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Iterative method for statistical characterization of neutron flux

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Abstract

The paper presents derivation of equation for the mathematical expectation of neutron flux distribution in a nuclear reactor and describes the iterative algorithm for solving the equation. The convergence criterion of the iterative process is defined. Some results of numerical evaluations are presented to illustrate main ideas of the paper.

Keywords: Statistics, Nuclear reactor.

Introduction

Information about statistical characteristics of neutron flux under random fluctuations of the reactor properties plays a significant role for solving the problems related with control of heat generation field and determination of the homogenized constants [1-3]. Usually, the most attention is given to the correlation function of neutron distribution. As to the mathematical expectation, some researchers assume, as a rule, that the mathematical expectation is very close to solution of appropriate neutron transport equation on averaged constants [4]. The paper demonstrates that, under certain conditions, this assumption can be wrong. In contrast to Ref. 3, where approximate equation for mean neutron flux was derived for diffusion medium with the randomly fluctuating parameters, the paper presents basic equation and iterative method for solving the equation for the reactor model where material parameter can be perturbed.

Method for determination of the mathematical expectation of neutron flux distribution in a nuclear reactor Iterative algorithm

If one-group diffusion approximation with randomly perturbed material parameter æ$_0{}^2(\vec{r})$ is used to characterize random distribution of neutron flux, then spatial profile of random neutron flux submits to the following equation:

$$\Delta\varphi(r) + [\chi_0^2(r) + \varepsilon(r)]\varphi(r) + \theta\varphi(r) = 0 \qquad (1)$$

$$\varphi|_s = 0;$$

where:

$\chi_0^2(r)$ – local material parameter of the unperturbed reactor;

$\varepsilon(r)$ – random perturbation of material parameter;

θ – parameter responsible for the reactor criticality (homogeneous control).

Solution of equation (1) can be presented as a sum of the mathematical expectation $\overline{\varphi}(r)$ and a random summand:

$$\varphi(r) = \overline{\varphi}(r) + \delta\varphi(r) \qquad (2)$$

We applied the following assumptions in respect to statistical properties of random functions:

$$\hat{M}[\varepsilon(r)] = 0;\ \hat{M}[\varepsilon(r)\varepsilon(r')] = k(r,r'); \qquad (3)$$

$$\hat{M}[\theta] = 0;\ \hat{M}[\varphi(r)] = \overline{\varphi}(r) \qquad (4)$$

where:

$k(\vec{r},\vec{r}\,')$ – the known correlation function of neutron noise;

$\hat{M}$ – operator of the mathematical expectation.

By using expression (2), equation (1) can take the form:

$$\Delta\overline{\varphi} + \Delta\delta\varphi + \theta\overline{\varphi} + \theta\delta\varphi + \chi_1^2\overline{\varphi} + \chi_1^2\delta\varphi + \varepsilon\overline{\varphi} + \varepsilon\delta\varphi = 0 \qquad (5)$$

By applying operation of the mathematical expectation to equation (5), the following equation can be obtained:

$$\Delta\overline{\varphi} + \chi_1^2\overline{\varphi} + \hat{M}[\varepsilon\delta\varphi] + \hat{M}[\theta\delta\varphi] = 0. \qquad (6)$$

By term-wise subtracting equation (6) from equation (5) with neglecting the second-order terms, i. e. by assuming that $\theta\delta\varphi = 0$ and $\varepsilon\delta\varphi = 0$, the following equation can be derived for random deviations of neutron flux:

$$\Delta\delta\varphi + \chi_1^2\delta\varphi + (\theta+\varepsilon)\overline{\varphi} - K_\varepsilon - K_\theta = 0. \qquad (7)$$

where: $\hat{M}[\varepsilon\delta\varphi] = K_\varepsilon$ $\hat{M}[\theta\delta\varphi] = K_\theta$

Equations (6), (7) constitute the set that can be iteratively solved to determine the mathematical expectation and deviations of neutron flux from the mathematical expectation:

$$\begin{cases} \Delta\overline{\varphi} + \chi_1^2\overline{\varphi} + K_\varepsilon + K_\theta = 0, \\ \Delta\delta\varphi + \chi_1^2\delta\varphi + (\theta+\varepsilon)\overline{\varphi} - K_\varepsilon - K_\theta = 0. \end{cases} \qquad (8)$$

It is convenient to introduce differential operator $\hat{L}$ for description of the iterative algorithm:

$$\hat{L} = \Delta + \chi_1^2; \qquad (9)$$

where Δ is the Laplacian.

Then, the set (8) can be re-written as follows:

$$\begin{cases} \hat{L}\overline{\varphi} = -(K_\varepsilon + K_\theta) \\ \hat{L}\delta\varphi = (K_\varepsilon + K_\theta) - (\theta + \varepsilon)\overline{\varphi} \end{cases} \qquad (10)$$

or

$$\begin{cases} \overline{\varphi} = -\hat{L}^{-1}\{K_\varepsilon + K_\theta\} \\ \delta\varphi = \hat{L}^{-1}\{K_\varepsilon + K_\theta - (\theta+\varepsilon)\overline{\varphi}\} \end{cases} \qquad (11)$$

where $\hat{L}^{-1}$ – the inverse operator to $\hat{L}$. It can be shown that $\hat{L}^{-1}$ operator exists as a single one.

The mathematical expectations of neutron flux at K-1 and K-th steps of the iterative algorithm are related with each other. This relationship can be derived by applying the following mathematical operations.

Upon completion of K-1 steps it became known that $s^{k-1} = K_{\varepsilon}^{(k-1)} + K_{\theta}^{(k-1)}$ and $\overline{\varphi}^{(k-1)}$. By using the second equation from the set (11), the following relationship can be obtained:

$$\delta\varphi^{k} = \hat{L}^{-1}\{s^{(k-1)} - (\varepsilon+\theta)\overline{\varphi}^{(k-1)}\} \qquad (12)$$

The correlation functions K_{ε}^{k} and K_{θ}^{k} at K-th step can be written in the following forms:

$$K_{\varepsilon}^{k} = \hat{M}[\varepsilon\delta\varphi^{k}] = \hat{M}[\varepsilon\hat{L}^{-1}\{s^{(k-1)} - (\varepsilon+\theta)\overline{\varphi}^{(k-1)}\}] \qquad (13)$$

With accounting for linearity of $\hat{L}^{-1}$ operator, expression (13) can be re-written as follows:

$$K_{\varepsilon}^{k} = \hat{M}[\varepsilon]\hat{L}^{-1}\{s^{(k-1)}\} - \hat{M}[\varepsilon\hat{L}^{-1}\{(\varepsilon+\theta)\overline{\varphi}^{(k-1)}\}]$$

Since $\hat{M}[\varepsilon] = 0$, then

$$K_{\varepsilon}^{k} = -\hat{M}[\varepsilon\hat{L}^{-1}\{(\varepsilon+\theta)\overline{\varphi}^{(k-1)}\}] \qquad (14)$$

Similarly:

$$K_{\theta}^{k} = \hat{M}[\theta\delta\varphi^{k}] = \hat{M}[\theta\hat{L}^{-1}\{s^{(k-1)} - (\varepsilon+\theta)\overline{\varphi}^{(k-1)}\}] = \hat{M}[\theta]\hat{L}^{-1}\{s^{(k-1)}\} - \hat{M}[\theta\hat{L}^{-1}\{(\varepsilon+\theta)\overline{\varphi}^{(k-1)}\}]$$

т. е.

$$K_{\theta}^{k} = -M[\theta\hat{L}^{-1}\{(\varepsilon+\theta)\overline{\varphi}^{(k-1)}\}] \qquad (15)$$

By summing up expressions (15), (16), we can obtain:

$$s^{k} = K_{\varepsilon}^{k} + K_{\theta}^{k} = -M[(\varepsilon+\theta)\hat{L}^{-1}\{(\varepsilon+\theta)\overline{\varphi}^{(k-1)}\}] \qquad (16)$$

According to the first equation from the set (11):

$$\overline{\varphi}^{k} = -\hat{L}^{-1}\{K_{\varepsilon}^{k} + K_{\theta}^{k}\}$$

or:

$$\overline{\varphi}^{k} = \hat{L}^{-1}\{\hat{M}[(\varepsilon+\theta)\hat{L}^{-1}\{(\varepsilon+\theta)\overline{\varphi}^{(k-1)}\}]\} \qquad (17)$$

So, equation (18) links the mathematical expectations of neutron flux obtained at K-1 and K-th steps of iterative algorithm.

The iterative process of determining the mathematical expectation of neutron flux can be organized, for example, by such a way. The mathematical expectation $\overline{\varphi}^{k}$ is presented in the form:

$$\overline{\varphi}^{k} = \sum_{i=2}^{\infty} B_i^k \psi_i + \varphi_1 \qquad (18)$$

where $\{\psi_i\}$ are the eigenfunctions of a boundary value problem:

$$\Delta\psi_i + \chi_i^2\psi_i = 0$$

$$\psi|_S = 0\,.$$

Then, the mathematical expectation $\overline{\varphi}^{(k-1)}$ can be written in the form:

$$\overline{\varphi}^{(k-1)} = \varphi_1 + \sum_{j=2}^{\infty} B_j^{(k-1)}\psi_j \qquad (19)$$

Let us consider the operation $\hat{L}^{-1}\{(\varepsilon+\theta)\overline{\varphi}^{(k-1)}\}$. By definition of $\hat{L}^{-1}$ operator, its action results in function y that submits to the following equation:

$$\Delta y + \chi_1^2 y = (\varepsilon+\theta)\overline{\varphi}^{(k-1)} \qquad (20)$$

If solution of equation (20) is looked for in the form $y = \sum_{l=2}^{\infty} A_l\psi_l$, then substitution of this expression into equation (20) results in the following equation:

$$\sum_{l=2}^{\infty} A_l(\chi_1^2 - \chi_l^2)\psi_l = (\varepsilon+\theta)\varphi_1 + (\varepsilon+\theta)\sum_{j=2}^{\infty} B_j^{k-1}\psi_j \qquad (21)$$

The unknown coefficients A_l can be determined by using the following expression:

$$A_l = \frac{\int_V(\varepsilon+\theta)\varphi_1\psi_l dr + \sum_{j=2}^{\infty} B_j^{k-1}\int_V(\varepsilon+\theta)\psi_j\psi_l dr}{(\chi_1^2 - \chi_l^2) < \psi_l^2 >}$$

Then, function y takes the form:

$$y = \sum_{l=2}^{\infty} \frac{\int_V(\varepsilon+\theta)\varphi_1\psi_l dr + \sum_{j=2}^{\infty} B_j^{k-1}\int_V(\varepsilon+\theta)\psi_j\psi_l dr}{(\chi_1^2 - \chi_l^2) < \psi_l^2 >}\psi_l \qquad (22)$$

As $y(r) = \hat{L}^{-1}\{(\varepsilon(r)+\theta)\overline{\varphi}^{(k-1)}\}$, then expression $M[(\varepsilon(r')+\theta)\hat{L}^{-1}\{(\varepsilon(r)+\theta)\overline{\varphi}^{(k-1)}\}]$ can be transformed as follows:

$$M[(\varepsilon(r')+\theta)y(r)] \qquad (23)$$

With accounting for expression (22), expression (23) takes the form:

$$M[(\varepsilon(r')+\theta)y(r)] = \sum_{l=2}^{\infty} \frac{\int_V M[(\varepsilon(r')+\theta)(\varepsilon(r)+\theta)]\varphi_1\psi_l dr + \sum_{j=2}^{\infty} B_j^{k-1}\int_V M[(\varepsilon(r')+\theta)(\varepsilon(r)+\theta)]\psi_j\psi_l dr}{(\chi_1^2 - \chi_l^2) < \psi_l^2 >}\psi_l$$

or

$$M[(\varepsilon(r')+\theta)\hat{L}^{-1}\{(\varepsilon(r)+\theta)\overline{\varphi}^{(k-1)}\}] = \sum_{l=2}^{\infty} \frac{\int_V R(r,r')\varphi_1\psi_l dr + \sum_{j=2}^{\infty} B_j^{k-1}\int_V R(r,r')\psi_j\psi_l dr}{(\chi_1^2 - \chi_l^2) < \psi_l^2 >}\psi_l \qquad (24)$$

where $R(r,r') = M[(\varepsilon(r')+\theta)(\varepsilon(r)+\theta)]$.

According to expression (17), the mathematical expectation $\overline{\varphi}^{k}$ must satisfy the following equation:

$$\Delta\overline{\varphi}^{k} + \chi_1^2\overline{\varphi}^{k} = M[(\varepsilon(r')+\theta)\hat{L}^{-1}\{(\varepsilon(r)+\theta)\overline{\varphi}^{(k-1)}\}] \qquad (25)$$

As $\overline{\varphi}^{k} = \sum_{i=2}^{\infty} B_i^k\psi_i + \varphi_1$, then:

$$\sum_{i=2}^{\infty} B_i^k(\chi_1^2 - \chi_i^2)\psi_i = \sum_{l=2}^{\infty} \frac{\int_V R(r,r')\psi_1\psi_l dr + \sum_{j=2}^{\infty} B_j^{k-1}\int_V R(r,r')\psi_j\psi_l dr}{(\chi_1^2 - \chi_l^2) < \psi_l^2 >}\psi_l \qquad (26)$$

The expansion coefficients B_i^k can be determined with application of the following formula:

$$B_i^k = \sum_{l=2}^{\infty} \frac{\int_V\int_V R(r,r')\psi_1(r)\psi_l(r)\psi_i(r')drdr' + \sum_{j=2}^{\infty} B_j^{k-1}\int_V\int_V R(r,r')\psi_j(r)\psi_l(r)\psi_l(r')drdr'}{(\chi_1^2 - \chi_l^2)(\chi_1^2 - \chi_i^2) < \psi_l^2 >< \psi_i^2 >} \qquad (27)$$

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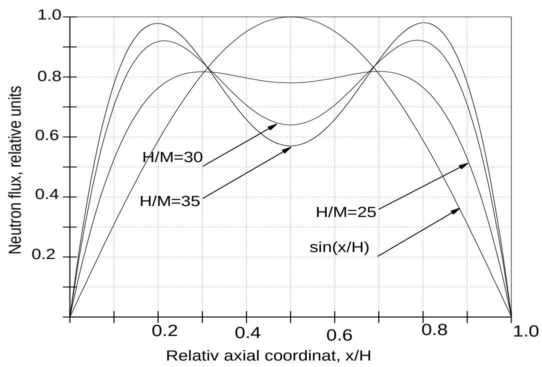
Then, the mathematical expectation of neutron flux at K-th iteration is described by the following series:

$$\overline{\varphi}^k = \varphi_1 + \sum_{i=2}^{\infty} \sum_{l=2}^{\infty} \frac{\iint\limits_{V\,V} R(r,r')\psi_1(r)\psi_l(r)\psi_l(r')\psi_i(r')drdr' + \sum_{j=2}^{\infty} B_j^{k-1} \iint\limits_{V\,V} R(r,r')\psi_j(r)\psi_l(r)\psi_l(r')\psi_i(r')drdr'}{(\chi_1^2-\chi_l^2)(\chi_1^2-\chi_i^2)<\psi_l^2><\psi_i^2>}\psi_i \quad (28)$$

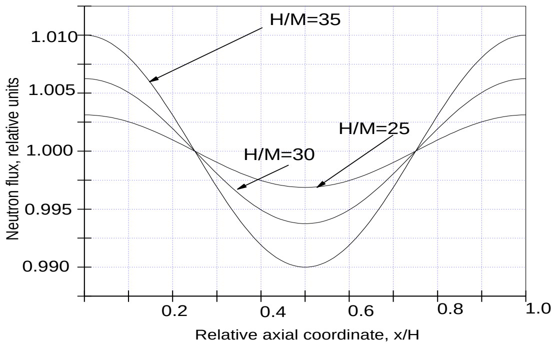
Results of numerical simulation

The formation of Axial profiles of neutron flux are presented in Fig. 1. for the reactor with zero boundary conditions (bare reactor) and for the reactor surrounded by "ideal" neutron reflector. It can be seen that as far as the reactor height increases, the difference between the mathematical expectation of neutron flux in the perturbed reactor and the mathematical expectation of neutron flux in the reactor with "averaged" properties becomes larger too. Axial profiles of neutron flux are presented in Fig. 2. for various perturbation levels of the reactor properties.

It became evident that the mathematical expectations of neutron flux in the reactor with random fluctuations of its properties did not coincide at all with axial profile of neutron flux in the reactor with "averaged" properties, and the larger reactor size resulted in the larger differences. It should be noted here that these differences depended on quality of the neutron reflector, namely the difference effect in the reactor surrounded by "ideal" neutron reflector is stronger that that in the bare reactor.



a)



b)

Fig. 1. Mathematical expectations of neutron flux in one-dimensional plane reactor (a – bare reactor; b – reactor with "ideal" neutron reflector). The perturbation level = 1%. The correlation function $K(r,r') = D_\varepsilon \delta(r-r')$.

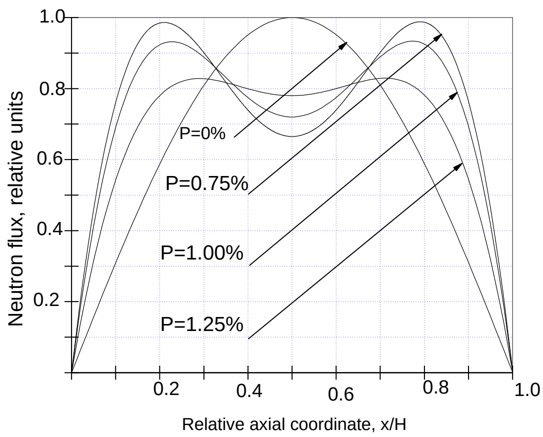


Fig. 2. Mathematical expectations of neutron flux in one-dimensional plane reactor without neutron reflector for various perturbation levels. The correlation function $K(r,r') = D_\varepsilon \delta(r-r')$ H/M =-35.

The following typical features of axial neutron distributions should be marked out:

- Spatial symmetry of axial profiles.
- Depression of neutron flux in a central zone.

Probably, the second feature confirms once more the well-known axial warping effect in the reactor halves for the physically large cores. The stronger correlation link between perturbations of the reactor properties and the larger perturbation levels resulted in the higher differences between the mathematical expectations of neutron flux and axial profiles of neutron flux in the reactor with "averaged" properties.

Convergence of the iterative algorithm

The following set of differential equations can be written for the true mathematical expectation of neutron flux $\overline{\varphi}$ and the mathematical expectation of neutron flux $\overline{\varphi}^i$ at i-th iteration:

$$\begin{cases} \hat{L}\overline{\varphi} = -(K_\varepsilon + K_\theta) \\ \hat{L}\delta\varphi = K_\varepsilon + K_\theta - (\varepsilon+\theta)\overline{\varphi} \end{cases} \quad (29)$$

$$\begin{cases} \hat{L}\overline{\varphi}^i = -(K_\varepsilon{}^i + K_\theta{}^i) \\ \hat{L}\delta\varphi^i = K_\varepsilon{}^{i-1} + K_\theta{}^{i-1} - (\varepsilon+\theta)\overline{\varphi}^{i-1} \end{cases} \quad (30)$$

where $\delta\varphi, K_\varepsilon, K_\theta$ – the true deviations from the mathematical expectation of neutron flux and the true correlation functions while $\delta\varphi^i, K_\varepsilon^i, K_\theta^i$ – the deviations from the mathematical expectation of neutron flux and the correlation functions at i-th iteration.

By subtracting the first equation of the set (29) from the first equation of the set (30), we obtained:

$$\hat{L}d\overline{\varphi}^i = dK_\varepsilon^i + dK_\theta^i \quad (31)$$

where:

$$dK_\varepsilon^i = K_\varepsilon^i - K_\varepsilon = -M[\varepsilon d\delta\varphi^i] \tag{32}$$

$$dK_\theta^i = K_\theta^i - K_\theta = -M[\theta d\delta\varphi^i] \tag{33}$$

By subtracting the second equation of the set (29) from the second equation of the set (30), we obtained:

$$\hat{L}d\delta\varphi^i = -(dK_\varepsilon^{i-1} + dK_\theta^{i-1}) - (\varepsilon+\theta)d\overline{\varphi}^{i-1} \tag{34}$$

Equations (31), (34) can be united into the single set:

$$\begin{cases} d\overline{\varphi}^i = \hat{L}^{-1}\{dK_\varepsilon^i + dK_\theta^i\} \\ d\delta\varphi^i = -\hat{L}^{-1}\{dK_\varepsilon^{i-1} + dK_\theta^{i-1} + (\varepsilon+\theta)d\overline{\varphi}^{i-1}\} \end{cases} \tag{35}$$

The deviations of the correlation functions and should be considered in more details. Taking expression for into account, we obtained:

$$dK_\varepsilon^i = -M[\varepsilon d\delta\varphi^i] = M[\varepsilon\hat{L}^{-1}\{dK_\varepsilon^{i-1} + dK_\theta^{i-1} + (\varepsilon+\theta)d\overline{\varphi}^{i-1}\}]$$

As $\hat{L}^{-1}$ and M are linear operators, then:

$$dK_\varepsilon^i = M[\varepsilon\hat{L}^{-1}\{dK_\varepsilon^{i-1} + dK_\theta^{i-1}\}] + M[\varepsilon\hat{L}^{-1}(\varepsilon+\theta)d\overline{\varphi}^{i-1}\}]$$

The first summand equals zero because $\hat{L}^{-1}\{dK_\varepsilon^{i-1} + dK_\theta^{i-1}\}$ is a non-random function that can be withdrawn from the mathematical expectation, and $M[\varepsilon]=0$.

Then:

$$dK_\varepsilon^i = M[\varepsilon\hat{L}^{-1}\{(\varepsilon+\theta)d\overline{\varphi}^{i-1}\}] \tag{36}$$

Similarly:

$$\begin{aligned} dK_\theta^i &= -M[\theta d\delta\varphi^i] = M[\theta\hat{L}^{-1}\{dK_\varepsilon^{i-1} + dK_\theta^{i-1} + (\varepsilon+\theta)d\overline{\varphi}^{i-1}\}] = \\ &= M[\theta\hat{L}^{-1}\{dK_\varepsilon^{i-1} + dK_\theta^{i-1}\}] + M[\theta\hat{L}^{-1}\{(\varepsilon+\theta)d\overline{\varphi}^{i-1}\}] \\ dK_\theta^i &= M[\theta\hat{L}^{-1}\{(\varepsilon+\theta)d\overline{\varphi}^{i-1}\}] \end{aligned} \tag{37}$$

By summing up expressions (36) and (37), we obtained:

$$dK_\varepsilon^i + dK_\theta^i = M[(\varepsilon+\theta)\hat{L}^{-1}\{(\varepsilon+\theta)d\overline{\varphi}^{i-1}\}]$$

By substituting this sum into the first equation of the set (35), the following result can be obtained:

$$d\overline{\varphi}^i = \hat{L}^{-1}\{M[(\varepsilon+\theta)\hat{L}^{-1}\{(\varepsilon+\theta)d\overline{\varphi}^{i-1}\}]\} \tag{38}$$

The iterative process is a convergent one if:

$$\frac{\left\|d\overline{\varphi}^i\right\|}{\left\|d\overline{\varphi}^{i-1}\right\|} < 1 \tag{39}$$

By using the well-known properties of norms of mathematical operators and functions, the norm of expression (38) can be evaluated:

$$\left\|d\overline{\varphi}^i\right\| = \left\|\hat{L}^{-1}\{M[(\varepsilon+\theta)\hat{L}^{-1}\{(\varepsilon+\theta)d\overline{\varphi}^{i-1}\}]\}\right\| \le \left\|\hat{L}^{-1}\right\|\left\|M[(\varepsilon+\theta)\hat{L}^{-1}\{(\varepsilon+\theta)d\overline{\varphi}^{i-1}\}]\right\| \le$$
$$\le \left\|\hat{L}^{-1}\right\|\left\|\varepsilon+\theta\right\|\left\|\hat{L}^{-1}\right\|\left\|\varepsilon+\theta\right\|\left\|d\overline{\varphi}^{i-1}\right\|$$

or

$$\left\|d\overline{\varphi}^i\right\| \le \left\|\hat{L}^{-1}\right\|^2 \left\|\varepsilon+\theta\right\|^2 \left\|d\overline{\varphi}^{i-1}\right\|$$

In accordance with the criterion (39), the following inequality must be correct for convergence of the iterative process:

$$\left\|\hat{L}^{-1}\right\|^2 \left\|\varepsilon+\theta\right\|^2 < 1 \tag{40}$$

The convergence domain of the iterative process can be found for the bare one-dimensional plane reactor (H – the reactor height; $\chi_1^2 = \left(\frac{\pi}{H}\right)^2$ -material parameter of the reactor; $\varepsilon(r)$ -random perturbation of material parameter).

If dimensionless variable $x = \frac{r}{H}$ is introduced, then operator $\hat{L}$ takes the form:

$$\hat{L} = \frac{d^2}{dx^2} + \pi^2,$$

$$\varphi_1 = \sin \pi x.$$

Norm of inverse operator $\hat{L}^{-1}$ equals $\frac{1}{\lambda_{\min}}$, where $\lambda_{\min}$ is a minimal non-zero eigenvalue of operator $\hat{L}$. In general, eigenvalues of operator $\hat{L}$ can be calculated as $\lambda = \pi^2(n^2-1), n = 1,2\ldots$ Evidently, minimal non-zero eigenvalue corresponds to n=2, i. e. :

$$\lambda_{\min} = 3\pi^2, \text{ and } \left\|\hat{L}^{-1}\right\|^2 = \frac{1}{9}\pi^4 \tag{41}$$

In accordance with the norm definition of a random function in Hilbert space L_2, the following norm can be evaluated:

$$\left\|\varepsilon(r)+\theta\right\|^2 = H^4\int_0^1 M[(\varepsilon(x)+\theta)^2]dx = H^4\int_0^1 M[(\varepsilon(x) - \frac{\int_0^1 \varepsilon \sin^2 \pi x dx}{\int_0^1 \sin^2 \pi x dx})^2]dx$$

Assuming that the perturbation parameter $\varepsilon(r)$ and the correlation function K(x, x') are known (for example, $K(x,x') = M[\varepsilon(x)\varepsilon(x')] = D_\varepsilon \delta(x-x')$), it is easy to obtain the following norm:

$$\left\|(\varepsilon+\theta)\right\|^2 = 0{,}5 D_\varepsilon H^4 \tag{42}$$

where D_ε – the perturbation dispersion.

The perturbation dispersion for material parameter can be presented as $D_\varepsilon \cong \frac{D[K_\infty]}{M^4}$, where $M^2 = (L^2+\tau)$ - migration area, K_∞ – neutron multiplication factor.

Assuming that K_∞=1, 03, dispersion of neutron multiplication factor can be evaluated as $D[K_\infty] \approx 2{,}121*10^{-4}p^2$, where p – the perturbation level of the reactor macroscopic cross-sections in percents.

So, $D_\varepsilon = \frac{2{,}121*10^{-4}p^2}{(L^2+\tau)^2}$, and expression (42) takes the form:

$$\left\|\varepsilon+\theta\right\|^2 = \frac{2{,}121*10^{-4}p^2}{2}\left(\frac{H}{\sqrt{L^2+\tau}}\right)^4 \tag{43}$$

With accounting for expression (2. 3. 35), the convergence criterion can be defined as follows:

$$\frac{2{,}121*10^{-4}}{2*9*\pi^4}\left(\frac{H}{\sqrt{L^2+\tau}}\right)^4 p^2 < 1$$

or

$$\frac{H}{\sqrt{L^2+\tau}} < \frac{10\pi\sqrt{3}}{\sqrt{p}} \qquad (44)$$

The convergence domain of the iterative process is shown in Fig. 3. Dependence of maximal relative error in determination of axial neutron distribution $\varepsilon = \max_x \frac{\sqrt{\int_0^1 (\overline{\varphi}_k(x) - \overline{\varphi}_{k-1}(x))^2 dx}}{\int_0^1 \overline{\varphi}_k(x) dx}$ on the number of iterations is shown in Fig. 4.

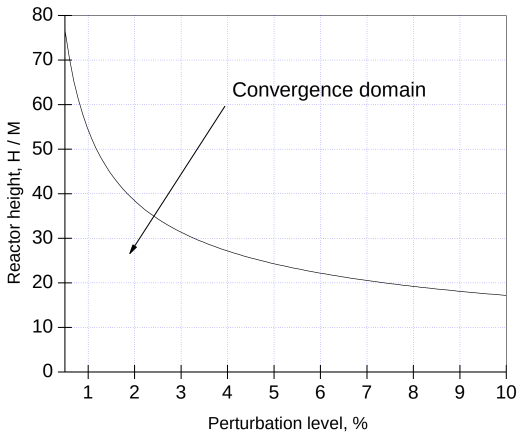


Fig. 3. Convergence domain of the iterative algorithm

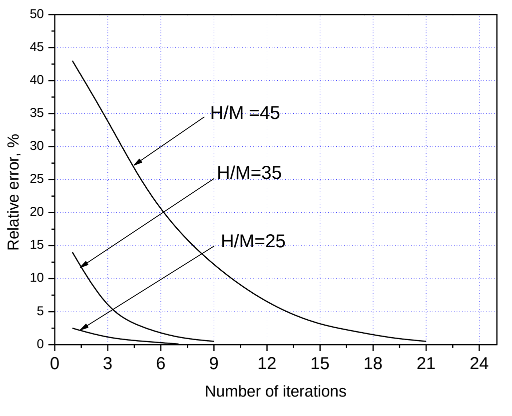


Fig. 4. Dependence of maximal relative error on the number of iterations

As is seen, the smaller reactor height resulted in the quicker convergence of the iterative algorithm. Numerical studies revealed that the iterative search for the mathematical expectation required substantially lower (by one order of magnitude) number of operations than that of the statistical experiment method.

Conclusion

In general, the following conclusions can be derived from the performed numerical studies:

1. The mathematical expectation of neutron flux does not coincide with the neutron distribution calculated with application of averaged macroscopic cross-sections.
2. The discrepancy increases when physical size of the reactor core, the perturbation level and the correlation level become larger.
3. The iterative algorithm for determination of the mathematical expectation appeared substantially quicker than the statistical experiment method. In addition, the iterative algorithm allowed us to evaluate the first-order correction to the linear model.

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