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General Production And Sales Time With Two-Units System And Manpower

P. Madhusoodhanan

Assistant professor, SRM University, Kattankulathur, Tamil Nadu, India madhupms1973@gmail. com

P. Sekar

Principal, C. Kandaswami Naidu College for Men, Anna Nagar, Chennai, Tamil Nadu, India

R. Ramanarayanan,

Professor of Mathematics(Retd), Vel Tech University, Chennai, Tamil Nadu, India

Abstract
A production and sale system is considered. During the operation time a machine produces random number of products. After operation time, sale time starts. Two models are treated. In model 1, the sales time of production is i. i. d random variables, when production is stopped and sales are done one by one. In model 2, the operation time is more than a threshold time, the sales are done altogether and when it is less than the threshold time, the sales are done one by one. Joint transforms of the variables, their means and Co-variances with numerical results are presented.

Mathematics Subject Classification: 91B70

Keywords: Storage system, Production and Sale, Repair and Recruitment, Joint transform.

I. Introduction

In manufacturing models to get the return on investment and to pay minimum interest, it is natural that when the production time is more, the sale time is made short so as to cut cost. It has been noticed that when the units produced are more, financial supports for the customers are provided to clear products early. These are widely felt in perishable commodity sectors where many banking institutions provide required finance for the purchase.

Storage systems of (s, S) type was studied by Arrow, Karlin and Scarf [1]. Such systems with random lead times and unit demand were treated by Danial and Ramanarayanan [2]. Models with bulk demands were analyzed by Ramanarayanan and Jacob [12]. Murthy and Ramanarayanan [8, 9, 10, 11] considered several (s, S) inventory systems. Kun-Shan Wu, Ouyang and Liang-Yuh [3] studied (Q, r, L) inventory model with defective items. Usha, Nithyapriya and Ramanarayanan [14] considered storage systems with random sales time depending on production. General Manpower and Machine system with Markovian production were analyzed by Hari kumar. k [4]. General Production and Sales System with SCBZ Machine Time and Manpower, Exponential Machine Time and Manpower were discussed by Madhusoodhanan, Sekar and Ramanarayanan[5, 6]. Snehalatha. M., Sekar. P and Ramanarayanan. R[13] were analyzed on Probabilistic Analysis of General Manpower SCBZ Machine System with Exponential Production, General Sales and General Recruitment. General production and Sales by Markovian Manpower and Machine System were analyzed by Madhusoodhanan. P, Sekar. P and Ramanarayanan. R[7]. In this paper, two models are treated. In model 1, the sales time of production is i. i. d random variables, when production is stopped and sales are done one by one. In model 2, the operation time is more than a threshold time, the sales are done altogether and when it is less than the threshold time, the sales are done one by one. Joint transforms of the variables, their means and Co-variances with numerical results are presented.

II. MODEL 1

2. 1 ASSUMPTIONS:

1. Inter production times of products are i. i. d random variables with Cdf F(x) and pdf f(x).
2. The sales time of products is i. i. d random variables with Cdf G(x) and pdf g(x). The sales start when production is stopped and the sales are done one by one.
3. The system producing the products has two units. When two units are working the failure rate is λ_1. The repair rate of a failed unit is μ_1. When one unit is working the failure rate is λ_2. The two unit system fails when the two units are in failed state.
4. The inter-departure time of employees attending the system has exponential distribution with parameter μ. The manpower system collapses with probability p when an employee leaves and with probability q it continues operation.
5. The production is stopped when two units system fails or when the manpower system collapses.

When the production is stopped the repairs and recruitments are done one by one. The repair/replacement times and the recruitment times are independent random variables with Cdfs R(y) and $R_1(y)$ and pdfs r(y) and $r_1(y)$ respectively.

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2. 2 ANALYSIS:

To study the above model the pdf and Cdf of 2 unit system are required. To present the same the following functions are needed.

$P_{00}(x)$= P(at time x the two units are good, the system does not fail during (0, x)/at time 0 the two units are good).

$P_{01}(x)$ = P(at time x one unit is working and the other is under repair, the system does not fail during (0, x) /at time 0 the two units are good).

$P_{10}(x)$ = P(at time x the two units are good, during (0, x) the system does not fail/ at time 0 one unit is working and the other is under repair).

$p_{i,2}(x)dx$ = P(the system fails for the first time during (x, x+dx)/at time 0 the two units are good),

i = 0, 1.

$p_{i2}(x)$ is pdf for i = 0, 1. It is noted that $P_{00}(x)$ satisfies the following.

$$P_{00}(x) = e^{-\lambda_1 x} + \int_0^x \lambda_1 e^{-\lambda_1 u} P_{10}(x-u)du \quad (1)$$

$$P_{10}(x) = \int_0^x \mu_1 e^{-\mu_1 u} e^{-\lambda_2 u} e^{-\lambda_1 (x-u)} du + \int_0^x \int_0^v \mu_1 e^{-\mu_1 v} e^{-\lambda_2 u} \lambda_1 e^{-\lambda_1 (v-u)} P_{10}(x-v)dv \quad (2)$$

The first term of the right side of equation (1) is the probability that the no failure occurs during (0, x) and the second term is the probability that one unit fails during (0, x) but the two unit system does not fail during (0, x) and the two units are good at time x.

The first term of the right hand side of equation (2) is the probability that the failed unit is repaired and no failure occurs during repair time and later. The second term is the probability that the failed unit is repaired and then a failure occurs, and two units are good at time x and the system does not fail during (0, x).

Using similar arguments it may be seen,

$$P_{0,1}(x) = \int_0^x \lambda_1 e^{-\lambda_1 u} e^{-\mu_1 (x-u)} e^{-\lambda_2 (x-u)} du + \int_0^x \int_0^v \lambda_1 e^{-\lambda_1 u} \mu_1 e^{-\mu_1 (v-u)} e^{-\lambda_2 (v-u)} P_{01}(x-v)dudv \quad (3)$$

The failure density $p_{02}(x)$ satisfies

$$p_{0,2}(x) = \int_0^x \lambda_1 e^{-\lambda_1 u} p_{12}(x-u)du \quad (4)$$

$$and\ p_{12}(x) = \lambda_2 e^{-\lambda_2 x} e^{-\mu_1 x} + \int_0^x e^{-\lambda_2 u} \mu_1 e^{-\mu_1 u} p_{02}(x-u)du \quad (5)$$

Using Laplace transform it may be noted that

$$P_{00}^*(s) = \frac{\lambda_2 + \mu_1 + s}{[s^2 + s(\lambda_1 + \lambda_2 + \mu_1) + \lambda_1 \lambda_2]} \quad (6)$$

$$P_{01}^*(s) = \frac{\lambda_1}{[s^2 + s(\lambda_1 + \lambda_2 + \mu_1) + \lambda_1 \lambda_2]} \quad (7)$$

$$p_{02}^*(s) = \frac{\lambda_1 \lambda_2}{[s^2 + s(\lambda_1 + \lambda_2 + \mu_1) + \lambda_1 \lambda_2]} \quad (8)$$

Here * denotes Laplace transform. When the Laplace transforms are inverted it may be obtained that

$$P_{00}(x) = \frac{1}{2} e^{-ax}(e^{bx} + e^{-bx}) + \frac{1}{4b}(\lambda_2 - \lambda_1 + \mu_1) e^{-ax}(e^{bx} - e^{-bx}) \quad (9)$$

$$P_{01}(x) = \frac{\lambda_1}{2b} e^{-ax}(e^{bx} - e^{-bx}) \quad (10)$$

$$p_{02}(x) = \frac{\lambda_1 \lambda_2}{2b} e^{-ax}(e^{bx} - e^{-bx}) \quad (11)$$

$$Here\ a = \frac{(\lambda_1 + \lambda_2 + \mu_1)}{2}$$

$$b = \frac{1}{2}\sqrt{(\lambda_1 - \lambda_2)^2 + \mu_1^2 + 2\mu_1(\lambda_1 + \lambda_2)}$$

$$and\ \lambda_1 \lambda_2 = a^2 - b^2$$

The failure time CDF of the two unit system H(x) is

$$H(x) = \int_0^x p_{02}(u)du = 1 - P_{00}(x) - P_{01}(x)$$

On simplification

$$H(x) = 1 - \left(\frac{a+b}{2b}\right) e^{-x(a-b)} - \left(\frac{b-a}{2b}\right) e^{-x(a+b)} \quad (12)$$

To study the production model, the joint probability density function f(x, y, z) of three variables $(X, \hat{R}, \hat{S})$, namely (i) X is the operation time which is the minimum of the life time of the two unit system and the man power service time, (ii) Y is the sum of repairs and recruitments times and (iii) Ŝ is the sales time of the products is to be written. The pdf is given by

$$f(x,y,z) = P_{02}(x)\left[\sum_{i=0}^{\infty} e^{-\mu x} \frac{(\mu x)^i}{\lfloor i} q^i r_2(y) © r_{1i}(y))\right] [\sum_{k=0}^{\infty} (F_k(x) - F_{k+1}(x)) g_k(z)]$$

$$+ \sum_{k=1}^{\infty} (P_{00}(x) r_{1i}(y) + P_{01}(x) r(y) © r_{1i}(y)) \left(e^{-\mu x} \frac{(\mu x)^{i-1}}{\lfloor i-1} q^{i-1} p\mu\right) [\sum_{k=0}^{\infty} (F_k(x) - F_{k+1}(x)) g_k(z)] \quad (13)$$

The first term of equation (13) is the part of the pdf that the two unit system fails, the products produced are sold one by one and the two repairs of the failed units and recruitments are done one by one. The second term of the equation (13) is the part of the pdf that the man power system collapses, the products produced are sold one by one and recruitments are done by one when the two unit system is good and one repair is carried out when one unit is in failed state. The letter © and suffix letters indicate the convolution of pdf or Cdf as the case may be. The triple Laplace transform

$$f^*(\xi,\eta,\varepsilon) = \int_0^\infty \int_0^\infty \int_0^\infty e^{-\xi x - \eta y - \varepsilon z} f(x,y,z)dxdydz$$

may be seen as

$$f^*(x,y,z) = \int_0^\infty e^{-\xi x} p_{02}(x) e^{-\mu x(1-qr_1^*(\eta))} r^{*2}(\eta) \sum_{k=0}^{\infty} [F_k(x) - F_{k+1}(x)] g^{*k}(\varepsilon) dx$$

$$+ \int_0^\infty e^{-\xi x} P_{00}(x) p\mu r_1^*(\eta) e^{-\mu x(1-qr_1^*(\eta))} \sum_{k=0}^{\infty} [F_k(x) - F_{k+1}(x)] g^{*k}(\varepsilon) dx$$

$$+ \int_0^\infty e^{-\xi x} P_{01}(x) p\mu r^*(\eta) r_1^*(\eta) e^{-\mu x(1-qr_1^*(\eta))} \sum_{k=0}^{\infty} [F_k(x) - F_{k+1}(x)] g^{*k}(\varepsilon) dx \quad (14)$$

On simplification we get

$$f^*(\xi,\eta,\varepsilon) =$$

$$\left\{\frac{\lambda_1 \lambda_2}{2b} r^{*2}(\eta) + \left[\frac{1}{2} + \frac{1}{4b}(\lambda_2 - \lambda_1 + \mu_1)\right] p\mu r_1^*(\eta) + \frac{\lambda_1}{2b} p\mu r^*(\eta) r_1^*(\eta)\right\} \frac{1}{\theta_1} \frac{(1 - f^*(\theta_1))}{(1 - g^*(\varepsilon) f^*(\theta_1))}$$

$$+ \left\{-\frac{\lambda_1 \lambda_2}{2b} r^{*2}(\eta) + \left[\frac{1}{2} - \frac{1}{4b}(\lambda_2 - \lambda_1 + \mu_1)\right] p\mu r_1^*(\eta) - \frac{\lambda_1}{2b} p\mu r^*(\eta) r_1^*(\eta)\right\} \frac{1}{\theta_2} \frac{(1 - f^*(\theta_2))}{(1 - g^*(\varepsilon) f^*(\theta_2))}$$

$$here\ \theta_1 = \xi + a - b + \mu(1 - qr_1^*(\eta)) \quad (15)$$

$$and\ \theta_2 = \xi + a + b + \mu(1 - qr_1^*(\eta)) \quad (16)$$

Now using differentiation

$$E(X) = -\frac{\partial}{\partial \xi} f^*(\xi,\eta,\varepsilon) \mid \xi = \eta = \varepsilon = 0. \textit{This gives}$$

$$E(X) = \left(\frac{a+b}{2b}\right)\left(\frac{1}{a-b+p\mu}\right) - \left(\frac{a-b}{2b}\right)\left(\frac{1}{a+b+p\mu}\right) \qquad (17)$$

Similarly

$$E(\hat{R}) = -\frac{\partial}{\partial \eta} f^*(\xi,\eta,\varepsilon) \mid \xi = \eta = \varepsilon = 0 \qquad \textit{gives}$$

$$E(\hat{R}) = \frac{(2\lambda_1\lambda_2 + \lambda_1\mu p)E(R) + \mu p(\lambda_1 + \lambda_2 + \mu_1 + \mu p)E(R_1)}{\lambda_1\lambda_2 + (\lambda_1 + \lambda_2 + \mu_1)\mu p + (\mu p)^2} \qquad (18)$$

The Laplace transform of the joint pdf of X *and* $\hat{S}$ *can be seen as*

$$f^*(\xi,0,\varepsilon) = \frac{1}{2b}(a+b)(a-b+\mu p)\frac{1}{(\xi+a-b+\mu p)}\frac{(1-f^*(\xi+a-b+\mu p))}{(1-g^*(\varepsilon)f^*(\xi+a-b+\mu p))}$$
$$-\frac{1}{2b}(a-b)(a+b+\mu p)\frac{1}{(\xi+a+b+\mu p)}\frac{(1-f^*(\xi+a-b+\mu p))}{(1-g^*(\varepsilon)f^*(\xi+a-b+\mu p))} \qquad (19)$$

Now

$$E(\hat{S}) = -\frac{\partial}{\partial \varepsilon} f^*(\xi,0,\varepsilon) \mid \xi = \varepsilon = 0 \; \textit{gives}$$

$$E(\hat{S}) = \frac{E(G)}{2b}\left[(a+b)\frac{f^*(a-b+\mu p)}{(1-f^*(a-b+\mu p))} - (a-b)\frac{f^*(a+b+\mu p)}{(1-f^*(a+b+\mu p))}\right] \qquad (20)$$

The product moment $E(X\hat{S})$ can be seen as

$$E(X\hat{S}) = \frac{\partial^2}{\partial\xi\partial\varepsilon} f^*(\xi,0,\varepsilon) \mid \xi = 0 = \varepsilon \; \textit{and}$$

$$E(X\hat{S}) = \frac{E(G)}{2b}\left[\begin{array}{l}\frac{(a+b)}{(a-b+\mu p)}\frac{f^*(a-b+\mu p)}{(1-f^*(a-b+\mu p))} - (a+b)\frac{f^{*\prime}(a-b+\mu p)}{(1-f^*(a-b+\mu p))^2} \\ -\frac{(a-b)}{(a+b+\mu p)}\frac{f^*(a+b+\mu p)}{(1-f^*(a+b+\mu p))} + (a-b)\frac{f^{*\prime}(a+b+\mu p)}{(1-f^*(a+b+\mu p))^2}\end{array}\right] (21)$$

From equations (21), (20) and (17)

$$Cov(X,\hat{S}) = E(X\hat{S}) - E(X)E(\hat{S}) \; \textit{may be written.}$$

III. MODEL 2

In this section we treat the previous model 1 with all assumptions (1), (3), (4), (5) and(6) except the assumption (2) for sales.

3. 1 ASSUMPTIONS FOR SALES:

(2. 1) When the operation time X is more than a threshold time U, the sales are done all together. It is assigned to an agent whose sale time distribution function is $G_1(z)$ and pdf $g_1(z)$.

(2. 2) When the operation time X is less than the threshold time U, the sales are done one by one with Cdf$G_2(z)$ and pdf $g_2(z)$.

(2. 3) The threshold U has exponential distribution with parameter δ.

3. 2 ANALYSIS:

Using the arguments given for model 1 the joint pdf of $(X,\hat{R},\hat{S})$ (Operation time, Repair-recruitment time, Sales time) is seen as follows.

$$f(x,y,z) = p_{02}(x)\left[\sum_{i=0}^{\infty} e^{-\mu x}\frac{(\mu x)^i}{\lfloor i} q^i r_2(y) © r_{1i}(y))\right]$$
$$[\sum_{k=0}^{\infty}(F_k(x) - F_{k+1}(x))((1-e^{-\delta x})g_1(z) + e^{-\delta x}g_{2,k}(z)]$$
$$+\sum_{k=1}^{\infty}(P_{00}(x)r_{1i}(y) + P_{01}(x)r(y) © r_{1i}(y))\left(e^{-\mu x}\frac{(\mu x)^{i-1}}{\lfloor i-1} q^{i-1}p\mu\right)$$
$$[\sum_{k=0}^{\infty}(F_k(x) - F_{k+1}(x))((1-e^{-\delta x})g_1(z) + e^{-\delta x}g_{2,k}(z)] \qquad (22)$$

Equation (22) is written using the same arguments given for model 1 for all the terms and noting the assumptions that the sale time pdf is $g_1(z)$ when (X>V) the operation time is greater than the threshold and that when the operation time is less than the threshold (X<V) the k products are sold one by one with pdf $g_{2,k}(z)$ where the suffix k indicates k-fold convolution of $g_2(z)$ with itself.

The triple Laplace transform can be seen as

$$f^*(\xi,\eta,\varepsilon) = \int_0^{\infty} e^{-\xi x}e^{-\mu x(1-qr_1^*(\eta))}[p_{02}(x)r^{*2}(\eta) + P_{00}(x)p\mu r_1^*(\eta) + P_{01}(x)p\mu r^*(\eta)r_1^*(\eta)]$$
$$\sum_{k=0}^{\infty}[F_k(x) - F_{k+1}(x)]((1-e^{-\delta x})g_1^*(\varepsilon) + e^{-\delta x}g_{2,k}(\varepsilon))dx \qquad (23)$$

Using equations (9), (10) and (11) the triple transform is as given below.

$$f^*(\xi,\eta,\varepsilon) =$$
$$\left\{\frac{\lambda_1\lambda_2}{2b}r^{*2}(\eta) + \left[\frac{1}{2} + \frac{1}{4b}(\lambda_2 - \lambda_1 + \mu_1)\right]p\mu r_1^*(\eta) + \frac{\lambda_1}{2b}p\mu r^*(\eta)r_1^*(\eta)\right\}$$
$$\left\{\frac{g_1^*(\varepsilon)}{\theta_1} - \frac{g_1^*(\varepsilon)}{\theta_1+\delta} + \frac{1}{(\theta_1+\delta)}\frac{(1-f^*(\theta_1+\delta))}{(1-g_2^*(\varepsilon)f^*(\theta_1+\delta))}\right\}$$
$$+\left\{-\frac{\lambda_1\lambda_2}{2b}r^{*2}(\eta) + \left[\frac{1}{2} - \frac{1}{4b}(\lambda_2 - \lambda_1 + \mu_1)\right]p\mu r_1^*(\eta) - \frac{\lambda_1}{2b}p\mu r^*(\eta)r_1^*(\eta)\right\}$$
$$\left\{\frac{g_1^*(\varepsilon)}{\theta_2} - \frac{g_1^*(\varepsilon)}{\theta_2+\delta} + \frac{1}{(\theta_2+\delta)}\frac{(1-f^*(\theta_2+\delta))}{(1-g_2^*(\varepsilon)f^*(\theta_2+\delta))}\right\} \qquad (24)$$

Here θ_1 *and* θ_2 *are as defined in equation*(16)

Since there is only change in the sales pattern compared to model 1 we note E(X) and $E(\hat{R})$ remain same as those of model 1. Now the joint pdf of X and $\hat{S}$ is given by

$$f^*(\xi,0,\varepsilon) = \frac{1}{2b}(a+b)(a-b+\mu p)\left\{\frac{g_1^*(\varepsilon)}{\theta_3} - \frac{g_1^*(\varepsilon)}{\theta_3+\delta} + \frac{1}{(\theta_3+\delta)}\frac{(1-f^*(\theta_3+\delta))}{(1-g_2^*(\varepsilon)f^*(\theta_3+\delta))}\right\}$$
$$-\frac{1}{2b}(a-b)(a+b+\mu p)\left\{\frac{g_1^*(\varepsilon)}{\theta_4} - \frac{g_1^*(\varepsilon)}{\theta_4+\delta} + \frac{1}{(\theta_4+\delta)}\frac{(1-f^*(\theta_4+\delta))}{(1-g_2^*(\varepsilon)f^*(\theta_4+\delta))}\right\} \qquad (25)$$

Here

$$\theta_3 = \xi + a - b + \mu p \quad \textit{and} \quad \theta_4 = \xi + a + b + \mu p \qquad (26)$$

Now $E(\hat{S})$ can be seen as

$$E(\hat{S}) = -\frac{\partial}{\partial\varepsilon} f^*(\xi,0,\varepsilon) \mid \xi = \varepsilon = 0$$

On simplification

$$E(\hat{S}) = \frac{\delta E(S_1)[\delta + 2a + \mu p]}{(\delta+a-b+\mu p)(\delta+a+b+\mu p)}$$
$$+\frac{E(S_2)}{2b}\left[\begin{array}{l}\frac{(a+b)(a-b+\mu p)}{(\delta+a-b+\mu p)}\frac{f^*(\delta+a-b+\mu p)}{(1-f^*(\delta+a-b+\mu p))} \\ -\frac{(a-b)(a+b+\mu p)}{(\delta+a+b+\mu p)}\frac{f^*(\delta+a+b+\mu p)}{(1-f^*(\delta+a+b+\mu p))}\end{array}\right] \qquad (27)$$

Now the product moment $E(X\hat{S})$ *can be obtained by*

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$$E(X\hat{S}) = \frac{\partial^2}{\partial\xi\partial\varepsilon} f^*(\xi,0,\varepsilon) \mid \xi = 0 = \varepsilon \text{ as follows}$$

$$\begin{aligned} E(X\hat{S}) &= E(S_1)\left[\frac{2a+\mu p}{(a-b+\mu p)(a+b+\mu p)} - \frac{(a+b)(a-b+\mu p)}{2b(\delta+a-b+\mu p)^2} + \frac{(a-b)(a+b+\mu p)}{2b(\delta+a+b+\mu p)^2}\right] \\ &+ E(S_2)\left(\frac{a+b}{2b}\right)(a-b+\mu p)\left[\begin{array}{l} \frac{f^*(\delta+a-b+\mu p)}{(\delta+a-b+\mu p)^2(1-f^*(\delta+a-b+\mu p))} \\ -\left(\frac{1}{\delta+a-b+\mu p}\right)\frac{f^{*\prime}(\delta+a-b+\mu p)}{(1-f^*(\delta+a-b+\mu p))^2} \end{array}\right] \\ &+ E(S_2)\left(\frac{a-b}{2b}\right)(a+b+\mu p)\left[\begin{array}{l} \frac{-f^*(\delta+a+b+\mu p)}{(\delta+a+b+\mu p)^2(1-f^*(\delta+a+b+\mu p))} \\ +\left(\frac{1}{\delta+a-b+\mu p}\right)\frac{f^{*\prime}(\delta+a+b+\mu p)}{(1-f^*(\delta+a+b+\mu p))^2} \end{array}\right] \end{aligned} \quad (28)$$

From equations (28), (27) and (17) the $Cov(X,\hat{S}) = E(X\hat{S}) - E(X)E(\hat{S})$ may be written.

4. NUMERICAL EXAMPLES

The usefulness of the results obtained is presented by numerical examples. The two models are considered together.

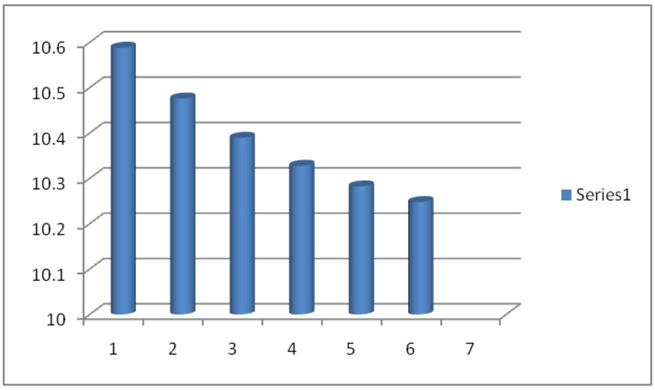
Since there is only change in sales pattern E(X), $E(\hat{R})$ and **E(** $\tilde{R}$ **)** are same for models 1 and 2.

4. 1 Numerical values for Model 1

Let p = 0. 5, a= 6, b=4, E(R) = 5, $E(R_1)$ =10, E(G)=20, μ_1 = 3, μ=10, 20, 30, 40, 50, 60, ν=2, 4, 6, 8, 10, 12, λ_1= 2, λ_2 = 5
Here 'f' is an exponential density function with parameter 'ν'

The table and graph for $E(\hat{R})$

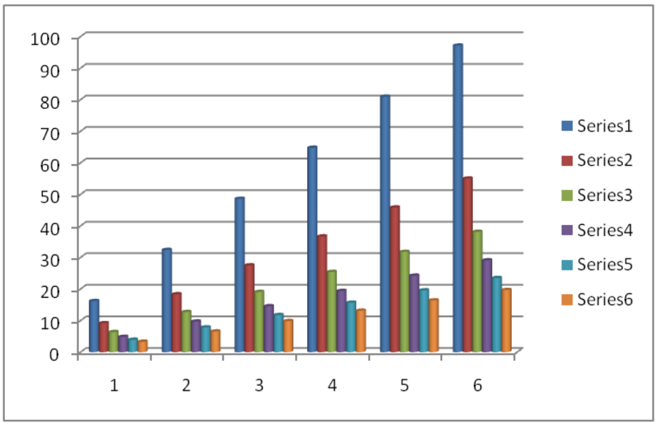
μ	$E(\hat{R})$
10	10.58824
20	10.47619
30	10.38961
40	10.32787
50	10.28249
60	10.24793



From the table and graph it is observed that, when μ increases the expected recruitment time $E(\hat{R})$ decreases.

The table and graph for $E(\hat{S})$

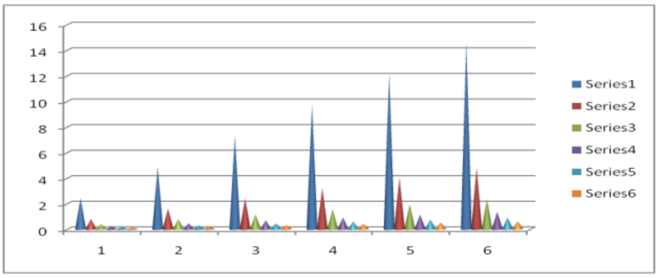
μ/ν	5	10	15	20	25	30
10	16.1905	32.381	48.5714	64.7619	80.9524	97.1429
20	9.16667	18.3333	27.5	36.6667	45.8333	55
30	6.35294	12.7059	19.0588	25.4118	31.7647	38.1176
40	4.84848	9.69697	14.5455	19.3939	24.2424	29.0909
50	3.91534	7.83069	11.746	15.6614	19.5767	23.4921
60	3.28125	6.5625	9.84375	13.125	16.4063	19.6875



From the table and graph it is observed that, when ν increases the expected sales time $E(\hat{S})$ increases and when μ increases the expected sales time $E(\hat{S})$ decreases.

The table and graph for $E(X,\hat{S})$

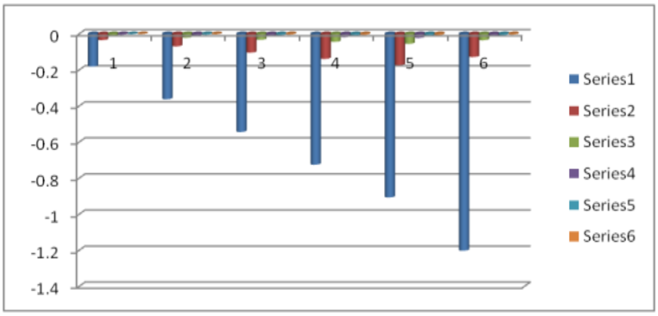
μ/ν	5	10	15	20	25	30
10	2.43991	4.87982	7.31973	9.75964	12.1995	14.6395
20	0.80556	1.61111	2.41667	3.22222	4.02778	4.83333
30	0.39253	0.78505	1.17758	1.5701	1.96263	2.35516
40	0.23049	0.46097	0.69146	0.92195	1.15243	1.38292
50	0.15106	0.30212	0.45318	0.60424	0.7553	0.90636
60	0.10645	0.21289	0.31934	0.42578	0.53223	0.63867



From the table and graph it is observed that, when ν increases the expected product moment of X and $\hat{S}$ $E(X,\hat{S})$ increases and when μ increases $E(X,\hat{S})$ decreases.

The table and graph for $Cov(X,\hat{S})$

μ/ν	5	10	15	20	25	30
10	-0.18141	-0.36281	-0.54422	-0.72562	-0.90703	-1.20238
20	-0.03472	-0.06944	-0.10417	-0.13889	-0.17361	-0.12828
30	-0.01107	-0.02215	-0.03322	-0.04429	-0.05536	-0.03567
40	-0.00459	-0.00918	-0.01377	-0.01837	-0.02296	-0.0142
50	-0.00224	-0.00448	-0.00672	-0.00896	-0.0112	-0.00682
60	-0.00122	-0.00244	-0.00366	-0.00488	-0.0061	-0.00369



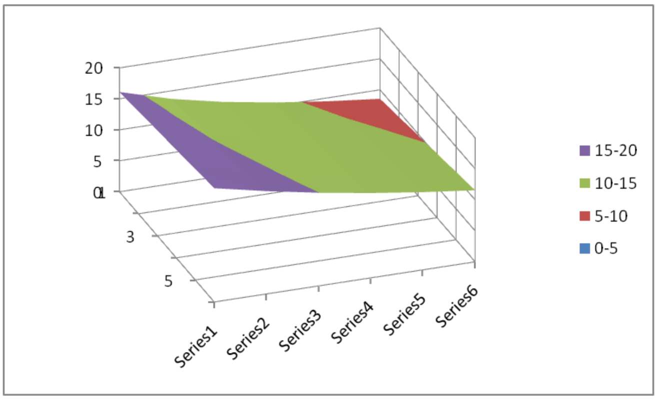
From the table and graph it is observed that when ν increases the Covariance decreases and μ increases, the $Cov(X,\hat{S})$ increases.

4. 2 Numerical values for model 2

Let p = 0. 5, a= 6, b=4, $E(S_1)$ =20, $E(S_2)$=10, δ=20, μ=10, 20, 30, 40, 50, 60, ν=2, 4, 6, 8, 10, 12, λ_1= 2, λ_2 = 5, μ_1=3, E(R) = 5, $E(R_1)$ =10 and 'f' is an exponential density function with parameter 'ν'

The table and graph for $E(\hat{S})$

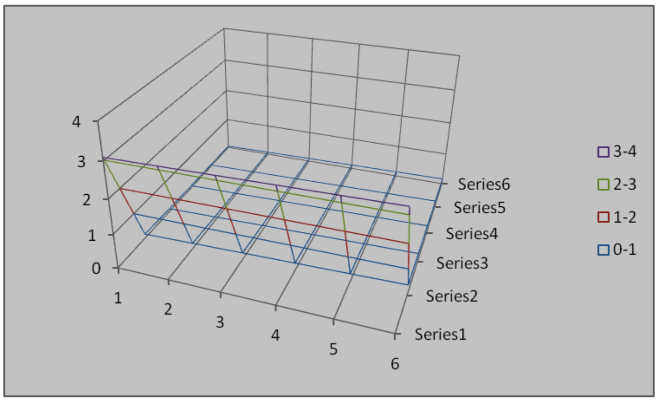
μ/ν	5	10	15	20	25	30
10	16.1085	16.5555	17.0026	17.4497	17.8968	18.3438
20	13.7012	14.2773	14.8535	15.4297	16.0059	16.582
30	11.9131	12.5349	13.1567	13.7785	14.4003	15.022
40	10.5342	11.1637	11.7932	12.4227	13.0522	13.6816
50	9.43941	10.0587	10.678	11.2973	11.9166	12.5359
60	8.54947	9.15023	9.75099	10.3517	10.9525	11.5533



From the table and graph it is observed that, when ν increases the expected sales time $E(\hat{S})$ increases and when μ increases the expected sales time $E(\hat{S})$ decreases.

The table and graph for $E(X,\hat{S})$

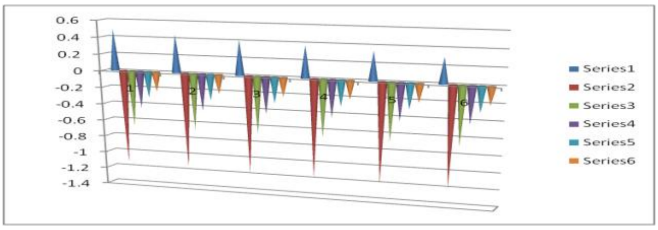
μ/ν	5	10	15	20	25	30
10	3.09497	3.13068	3.16639	3.2021	3.23781	3.27351
20	0.11811	0.15607	0.19403	0.232	0.26996	0.30793
30	0.08619	0.12128	0.15637	0.19147	0.22656	0.26165
40	0.06701	0.09813	0.12925	0.16037	0.19149	0.2226
50	0.05402	0.08126	0.10851	0.13576	0.16301	0.19026
60	0.04462	0.06843	0.09225	0.11607	0.13989	0.16371



From the table and graph it is observed that, when ν increases $E(X,\hat{S})$ increases and when μ increases $E(X,\hat{S})$ decreases.

The table and graph for $Cov(X, \hat{S})$

μ/ν	5	10	15	20	25	30
10	0.486938	0.450263	0.413587	0.376912	0.340236	0.303561
20	-1.13783	-1.15269	-1.16754	-1.18239	-1.19724	-1.21209
30	-0.67064	-0.67505	-0.67946	-0.68387	-0.68828	-0.69269
40	-0.44374	-0.44314	-0.44254	-0.44194	-0.44135	-0.44075
50	-0.31557	-0.31257	-0.30957	-0.30657	-0.30357	-0.30057
60	-0.23591	-0.23181	-0.2277	-0.2236	-0.21949	-0.21538



From the table it is observed that when ν increases, the $Cov(X, \hat{S})$ increases and when μ increases the $Cov(X, \hat{S})$ decreases.

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