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Operators induced by fuzzy stack on a fuzzy topological space

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Abstract –
In the present paper we introduce operators$\emptyset_s$, $\psi_s$induced by given fuzzy stack S and fuzzy topology τ. With the help of operators and the collectionτ^s, a new fuzzy topology $\tau^{S^*}$is obtained. It is shown that $\tau \subseteq \tau^s \subseteq \tau^{S^*}$.We have also shown various relations between the fuzzy stack oriented collections and given fuzzy topology. A suitability condition is applied on fuzzy stack, which makes the fuzzy topology better defined.

Key words - Fuzzy grill, $\emptyset_S$ - operator, ψ_S– operator

1.INTRODUCTION

Choquet [**2**] introduced the notion of grills in general topology. In fuzzy setting, the notion of fuzzy grill and fuzzy stack was introduced by K.K. Azad[1]. Since then fuzzy grills and fuzzy stacks are studied by many authors in topologicalproximity and nearness spaces[**4**, **5**, **8**, **10**, **12**].
In 2007, Roy and Mukherjee [5] introduced an operator defined by a grill on a topologicalspace. Das & Mukherjee [4] introduced an operator by fuzzy grill on fuzzy topological space and showed that it satisfy Kuratowski closure axioms.
In 2012, Min and Kim [11] studied operators induced by stacks on a topological space. In the present paper, we introduced operators with the help of fuzzy stack on a given fuzzy topology. It is obviously a generalized notion of operator defined by fuzzy grill on a fuzzy topological space in [4]. In section 3, the operator $\emptyset_s$is defined with the help of a fuzzy stack S and fuzzy topology $\tau$and some of the basic properties are studied. Further the operator $\psi_s$is defined by operator $\emptyset_s$.The collection τ^s of fuzzy sets induced by operator ψ_s are studied. In particular, we show that collection of fuzzy sets τ^s induced by ψ_s is a super collection of given fuzzytopology τ. In section 4, we applied some suitability condition on fuzzy stack. If S is a fuzzy stack on I^X and τ is a fuzzy topology on I^Xthen, for $V\in \tau$, $B \notin S \Rightarrow V–B\in \tau^s$, it is shown with the help of suitability condition that$\tau^s = \{V - B : V \in \tau, B \notin S\}$where $V, B \in I^X$.

2.PRELIMINARIES

A fuzzy set$A$ in a set $X$is a function on X in to the closed unit interval $[0,1]$ of the real line. Support of $A = \{x \in X: A(x) > 0\}$ is denoted by Supp A.The fuzzy sets in X taking on the constant values 0 and 1 are denoted by 0_X and 1_Xrespectively.For $A, B \in I^X, A \le B$ *if* $A(x) \le B(x)$ for each$x \in X$.By AqB, we mean that A is quasi - coincident with Bi.e.AqB implies$A(x) + B(x) > 1$forsome $x \in X$.The negations of these statements are denoted by$A\bar{q}B$.A fuzzy singleton or a fuzzy point with support $X$and value $p(0 < p \le 1)$ is denoted by x_p.The collection of all open $q-nbds$ of any fuzzy point x_p is denoted by $Q(x_p)$.A subfamily β of the fuzzy topology τ of an fts (X,τ) is a base for τ iff for each fuzzy singleton x_p in (X,τ) and for each open q$-$nbd U of x_p, $x_p q$B for some $B \in \beta$with $B \le U$. A fuzzy point x_p is called an adherence point of fuzzy set A if every open $q-nbd$of x_p is a quasi – coincident with A and $Cl(A)$ is the union of all adherence points of A.$M(A)$ is set of molecules of I^Xin A.$VM(\downarrow A) = A$, for $A \in I^X$where$\downarrow A = \{b \in P : \exists\, a \in A\, s.t.\, b \le a\}$, P is a Poset.

3 COLLECTION τ^s INDUCED BY FUZZY STACK *S*

Definition 3.1 - Let (I^X,τ) be a fuzzy topological space and S be a fuzzy stack on I^X. For$A \in I^X$, a mapping $\emptyset_S : I^X \to I^X$ denotedby$\emptyset_S(A) = \vee\{x_p : A * U \in S$ for all $U \in Q(x_p)\}$is called operator associated with fuzzy stack S and fuzzy topology on I^X.

Remarks 3.2 - For $A \in I^X$, $x_p \nleq \emptyset_S(A)$ iff $\exists\, U \in Q(x_p)$such that $U * A \notin S$.

Proposition 3.3 - Let (I^X,τ) be a fuzzy topological space and S be a fuzzy stack on I^X.

(i) For $A, B\in I^X$, $A \le B \Rightarrow \emptyset_S(A) \le \emptyset_S(B)$.

(ii) If $S_1$and S_2 are two fuzzy stack on I^X with$S_1 \subseteq S_2$, then $\emptyset_{S_1}(A) \le \emptyset_{S_2}$ (A) for $A\in I^X$.

(iii) For any fuzzy stack Son I^X and $A\in I^X$, if $A \notin S$, then $\emptyset_S(A) = 0_X$.

Proof –

(i) Let $x_p \le \emptyset_S(A)$, then for all $U\in Q(x_p)$, $A * U\in S$. Since $A \le B \Rightarrow A * U \le B * U$; but S is fuzzy stack so $B * U \in S$ for all $U \in Q(x_p)$, this implies $x_p \le \emptyset_S(B)$.

(ii) Suppose$x_p \le \emptyset_{S_1}(A)$, this implies that for all $U\in Q(x_p)$, $A * U \in S_1 \subseteq S_2$, then $x_p \le \emptyset_{S_2}(A)$.

(iii) Let $x_p \le \emptyset_S(A)$& $A\notin S \Rightarrow$ for all $U\in Q(x_p)$, $A * U \in S$, so we arrive at a contradiction. Thus $\emptyset_S(A) = 0_X$if $A\notin S$.

Remark 3.3.1 - $\emptyset_S(0_X) = 0_X$

Example 3.3.2 - $\emptyset_S(1_X) \neq 1_X$
We will prove $\exists$ a x_p such that $x_p \nless \emptyset_S(1_X)$ but $x_p \leq 1_X$. Let $X = \{a, b\}$, $\tau = \{c, 0_X, 1_X\}$ where $c(a) = 0.2$, $c(b) = 0.3$. Let $S = \{1_X all\ fuzzy\ sets\ T\ s.t. 0.3 \leq T(a) \leq 1, 0.4 \leq T(b) \leq 1\}$, here, S is a fuzzy stack and τ is a fuzzy topology.
Proposition 3.4 - Let (I^X, τ) be a fuzzy topological space and S be a fuzzy stack on I^X. Then, for $A \in I^X$.
(i) $\emptyset_S(A) \leq cl(A)$
(ii) $cl(\emptyset_S(A)) \leq \emptyset_S(A)$

Proof -
(i) Suppose that $x_p \nleq cl(A)$, then there exist an open q-nbd U of x_p in X s.t. $U\bar{q}A$, that is, $A(y) + U(y) \leq 1$ for each $y \in X$ and hence $A * U = O_X \notin S$, then $x_p \nleq \emptyset_S(A)$.
(ii) Let $x_p \leq cl(\emptyset_S(A)$ and $U \in Q(x_p)$, then $\emptyset_S(A) q U$ that is, there exist $y \in X$ such that $\emptyset_S(A)(y) + U(y) > 1$, let $\emptyset_S(A)(y) = t$, this implies $y_t \leq \emptyset_S(A)$ and $U \in Q(y_t) \Rightarrow A * U \in S \Rightarrow x_p \leq \emptyset_S(A)$.

Corollary 3.4.1 - For a fuzzy topological space (I^X, τ), $\emptyset_S(\emptyset_S(A)) \leq \emptyset_S(A)$ where $A \in I^X$.

Proof - By 3.4 (i) $\emptyset_S(\emptyset_S(A)) \leq cl(\emptyset_S(A))$ and by 3.4 (ii) $cl(\emptyset_S(A)) \leq \emptyset_S(A)$, this implies $\emptyset_S(\emptyset_S(A)) \leq \emptyset_S(A)$.

Proposition 3.5 - For fuzzy topological space (I^X, τ), $\emptyset_S$ is closed for $A \in I^X$.

Proof - By 3.4 (ii), $cl(\emptyset_S(A)) \leq \emptyset_S(A)$ and by definition of closure, $\emptyset_S(A) \leq cl(\emptyset_S(A))$. Then $cl(\emptyset_S(A)) = \emptyset_S(A)$. This shows that $\emptyset_S(A)$ is closed for $A \in I^X$.

Definition 3.6 - Let S be a fuzzy stack on fuzzy topological space (I^X, τ). We define $\psi_S : I^X \to I^X$ by $\psi_S(A) = A \vee \emptyset_S(A)$ where $A \in I^X$.

Proposition 3.7 - The above defined operator ψ_S satisfy the following conditions
(i) $\psi_S(0_X) = 0_X$
(ii) $A \leq \psi_S(A)$ for $A \in I^X$
(iii) For $A, B \in I^X$, $A \leq B \Rightarrow \psi_S(A) \leq \psi_S(B)$
(iv) For $A, B \in I^X$, $\psi_S(A \wedge B) \leq \psi_S(A) \wedge \psi_S(B)$

Proof - (i) & (ii) Follows from definition
(iii) For $A, B \in I^X$, $\psi_S(A) = A \vee \emptyset_S(A) \leq B \vee \emptyset_S(B) = \psi_S(B)$, so $A \leq B$ implies $\psi_S(A) \leq \psi_S(B)$.
(iv) For $A, B \in I^X$, $A \wedge B \leq A, B$. Using proposition 3.7, $\psi_S(A \wedge B) \leq \psi_S(A), \psi_S(B)$, which gives $\psi_S(A \wedge B) \leq \psi_S(A) \wedge \psi_S(B)$.

Example 3.7.1 - Let (I^X, τ) be a fuzzy topological space where $X = \{a, b\}$, $\tau = \{c, 0_X, 1_X)$, $c(a) = 0.5$, $c(b) = 0.2$
S be a fuzzy stack containing 1_X and all fuzzy set S s.t. $S(a) > 0.1$, $S(b) > 0.2$, we will prove that $\psi_S(A \wedge B) \neq \psi_S(A) \wedge \psi_S(B)$, $A, B \in I^X$ s.t. Let $A(a) = 0.1$, $A(b) = 0.5$, $B(a) = 0.3$, $B(b) = 0.3$. Here $A \notin S$ so $\emptyset_S(A) = 0_X$, thus $\psi_S(A) = A \vee \emptyset_S(A) = A$. $(A \wedge B)(a) = 0.1$, $(A \wedge B)(b) = 0.3 \Rightarrow A \wedge B \notin S$ so $\emptyset_S(A \wedge B) = 0_X$, thus $\psi_S(A \wedge B) = (A \wedge B)$, $B(a) = 0.3$, $B(b) = 0.3$; $\emptyset_S(B) = a_{p_1} \vee b_{p_2}$ where $p_1 \leq 0.5$, $p_2 \leq 0.8$, $\psi_S(B)(a) = (B \vee \emptyset_S(B))(a) = 0.5$, $\psi_S(B)(b) = 0.8$, $\psi_S(A \wedge B) \neq \psi_S(A) \wedge \psi_S(B)$.

Example 3.7.2 - Let (I^X, τ) be a fuzzy topological space where $X = \{a, b\}$, $\tau = \{c, 0_X, 1_X)$, $c(a) = 0.5$, $c(b) = 0.2$. Let S be a fuzzy stack containing 1_X and all fuzzy set S such that $S(a) > 0.1$, $S(b) > 0.2$. We will prove that $\psi_S(A \vee B) \neq \psi_S(A) \vee \psi_S(B)$, $A, B \in I^X$ such that Let $A(a) = 0.1$, $A(b) = 0.3$, $B(a) = 0.3$, $B(b) = 0.1$. $\psi_S(A) = A \vee \emptyset_S(A) = A \vee 0_X = A$, similarly $\psi_S(B) = B \vee \emptyset_S(B) = B (\because B \notin S)$. But $(A \vee B)(a) = 0.3$, $(A \vee B)(b) = 0.3$, $\psi_S(A \vee B)(a) = 0.5$, $\psi_S(A \vee B)(b) = 0.8$, $\psi_S(A \vee B) \neq \psi_S(A) \vee \psi_S(B)$.

Definition 3.8 - Let (I^X, τ) be a fuzzy topological space and S be fuzzy stack on I^X, then
$\tau^S = \{U \in I^X : \psi_S(1_X - U) = (1_X - U)\}$

Proposition 3.9.1 - Let (I^X, τ) be fuzzy topological space and S be a fuzzy stack on I^X. Then
(i) $0_X, 1_X \in \tau^S$
(ii) If $U_p \in \tau^S$ for $p \in J$, then $\vee U_p \in \tau^S$

Proof -
(i) Since $\psi_S(0_X) = 0_X$ and $\psi_S(1_X) = 1_X \Rightarrow 1_X, 0_X \in \tau^S$.
(ii) Suppose $U_p \in \tau^S$ for all $\propto \in J$. Let $U_p \in \tau^S$, then $\psi_S(1_X - U_p) = 1_X - U_p$ and so $(1_X - U_p) \vee \emptyset_S(1_X - U_p) = (1_X - U_p)$. So $\emptyset_S(1_X - U_p) \leq 1_X - U_p$. This shows $\emptyset_S(1_X - \vee U_p) \leq \emptyset_S(1_X - U_p) \leq 1_X - U_p$, which gives $\emptyset_S(1_X - \vee U_p) \leq \wedge (1_X - U_p) = (1_X - \vee U_p$. So $\psi_S(1_X - \vee U_p) = (1_X - \vee U_p)$. Hence $\vee U_p \in \tau^S$.

Remark 3.9.2 - Let (I^X, τ) be a fuzzy topological space and S be a fuzzy stack on I^X, elements of τ^S are said to be $\tau^S - open$. If the complement of a fuzzy set is $\tau^S - open$, then fuzzy set is called $\tau^S - closed$.

Theorem 3.10 - Let (I^X, τ) be a fuzzy topological space and S be a fuzzy stack on I^X. Then for $V \in \tau$ and $A \notin S$, $V - A \in \tau^S$.

Proof - Suppose that $U = V - A$ s.t. $V \in \tau$ and $A \notin S$. We show that $\emptyset_S(1_X - U) \leq 1_X - U$. Let there exists a fuzzy point x_p such that $x_p \leq \emptyset_S(1_X - U)$, but $x_p \nleq 1_X - U$, which implies that for each $W \in Q(x_p)$, $W + (1_X - U) - 1 \in S$. Since $W + 1 - U - 1 = W - U \in S$, $W - (V - A) \in S$, but $x_p \nleq 1_X - U \Rightarrow p > (1_X - U)(x) = 1 - (V - A)(x) \Rightarrow p + V(x) > 1 + A(x) \geq 1$
[In fact $V(x) \leq A(x)$ is not possible since in that case $(V - A)(x) = 0$, this implies $1 - (V - A)(x) = 1 - 0 < p$. Thus $V(x) > A(x)$ and $(V - A)(x) = V(x) - A(x)$. Then $p > 1 - (V - A)(x) = 1 - V(x) + A(x) \Rightarrow p + V(x) > 1 + A(x) \geq 1$.]
Hence $V \in Q(x_p)$. As $V - (V - A) \in S$ but $V - (V - A) \leq A \Rightarrow A \in S$, contradiction to $A \notin S$. Then $\emptyset(1_X - U) \leq 1_X - U$. Thus $V - A \in \tau^S$.

Corollary 3.10.1 - Let S be a fuzzy stack on (I^X, τ), then $\tau \subseteq \tau^S$

Definition3.11 -
For $A \in I^X$, we define $i_{\tau}s: I^X \to I^X$, $c_{\tau}s: I^X \to I^X$, suchthat $i_{\tau}s(A) = V\{U \in I: U \leq A$ and $U\ \tau^S\}$, $c_{\tau}s(A) = \wedge \{F \in I^X : A \leq F$ and $1_X - F \in \tau^S\}$.

Remark 3.11.1 - Let(I^X, τ) be a fuzzy topological space and S be a fuzzy stack on I^X. For $A \in I^X$, $x_p \leq i_{\tau}s(A)$ if and only if there exists a τ^S –open set U such that $x_p \leq U$ and $U \leq A$.

Corollary 3.11.2 - Let (I^X, τ) be a fuzzy topological space and S be a fuzzy stack on I^X. If for $A \in I^X$, $x_p \leq i_{\tau}s(A)$, then there exist $W \in Q\ (x_p)$such that$(1_X - A) * W \notin S$.

Proof– Let $x_p \leq i_{\tau}s(A)$ then there exists a τ^S –open set U such that $x_p \leq U$ and $U \leq A$. Then$1_X - A \leq 1_X - U = \psi_S(1_X - U)$. But$x_p \nleq \emptyset_S(1_X - U) \leq (1_X - U)$.So, there exists $W \in Q\ (x_p)$such that$(1_X - U) * W \notin S \Rightarrow (1_X - A) * W \notin S$.

Theorem 3.12 - Let (I^X, τ) be a fuzzy topological space and S be a fuzzy stack on I^X. For $A, B \in I^X$
(i) $i_{\tau}s(0_X) = 0_X$, $i_{\tau}s(1_X) = 1_X$
(ii) $i_{\tau}s(A) \leq A$
(iii) *if* $A \leq B$, then$i_{\tau}s(A) \leq i_{\tau}s(B)$
(iv) $i_{\tau}s(i_{\tau}s(A)) = i_{\tau}s(A)$
(v) $i_{\tau}s(A \wedge B) = i_{\tau}s(A) \wedge i_{\tau}s(B)$

Theorem 3.13 - Let (I^X, τ)be a fuzzy topological space and S be a fuzzy stack on I^X. For $A, B \in I^X$,
(i) $c_{\tau}s(1_X) = 1_X$, $c_{\tau}s(0_X) = 0_X$
(ii) $A \leq c_{\tau}s(A)$
(iii) If $A \leq B \Rightarrow c_{\tau}s(A) \leq c_{\tau}s(B)$
(iv) $c_{\tau}s(c_{\tau}s(A)) = c_{\tau}s(A)$
(v) $c_{\tau}s(A \vee B) = c_{\tau}s(A) \vee c_{\tau}s(B)$

Remark 3.13.1 - $c_{\tau}s$ is a Kuratowski closure operator

Theorem 3.14 - Let (I^X, τ) be a fuzzy topological space and S be a fuzzy stack on I^X. For $A, B \in I^X$, A is τ^S –*closed* iff $\psi_S(A) = A$

Proof - For $A \in I^X$, A is τ^S –closed iff $1_X - A$ is τ^S –*open* iff$\psi_S(1_X - (1_X - A)) = (1_X - (1_X - A))$ iff $\psi_S(A) = A$

Theorem 3.15 - Let (I^X, τ)be a fuzzy topological space and S be a fuzzy stack on I^X. For $A \in I^X$,
(i) $A \notin S \Rightarrow 1_X - A \in \tau^S$
(ii) $\emptyset_S(A)$ *is* τ^S –*closed*
(iii) $\psi_S(A) \leq c_{\tau}s(A)$

Proof -
(i) Let $A \notin S$, then$\emptyset_S(A) = 0_X$, $\psi_S(1_X - (1_X - A)) = \psi_S(A) = A \vee \emptyset_S(A) = A = 1_X - (1_X - A)$, this implies $1_X - A \in \tau^S$.
(ii) Since $\psi_S(\emptyset_S(A)) = \emptyset_S(A) \vee \emptyset_S(\emptyset_S(A)) = \emptyset_S(A)$, which gives$\emptyset_S(A)$ *is*τ^S *- closed.*
(iii) Since $\psi_S(A) \leq \psi_S(c_{\tau}s(A))$, we have$\psi_S(A) \leq \psi_S(c_{\tau}s(A)) = c_{\tau}s(A)$

Definition 3.16 - Define $\tau^{S^*} = \{A \in I^X : c_{\tau}s(1_X - A) = (1_X - A)\}$ also$\tau^{S^*}$is a fuzzy topology on I^X, where $c_{\tau}s$ is a Kurotowski closure operator.

Note 3.16.1 - $\tau \subseteq \tau^{S^*} \subseteq \tau^S$

Example 3.16.2 - In an fuzzy topological space(I^X, τ), the trivial fuzzy stack is $I^X / \{0_X\}$. If we take $S = I^X / \{0_X\}$, this implies for $A \in I^X$, $\emptyset_S(A) = \psi_S(A) = cl(A) = c_{\tau}s(A)$, In this case$\tau = \tau^{S^*} = \tau^S$.

Proposition3.17 - Let (I^X, τ)be a fuzzy topological space and S_1 and S_2be fuzzy stack on I^X. For $A \in I^X$,
(i) If$S_1 \subseteq S_2$, then$\tau^{S_2} \subseteq \tau^{S_1}$
(ii) If$S_1 \subseteq S_2$, then$c_{\tau}s_1(A) \leq c_{\tau}s_2(A)$
(iii) $\tau^{S_2{}^*} \subseteq \tau^{S_1{}^*}$, where$\tau^{S^*} = \{A : c_{\tau}s(1_X - A) = (1_X - A)\}$

Proof -
(i) Since$\tau^S = \{A: \psi_S(1_X - A) = (1_X - A)\}$. Let $U \in \tau^{S_2}$, this implies $\psi_{S_2}(1_X - U) = (1_X - U) \Rightarrow \emptyset_{S_2}(1_X - U) \leq (1_X - U)butS_1 \subseteq S_2$, so$\emptyset_{S_1}(1_X - U) \leq \emptyset_{S_2}(1_X - U) \leq (1_X - U) \Rightarrow \psi_{S_1}(1_X - U) = (1_X - U) \Rightarrow U \in \tau^{S_1}$
(ii) Follows from definition.
(iii) Let $U \in \tau^{S_2{}^*}$, then$c_{\tau}s_2(1_X - U) = (1_X - U)$So, $c_{\tau}s_1(1_X - U) \leq c_{\tau}s_2(1_X - U) = (1_X - U)$, which gives$c_{\tau}s_1(1_X - U) \leq (1_X - U)but(1_X - U) \leq c_{\tau}s_1(1_X - U) \Rightarrow c_{\tau}s_1(1_X - U) = (1_X - U) \Rightarrow U \in \tau^{S_1{}^*}$

4. FUZZY TOPOLOGY SUITABLE FOR FUZZY STACK

In this section, fuzzy stack satisfying a certain condition are considered. We applied some suitable condition on fuzzy stack S which makes collection τ^Ssuitable and compatible. Under this suitability condition, imposed on concerned stack, the collection τ^Swill become $\{V - \beta : V \in \tau, B \notin S\}$.

Definition 4.1 - Let S be a fuzzy stack on fuzzy on fuzzy topological space (I^X, τ). Then the fuzzy topologyτ is suitable for fuzzy stack S, if corresponding to each $x_\alpha \leq A$, there exit $U \in Q\ (x_p)$ such that $A * U \notin S$, then $A \notin S$, for $A \in I^X$.

Definition 4.2 - For every fuzzy set A in X, we define fuzzy set $\widetilde{A}$ by $\widetilde{A} = \{x_\alpha \leq A / x_\alpha \nleq \emptyset_S(A)\}$.

Remark 4.2.1 - $\widetilde{A} \wedge \emptyset_S(A) = 0_X$, for $A \in I^X$.

Remark 4.2.2 - $\widetilde{A} \wedge \emptyset_S(\widetilde{A}) = 0_X$, for $A \in I^X$.

Proposition 4.3 - For any fuzzy set A, $A = \widetilde{A} \vee (A \wedge \emptyset_S(A))$.

Proposition 4.4 - Let S be a fuzzy stack on fuzzy topological

(I^X, τ). Then τ is suitable for fuzzy stack S iff $A \wedge \emptyset_S(A) = 0_X$, implies $\emptyset_S(A) = 0_X$.

Proof - Since τ is suitable for fuzzy stack S. Let $A \wedge \emptyset_S(A) = 0_X$, so for each $x_\alpha \leq A$, we have $x_\alpha \nleq \emptyset_S(A)$. Hence there exist $U \in Q\ (x_\alpha)$ such that $A * U \notin S$. Since τ is suitable for S, $A \notin S \Rightarrow \emptyset_S(A) = 0_X$. Converselyif $A \wedge \emptyset_S(A) = 0_X \Rightarrow A \notin S$. So τ is suitable for S. This completes the proof.

Proposition 4.5 - If $A \wedge \emptyset_S(A) = 0_X$, then$\emptyset_S(\widetilde{A}) = 0_X$.

Remark 4.5.1 - If τ is suitable for S, then $\emptyset_S(\widetilde{A}) = 0_X$.

Proposition 4.6 - For $A \in I^X$, if $\emptyset_S(A \wedge \emptyset_S(A)) = \emptyset_S(A)$, Then if $A \wedge \emptyset_S(A) = 0_X \Rightarrow \emptyset_S(A) = 0_X$.

Proof - Let $A \wedge \emptyset_S(A) = 0_X$. By given $\emptyset_S(A) = \emptyset_S(A \wedge \emptyset_S(A)) = \emptyset_S(0_X)$.

Proposition 4.7 - For every set$A$ in X, if A contains no non empty set B with $B \leq \emptyset_S(B) \Rightarrow A \notin S$. Then $\widetilde{A} \notin S$ for $A \in I^X$.

Proof - For $A \in I^X$, $\widetilde{A} = \{x_\alpha \leq A : x_\alpha \nleq \emptyset_S(A)\}$. We claim that $\widetilde{A}$does not contain any non empty fuzzy set B s.t.$B \leq \emptyset_S(B)$.Then, $\widetilde{A} \notin S$, if possible, let B be non empty fuzzy set contained in $\widetilde{A}$ such that $B \leq \emptyset_S(B)$. Let $x_\alpha \leq$ B, then$x_\alpha \leq \widetilde{A}$. But $B \leq \widetilde{A}$and so$x_\alpha \leq \emptyset_S(B) \leq \emptyset_S(\widetilde{A})$. Also, $x_\alpha \leq \emptyset_S(B) \leq \emptyset_S(\widetilde{A})$. Hence$x_\alpha \leq \widetilde{A} \wedge \emptyset_S(\widetilde{A})$, but $\widetilde{A} \wedge \emptyset_S(\widetilde{A}) = 0_X$, a contradiction.

Theorem 4.8 - Let (I^X, τ) be a fuzzy topological space and S be a fuzzy stack on I^X such that τ is suitable of S, then $\tau^S -closed$ fuzzy set can be expressed as a join of fuzzy set which is closed in (I^X, τ) and a fuzzy set which is not in fuzzy stack S.

Proof - Suppose A is a $\tau^S -closed$ set, then $A = A \vee \emptyset_S(A)$, this implies $\emptyset_S(A) \leq A$. Using proposition 4.3, $A = \widetilde{A} \vee \emptyset_S(A)$, but $\widetilde{A} \notin S$.

Theorem 4.9 - Let (I^X, τ) be a fuzzy topological space and S be a fuzzy stack on I^X such that τ is suitable for S. Let $\sigma = \{0_X, 1_X\}$ denote fuzzy indiscrete topology on I^X, then $\tau^S = \sigma^S \cup \tau$.

Proof - Since$\sigma = \{0_X, 1_X\}$ is fuzzy indiscrete topology on I^X. So 1_X is only σ - q*nbd* of every fuzzy point x_α in X. Then for $A \in I^X$, $x_\alpha \leq \emptyset_S(A) \Leftrightarrow A * 1_X \in S \Leftrightarrow A \in S$.Thus for every $A \in S$, $\emptyset_S(A) = 1_X$ and for every fuzzy set $A \notin S$, $\emptyset_S(A) = 0_X$. This implies $\psi_S(A) = A$ if $A \notin S$.$\sigma^S = \{U : 1_X - U \notin S\}$, clearly $\sigma^S \subseteq \tau^S$ since for $V \in \tau^S$. Since $\tau \subseteq \tau^S$, $\sigma^S \subseteq \tau^S \Rightarrow \tau \cup \sigma^S \subseteq \tau^S$.Conversely, let $U \in \tau^S$, so $1 - U = F \vee B$ where F is τ - closed and $B \notin S$ (using theorem 4.8). This gives, $U = (1_X - F) \wedge (1_X - B)$ where $1_X - F$ is τ - open and $1_X - B$ is σ^S - open so $U \in \tau\ U\ \sigma^S$.

Theorem 4.10 - Let S be fuzzy stack on (I^X, τ) with τ is suitable for S then $\tau^S = \{V - B: V \in \tau, B \notin S\}$.

Proof Let A be τ^S - closed fuzzy set then τ^S - $cl\ (A) = A = A\ V\ \emptyset_S(A) \Rightarrow \emptyset_S(A) \leq A$. Since $A = \widetilde{A} \vee (A \wedge \emptyset_S(A)$, we have $A = \widetilde{A} \vee (A \wedge \emptyset_S(A)$. Using definition of$\widetilde{A}$, $\emptyset_S(A) \wedge \widetilde{A} = 0_X$. Since τ is suitable for $S \Rightarrow \widetilde{A} \notin S$.$A = \widetilde{A}\ V\ \emptyset_S(A)$ where $\emptyset_S(A)$ is τ - closed and $\widetilde{A} \notin S$.We can write $A = \emptyset_S(A) + \widetilde{A}$.$1_X - A = 1_X - [\emptyset_S(A) + \widetilde{A}] = \{[1_X - \emptyset_S(A)] - \widetilde{A}\}$, here $1_X - \emptyset_S(A) \in \tau$ and $\widetilde{A} \notin S$.

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