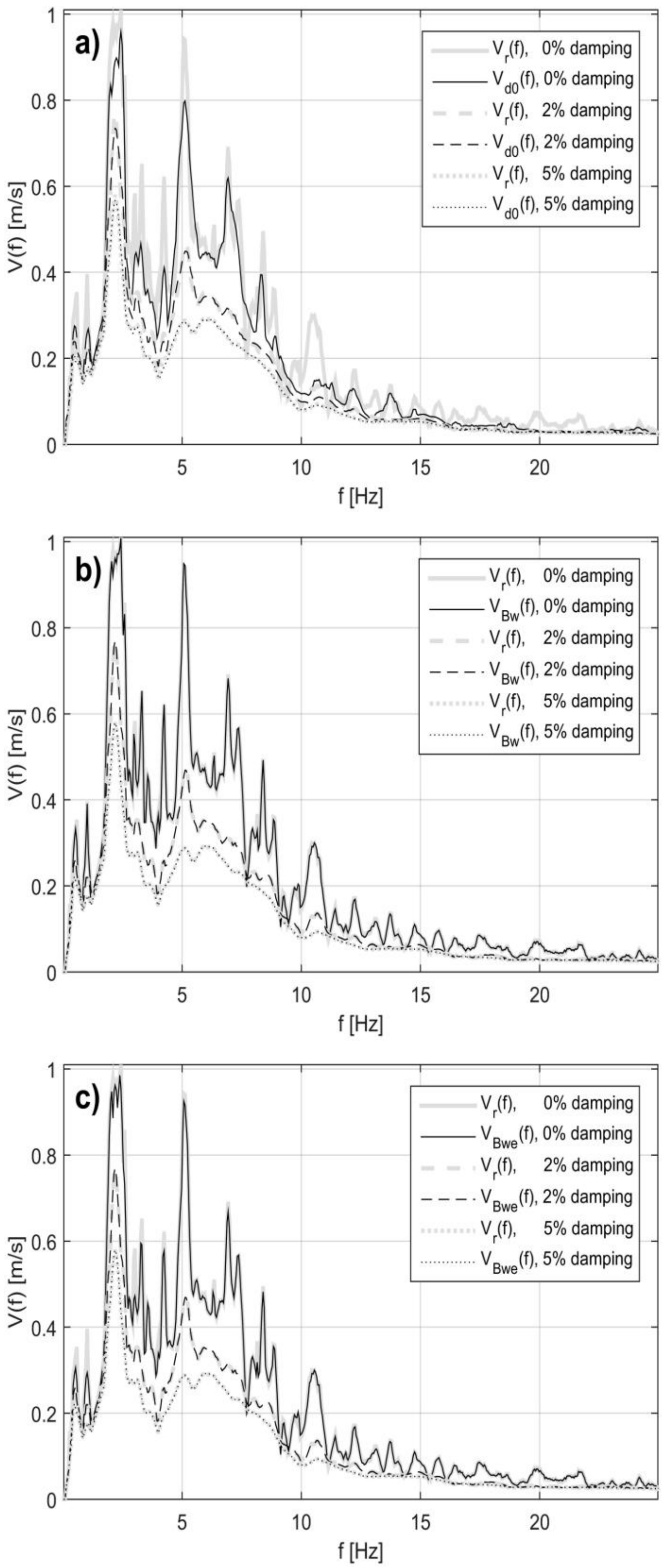

Fig. 6 - The Forgaria-Cornino pseudo-velocity spectra at different damping ratio compared with the correspondent spectra of the tapered records $r_t(t)$. The gaps in a), for the shortest equivalent stationary duration d_0, are significant only in case of null damping ratio and only for second order peaks located in the low and high frequency range.

References

1. G.W. Housner, "Intensity of Earthquake ground shaking near the causative fault", Proc. 3rd WCEE,, New Zealand, 1965.
2. B. A. Bolt, "Duration of Strong Ground Motion", Proc. 5th WCEE, pp.1304-1313, Rome, 1974.
3. M. D., Trifunac, A. C. Brady, "A study of Duration of Strong Earthquake Ground Motion", BSSA, Vol.65, pp. 581-626, June 1975.
4. R. Dobry, I.M. Idriss, C. Y. Chang, E. Ng, "Influence of Magnitude, site conditions and distance on significant duration of earthquakes", Proc. 6th WCEE, New Dehli, India, 1977.
5. M. W. McCann Jr, Haresh C. Shah, "Determining Strong-Motion Duration of Earthquakes", BSSA Vol.69, pp.1253-1265, August 1979.
6. E. H. Vanmarcke, S. P. Lai, "Strong-Motion Duration and RMS Amplitude of Earthquake Records", BSSA, Vol.70, pp. 1293-1307, August 1980.
7. F. Sabetta, "Analisi di quattro definizioni di durata applicate ad accelerogrammi relativi a terremoti italia", Rapporto Tecnico ENEA, ENEA-RT/AMB (83) 4, 1983.
8. A.Kameda, "Stochastic Proces Model of Strong Earthquake Motions of Inelastic Structural Response", Proc. of the U.S.-SE. Asia Symp. on Eng. for Natural Hazards Protect., Manila, 1977.
9. M.D. Trifunac, "Earthquake response spectra for performance based design-A critical review", Soil Dynamics and Earthquake Engineering Vol.37, pp.73-83, 2012.
10. S.C. Potnis, R.S. Desai, I.D. Gupta, "Estimation of strong ground motion stationary duration from stochastic structural response analysis", Proc. 14th WCEE, Beijing, China 2008.
11. I.M. Taflampas et al., "A new definition of strong motion duration related parameters affecting the response of medium-long period structures", 14th WCEE, Beijing, China 2008.
12. F.Carli, "Parameter estimation of evolutionary stochastic model for dynamic loads", IUTAM Symp. on Identification of Mechanical Systems, Wuppertal, Germany 1993.
13. F. Carli, C. Carino, "An evolutionary approach for the synthesis of spectrum compatible and site-dependent accelerograms", Proc. 3rd Int. Conf. on Soft Comp. Tech. in Civil, Structural & Envionmental Eng, Civil-Comp Press, 2013.
14. F. Carli, "Generazione di accelerogrammi spettrocompatibili. Atti "L'Ingegneria Sismica in Italia", Taormina, Italia 1997.
15. D.E. Newland, "An Introduction to random vibration, spectral and wavelet analysis", Longman, NewYork, 1983).
16. J.S. Bendat, A.G. Piersol, "Random Data: Analyses and Measurement Procedures", Whiley, New York, 1971.
17. L. Luzi, S. Hailemikael, D. Bindi D, F. Pacor, F. Mele, F. Sabetta,"ITACA (ITalian ACcelerometric Archive): A Web Portal for the Dissemination of Italian Strong-motion Data", Seismological Research Letters, 79(5), pp.716-722, 2008.
18. D. von Seggern, "CRC Standard Curves and Surfaces", CRC Press, Boca Raton, FL, 1993.
19. Weisstein, Eric W. "Boxcar Function." From MathWorld A Wolfram Web Resource.
20. J.W. Tukey, "An introduction to the calculations of numerical spectrum analysis", Spectral Analysis of Time Series, pp.25-46, 1967.