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Application of arc-length method to calculate deflected state of the compressed strut with random initial imperfection

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Abstract.
This article examines the question of post-critical deformation of the compressed strut of variable cross-section, with random initial imperfections. These struts are widely used in various industries, as well as in elements of quasi-zero stiffness vibration isolation systems. Given the high complexity of accounting for accurate profile of initial imperfections the detailed analysis of its impact on stress-strain state of the strut is required.

Keywords: arc-length method, continuation method, post-critical deformation, random initial imperfections.

In many industries struts with constant or variable cross section along the length of the member are used as a support member, perceiving axial load [1, 2, 3, 6, 7]. From structural mechanics theory [3-5] it is known that the presence of initial curvature of the central compressed strut leads to higher stresses in the critical section, compared with a perfectly straight strut. Moreover, in many engineering structures, including quasi-zero stiffness vibration isolators [6, 7] variable cross-section struts, compressed above the critical force, are used. Since any strut has the initial imperfection, the calculation method, especially for over-loaded or critical elements, should account for this factor. However, in practice, the initial curvature of the strut is considered to be in the form of a harmonic function. Hence it is of great interest the description of the process of geometrically nonlinear deformation of compressed struts with initial curvature, specified as a random function.

We will consider the central compressed strut of length L under the action of compressive axial force P, the value of which may exceed the critical Euler force. The calculation scheme is presented in Fig. 1.

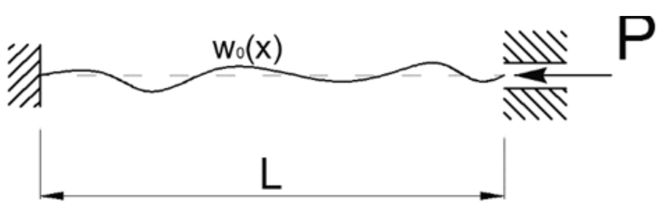

Fig. 1. Design diagram.

We assume that the initial curvature of the strut $w_0(x)$ has the standard normal distribution. The initial system of differential equations describing large displacement of the strut of variable cross section has the form:

$$\begin{cases} \dfrac{dx}{ds} = \cos\theta; \dfrac{dy}{ds} = \sin\theta; \dfrac{d\theta}{ds} = \dfrac{m}{f(s)} + \dfrac{d\theta_0}{ds} \\ \dfrac{dq}{ds} = t\dfrac{d\theta}{ds}; \dfrac{dt}{ds} = -q\dfrac{d\theta}{ds}; \dfrac{dm}{ds} = q \end{cases} \quad (1)$$

where

$$x = \frac{X}{L}; y = \frac{Y}{L}; s = \frac{S}{L}; q = \frac{QL^2}{EI_1}; \mathrm{t} = \frac{TL^2}{EI_1}; m = \frac{ML}{EI_1},$$

θ_0-the initial curvature of the beam.

To account for variable cross-section in the calculation, the following relation is used:

$$EI(s) = EI_1 f(s) \quad (2)$$

In Eq. (2) considered: EI_1-rigidity of one of the sections of the beam, f(s) is a dimensionless function that sets the change of the stiffness of the strut along its length.

Boundary conditions for the case of a rigidly clamped strut are given by:

$$s = 0; x(0) = y(0) = \theta(0) = 0$$
$$s = 1; y(1) = \theta(1) = t(1) = 0 \quad (3)$$

This boundary value problem is solved by the shooting method via reducing its to initial value problem. Due to the fact that there are 6 unknowns in in Eq. (1), and from boundary conditions (3) we know only 3 of them, the remaining 3 unknowns are determined using the shooting procedure. The minimization of the residual in this case is achieved by the method of arc-length continuation [8-10], where for the leading argument we use the arc length of the curve of equilibrium states. Method for solution of this problem was developed in [11]. The initial curvature of the rod was set by a random number generator and multiplied by the scale-factor A, which was assumed to be 10^{-10} and 10^{-9}. There were performed ten trials for each type of the strut. Calculations were performed for the strut with a linearly varying cross-section, schematically represented in Fig. 2.

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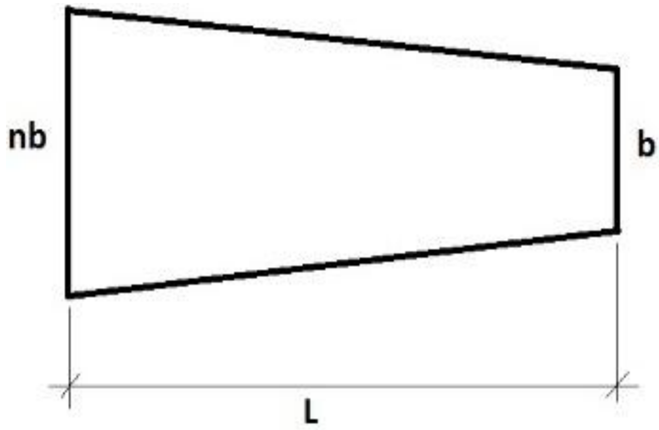
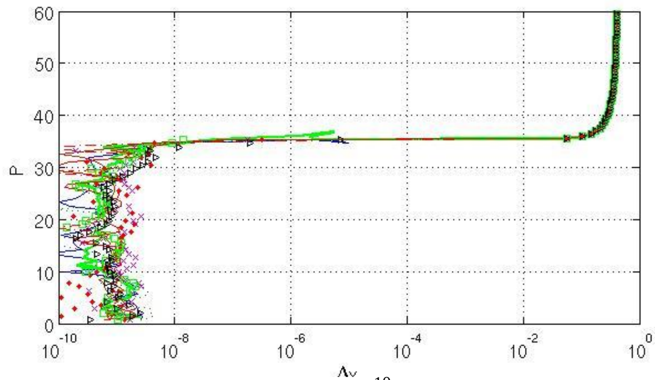
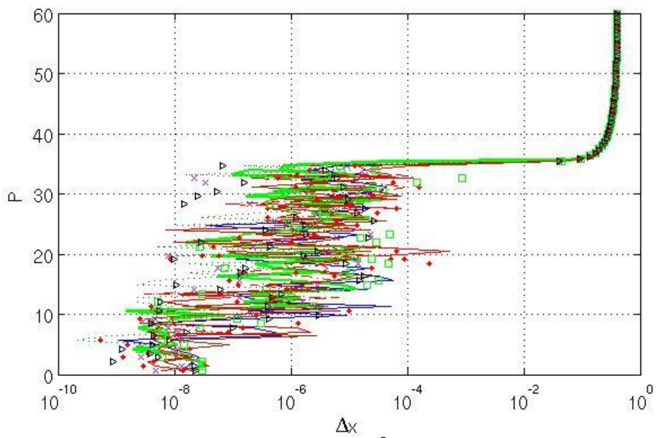


Fig. 2. Cross-section of the strut

In accordance with Fig. 2 the calculations were made for n = 0, 8; 1, 0 and 1, 5. According to the results of the calculations elastic characteristics of the strut in the axis "axial load — axial displacement" was plotted. Fig. 3-5 shows the calculation results for struts with n = 0, 8; 1, 0 and 1, 5 respectively. The characters in Fig. 3-5 represent deformation curves corresponding to different shapes of the initial curvature of the strut.

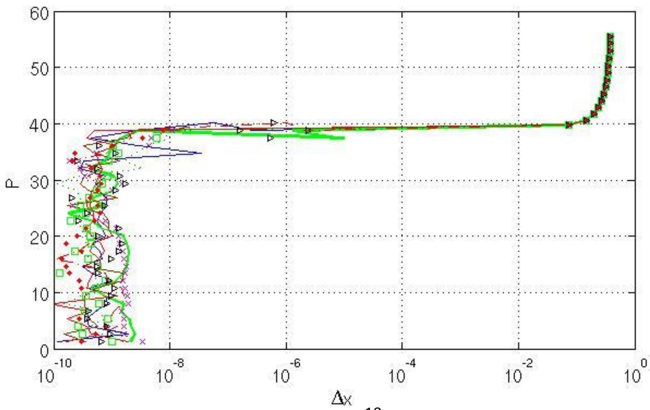


a) A = 10^{-10}

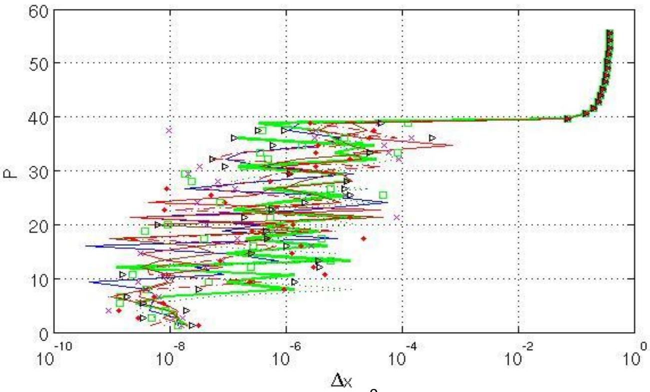


б) A = 10^{-9}

Fig. 3. Elastic curve for strut with n = 0, 8.

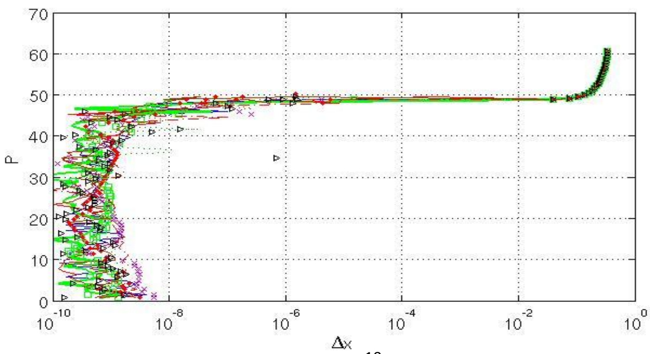


a) A = 10^{-10}

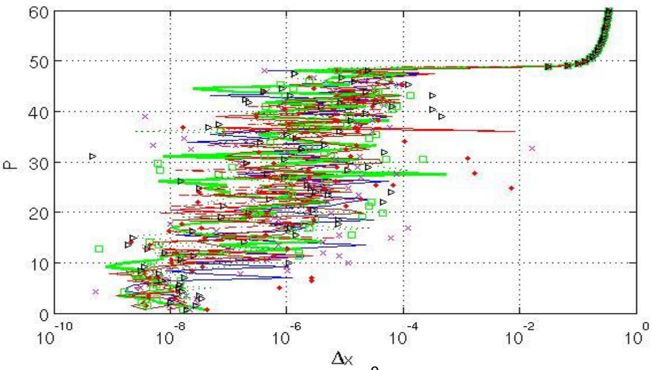


б) A = 10^{-9}

Fig. 4. Elastic curve for strut with n = 1, 0.



a) A = 10^{-10}



б) A = 10^{-9}

Fig. 5. Elastic curve for strut with n = 1, 5.

Analysis of the results shows that a small initial curvature of the strut, corresponding to the multiplier A = 10-10 practically does not affect the strain state of a strut of any of the considered cross sections. The difference between the curves in Fig. 3 disappears from $\Delta x = 10^{-5}$. The initial curvature of the rod, corresponding to the multiplier $A = 10^{-9}$ affects the deformed state of the rod up to $\Delta x = 10^{-3}$. It should be mentioned, that when a compressive force exceeds a critical value the difference between the curves corresponding to different initial curvatures becomes almost not distinguishable.
It should also be noted that with the increase of the width of the cross section (increasing the value of n) there is an increase in scatter of the values of displacement at a fixed value of the load. It is testimony to the fact that with the increase of the parameter n the stiffness of the strut also increases, and thus the sensitivity to any imperfections.
To determine the optimal values of the scaling coefficient we have conducted experimental studies of post-critical deformation of struts with different cross-sections. Fig. 6 presents the experimental setup with the sample for testing.

Fig. 6. Experimental set-up

The test sample was clamped in the grippers of the press and controlled by its axial displacement when the load increases, as well as lateral movement in the mid and quarter span. Fig. 7 presents the comparison between the average values of the calculation results corresponding to different scaling factors, with the averaged experimental data of the axial load of steel struts of constant cross section on the press.

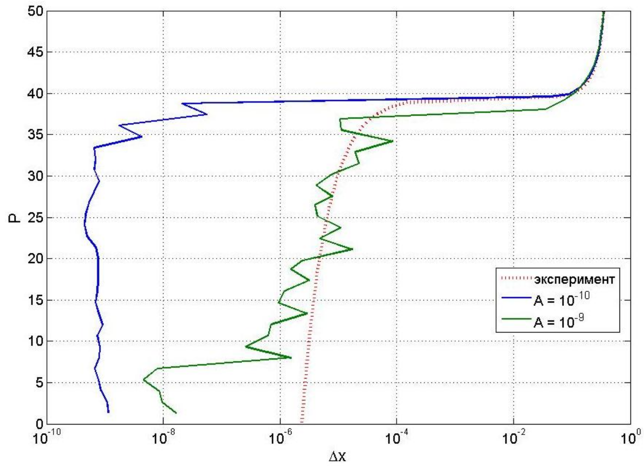


Fig. 7. Comparison of numerical and experimental data.

Analysis of the results of Fig. 7 show that the best correlation with experimental data have averaged curves for the initial curvature with multiplier $A = 10^{-9}$. The observed spread of values is associated with a small number of trials of the strut, when increasing the number of trials up to 100, it is greatly reduced. For any value of the scaling coefficient of the elastic curves of the compressed strut coincide when $P > P_{crit}$ where P_{crit}-critical Euler force. This is because when exceeding the critical Euler force encountered displacements becomes large (comparable to the cross-sectional dimensions) [3].
Thus, we can conclude that for large displacements of post-critically compressed struts initial imperfections will have no effect on theirs stress-strain state. However, this is not true for small displacements. From the results of the comparison with experimental data it is found that the best convergence have averaged elastic curves for the initial curvature with multiplier $A = 10^{-9}$ and a large number of tests.

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