

Reduction of PAPR in SC-FDMA System using Pulse Shaping Technique

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Abstract

Single carrier frequency division multiple access (SC-FDMA) technique has been adopted by long term evolution (LTE) standard for its uplink transmission. It has low peak to average power ratio (PAPR) than that of conventional orthogonal frequency division multiple access (OFDMA) technique. In this paper effect of pulse shaping on the PAPR performance of SC-FDMA system using raised-cosine filter have been investigated. Compared to other pulse shaping methods raised-cosine filter has low computational complexity with comparative PAPR performance. Apart from reduction in PAPR Pulse shaping is also used to reduce the bandwidth of the transmitted signal better suited for the communication channel and reduce the effect of intersymbol interference (ISI). The result obtained for interleaved frequency division multiple access (IFDMA) and localized frequency division multiple access (LFDMA) techniques show that the PAPR value decreases with increase in roll-off factor. The variation in PAPR is more in IFDMA than that of LFDMA. The reduction in PAPR value with 1024 subcarriers and 64 QAM modulated IFDMA signal at 0.01 value of CCDF is 2.9 dB with change in roll-off factor from 0.0 to 0.9. Whereas, it is only 1.0 dB for the case of LFDMA signal. Similar trends follow for 4 QAM and 16 QAM modulation formats.

Keywords: Peak-to-average power ratio, pulse shaping, single carrier frequency division multiple access, raised cosine filter, IFDMA, DFDMA, LFDMA.

Introduction

Single carrier frequency division multiple access (SC-FDMA) technique has been proposed to be used in the long term evolution (LTE) for its uplink transmission owing to low PAPR value [1-6]. In a conventional OFDMA system, subcarriers are partitioned and assigned to multiple mobile users. But in the SC-FDMA technique which is used for the uplink transmission, each user uses a subset of subcarriers to transmit its own data. The subcarriers which are not used for the data transmission are filled with zeros [7-9].

SC-FDMA is a linearly precoded OFDMA and has lower PAPR than OFDMA due to precoding [10-12]. There are two different approaches of assigning subcarriers among users, Distributed FDMA (DFDMA) and Localized FDMA (LFDMA) [13]. In DFDMA, N DFT outputs are distributed over the entire band of total M subcarriers with zeros filled in

N-M unused subcarriers. But in the LFDMA, DFT outputs are allocated to N consecutive subcarriers in M subcarriers and remaining are filled with zeros. If distribution of DFT outputs in DFDMA is done uniformly with equal distance then it is referred to as interleaved FDMA (IFDMA). For the LFDMA the value of PAPR is larger than DFDMA. Among the three different techniques IFDMA shows the lowest PAPR value for the given modulation format. If the PAPR is large, it requires a high back off of transmit power amplifier and leads to a low power efficiency [14]. The low power efficiency problem is undesirable mainly for the uplink transmission, owing to the low-cost and low power consumption requirements on the user mobile terminal [15-18].

In order to reduce the PAPR of SC-FDMA signals, several studies have been reported in the literature, such as space-frequency block coding (SFBC) for multiple transmit antennas, flexible PAPR reduction scheme, Partial transmit sequence (PTS) and selected mapping (SLM), etc. [19-20]. But, these techniques need high computational complexity apart from requirement of transmission of the side information [21]. Pulse shaping (PS) is used to band limit the transmitted signal in SC-FDMA system. Compared to other PAPR reduction techniques, PS is an effective and simple method. But, it may increase the PAPR of the transmitted signals. As a result, there is a trade-off between PAPR performance and bandwidth efficiency in PS based SC-FDMA systems. Different PAPR reduction technique using PS based SC-FDMA have been reported in the literature such as, using a piece-wise linear Nyquist filter [22], parametric linear combination pulses [23-24], improved Nyquist filter [25-27] and two-parameter Nyquist pulses with better performance [28]. In all these methods reported the reduction in PAPR has been obtained at the cost of increased computational complexity. But the raised cosine pulse shaping filter investigated in this paper has low computational complexity with comparable PAPR performance.

SC-FDMA Transceiver

SC-FDMA spread system consists of a data source, encoder, and baseband QAM modulation, serial to parallel convertor, N point DFT spreading, N point to M point sub carrier mapping, M point IDFT despreading, reconvert parallel data into serial form, adding cyclic prefix, using digital to analog convertor and finally RF modulation for converting baseband signal to passband signal before transmitting through the

channel [12]. In the receiver, reverse process of transmission is performed. Block diagram of SC-FDMA is shown in figure 1. If DFT of the same size as that of IDFT is used for spreading code then, the OFDMA system becomes equivalent to the single carrier FDMA (SC-FDMA) system because the DFT and IDFT operations virtually cancel each other. Then the transmit signal will have the same PAPR as that of a single-carrier system.

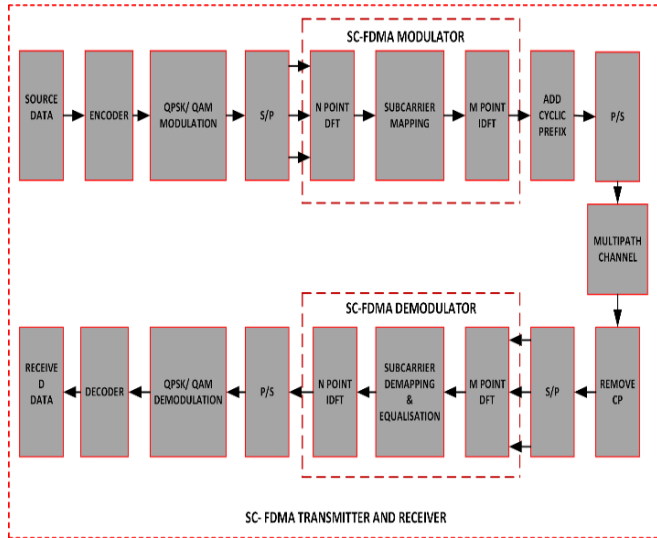


Fig.1: Block diagram of SC-FDMA system

Pulse Shaping With Nyquist Filter

The response of the baseband PAM transmission can be written as in equation (1)

$$p(t) = \sum_{n=-\infty}^{\infty} a_n h(t - nT) \quad (1)$$

Where T is the symbol period, a_n is the transmitted symbol, $h(t)$ is the impulse response of the filter and $p(t)$ is output response. The impulse response of the filter should be such that it should occupy minimum bandwidth and should have zero inter-symbol-interference.

In order to recover symbols from $p(t)$, sampling of the waveform has to be done at the multiples of symbol interval T . The waveform sampled at the interval T can be written as in equation (2).

$$p(kT) = \sum_{n=-\infty}^{\infty} a_n h(kT - nT) \quad (2)$$

Equation (2) can be broken down in two parts as in equation (3).

$$p(kT) = h(0) a_n + \sum_{n \neq k} a_n h(kT - nT) \quad (3)$$

To have zero inter symbol interference the second term in the equation (3) should be zero. According to Nyquist first criteria for distortionless transmission, the impulse response, $h(t)$ of the filter is given by equation (4).

$$h(nT) = \begin{cases} 1; & n = 0 \\ 0; & n = \pm 1, \pm 2, \dots \end{cases} \quad (4)$$

Where, T is the symbol period. The Nyquist pulse in time domain is given by equation (5).

$$P(t) = \text{sinc}\left(\frac{t}{T}\right) \frac{\cos\left(\frac{\pi t}{2T}\right)}{1 - 4\alpha^2 \frac{t^2}{T^2}} \quad (5)$$

Where, $\text{sinc}\left(\frac{t}{T}\right)$ is sinc function defined by equation (6).

$$\text{sinc}(x) \cong \frac{\sin(\pi x)}{(\pi x)} \quad (6)$$

The frequency domain representation, $H(f)$ of equation (5) is given by the Fourier transform of $h(t)$ as in equation (7).

$$\frac{1}{T} \sum_{k=-\infty}^{\infty} H\left(f + \frac{k}{T}\right) = 1 \quad (7)$$

The excess bandwidth of Nyquist inter symbol interference (ISI) free pulse is determined by the roll off factor, α which has value between 0 and 1 ($0 \leq \alpha \leq 1$).

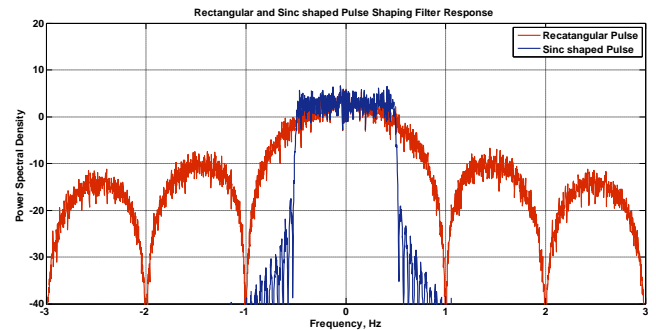


Fig.2: Response of rectangular and sinc shaped pulse shaping filter

Figure (2), indicates the response of rectangular and sinc shaped pulse shaping filter in frequency domain. From figure it can be observed that the sinc shaped filter does not result in a perfectly bandlimited spectrum in frequency domain. The bandwidth increases with increase in value of roll-off factor but the wave shape is smoother in time domain. As we know that there are more distortions at the edge of the rectangular pulses, the sinc shaped pulse promises distortionless transmission.

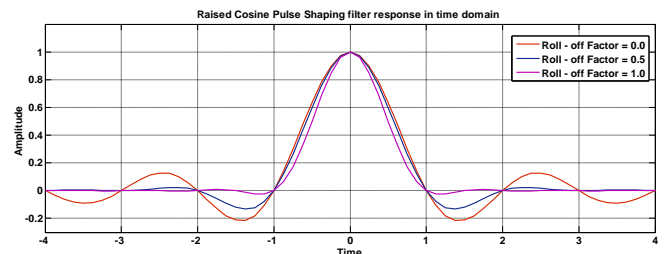


Fig.3: Raised Cosine Pulse Shaping filter response in time domain

The time domain and frequency domain responses of the sinc shaped pulse are shown in figure 3 and 4 respectively with different roll off factor.

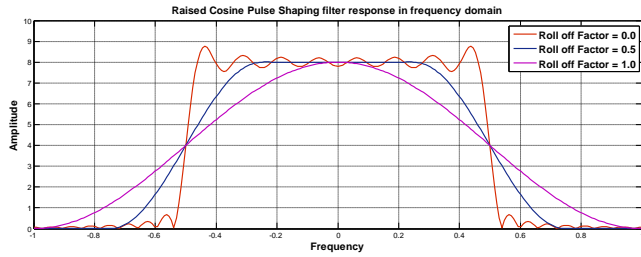


Fig.4: Raised Cosine Pulse Shaping filter response in frequency domain

It is observed from figure 3 that as the roll-off factor increases the amplitude of sidelobes (tails) decreases which is one of the important reasons for decrease in the value of peak to average power ratio of the transmitted passband signal. Further, it is noted that slower is the decay rate of tails, smaller is the value of PAPR. But the decrease in PAPR value is obtained at the cost of increase in the bandwidth when the value of roll-off factor is increased as shown in figure (4).

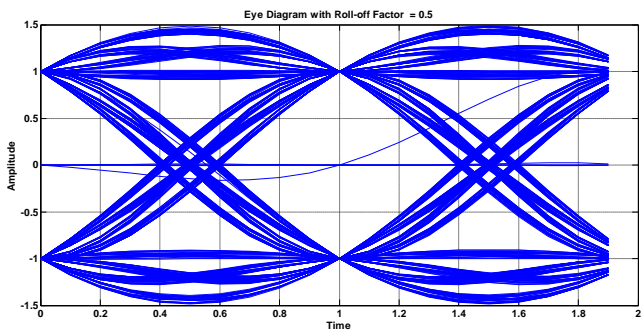


Fig.5: Eye diagram with roll-off factor = 0.5

Figure 5 and 6 shows the eye diagram of the raised cosine filter with roll-off factor 0.5 and 1.0 respectively. It is observed that the eye is more open with roll-off factor 1.0 than the eye with the roll-off factor 0.5.

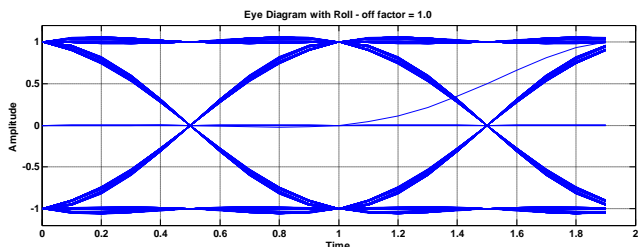


Fig. 6: Eye diagram with roll-off factor = 1.0

This is because of the fact that with 1.0 roll-off factor tails of the raised cosine filters decays at slower rate and has smaller

amplitude of sidelobes than that with 0.5 roll-off factor. It can be deduced from figure 6 that raised cosine filter with roll-off factor 1.0 will have less effect of ISI than that with 0.5.

PAPR of SC-FDMA Signal using Pulse Shaping Filter

Pulse shaping is a simple and effective method of PAPR reduction of SC-FDMA signals by weighting the outputs of the discrete Fourier transform (DFT) precoder with weighting window functions which has low sidelobe time-domain response. A linear filtering operation known as pulse shaping, which is used to reduce the out-of-band signal energy, is performed in transmitter through convolution between the modulated subcarriers and the filters impulse response.

In this paper raised-cosine pulse shaping filter has been used to reduce the Peak to Average Power Ratio (PAPR) value of single carrier frequency division multiple access (SC-FDMA) system. PAPR is a measure used to calculate the fluctuations in the envelope of signals. The average power and peak power for a given sample $\{x_m\}$ of an OFDM system is given by equation (8) and (9) respectively.

$$P_{av} = \frac{1}{F_s} \sum_{n=0}^{F_s-1} x_m^2 \quad (8)$$

$$P_{peak} = \max_m \{x_m^2\} \quad (9)$$

It is defined as the ratio of maximum (peak) power to average power of the signal. Complementary cumulative distribution function (CCDF) is used to find out the probability that the PAPR (crest factor) exceeds a particular value as given in equation (10).

$$\begin{aligned} F_{z_{max}}(z) &= P(z_{max} > z) = 1 - P(z_{max} \leq z) \\ &= 1 - F_{z_{max}}(z) = 1 - (1 - e^{-z^2})^N \end{aligned} \quad (10)$$

The raised-cosine Nyquist pulse with roll off factor, α is given by its even frequency spectrum and is defined by equation (11).

$$S(f) = \begin{cases} T; & 0 \leq f < B(1 - \alpha) \\ \frac{T}{2} \{1 + \cos(\frac{\pi}{2B\alpha}(f - B(1 - \alpha)))\}; & B(1 - \alpha) \leq f \leq B(1 + \alpha) \\ 0; & B(1 + \alpha) < f \end{cases} \quad (11)$$

TABLE 1: parameters used for simulation

No of Subcarriers	1024
FFT Size	2048
Spreading Factor	2
No of Blocks for Iteration	5000
Oversampling factor	8
Roll-off factors of RC filter for pulse shaping	0.0, 0.3, 0.6, 0.9
Baseband modulation	4QAM, 16QAM, 64QAM

Table-1 depicts the parameter used for Matlab simulation of the SCFDMA signal for 4QAM, 16QAM and 64QAM baseband modulation format with 1024 number of subcarriers.

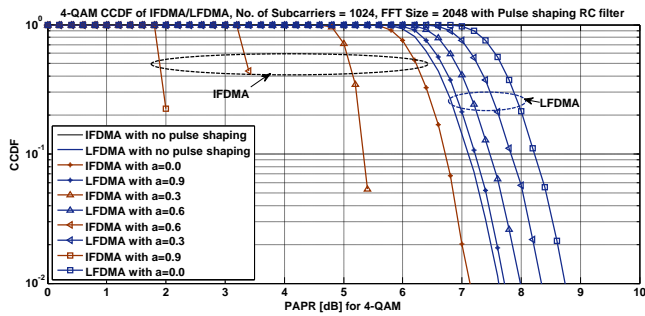


Fig.7: Effect of pulse shaping filter for 4 QAM signal

Figure 7 shows the Matlab simulated result on effect of raised cosine pulse shaping filter on 4 QAM baseband modulated IFDMA and LFDMA signal.

It can be observed that at 0.01 CCDF the value of PAPR is 2.4 dB for IFDMA signal and 7.7 dB for LFDMA with roll-off factor 0.9. It increases to 7.2 and 8.7 dB when roll-off factor is reduced to 0.0 for the case of IFDMA and LFDMA respectively.

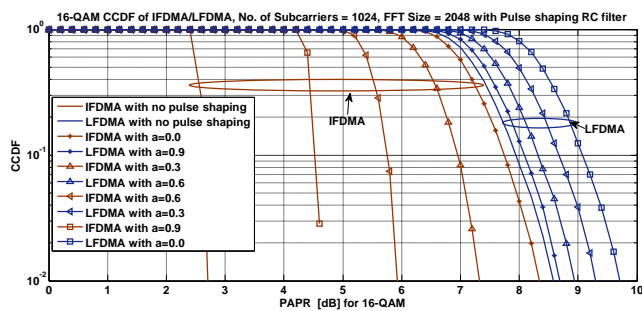


Fig. 8: Effect of pulse shaping filter for 16 QAM signal

Similar observations are obtained in figure 6 and 7 for the case of 16 QAM and 64 QAM signals respectively. From these figures it can be observed that effect of pulse shaping is more on IFDMA than that of LFDMA. Further, the PAPR value decreases with increase in the value of roll-off factor.

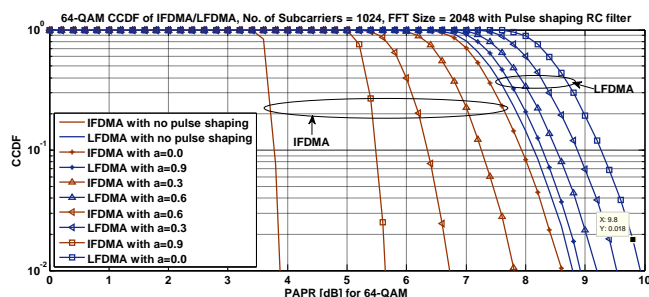


Fig. 9: Effect of pulse shaping filter for 64 QAM signal

Table 1 describes the PAPR value of SC-FDMA system at 0.01 CCDF with change in roll-off factor, α . It can be deduced

from the table that for 4 QAM signal, there is 4.8 dB reductions in PAPR value for IFDMA system with increase in roll off factor by 0.9.

Table 2: Effect of roll-off factor on PAPR performance

SC-FDMA	IFDMA			LFDMA		
Roll-off factor	4 Q A M (dB)	16 Q A M (dB)	64 Q A M (dB)	4 Q A M (dB)	16 Q A M (dB)	64 Q A M (dB)
Without Pulse Shaping	0.0	2.8	3.9	7.6	8.6	8.7
$\alpha=0.0$	7.2	8.4	8.6	8.7	9.7	9.9
$\alpha=0.3$	5.6	7.4	7.8	8.4	9.3	9.5
$\alpha=0.6$	3.8	5.9	6.7	8.0	8.9	9.2
$\alpha=0.9$	2.4	4.7	5.7	7.7	8.7	8.9

It is only 1.0 dB for the case of LFDMA system for the same value of increase in the roll off factor. A similar trend follows for 16 QAM and 64 QAM modulation format. It can be also deduced that with 4QAM signal without pulse shaping the PAPR value for IFDMA and LFDMA are 0.0 and 7.6 dB respectively. But PAPR values increases to 2.4 and 7.7 dB when the signal is passed through a raised cosine pulse shaping filter with 0.9 roll-off factor. This increase in the PAPR value is at the cost of reduction in bandwidth and effect of intersymbol interference (ISI).

Conclusion

In this paper effect of raised-cosine pulse shaping filter has been investigated used to reduce the PAPR of single carrier frequency division multiple access (SCFDMA) system. Pulse shaping is generally used to reduce the bandwidth of the transmitted signal better suited for the communication channel and reduce the effect of intersymbol interference. Investigation on effect of raised cosine pulse shaping filter on 4 QAM, 16 QAM and 64 QAM baseband modulated IFDMA and LFDMA signal have been done. From the result obtained it can be observed that effect of pulse shaping is more on IFDMA than that of LFDMA. Further, the PAPR value decreases with increase in roll-off factor. For IFDMA system the PAPR value at 0.01 of CCDF value with 1024 number of subcarriers with change in roll-off factor from 0.0 to 0.9, the reductions in PAPR value obtained is 2.9 dB for 64 QAM modulation format. But it is only 1.0 dB for the case of LFDMA system. A similar trend follows for 4 QAM and 16 QAM modulation formats.

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