International Journal of Applied Engineering Research ISSN 0973-4562 Volume 10, Number 19 (2015) pp 40734-40738

Noisy quantum mixed state pattern classification based on the Grover's search algorithm

K. Aghayar

Assistant Professor, Department of Physics, Faculty of sciences, Urmia University, Urmia, Iran

E. Rezaei Fard

Master of Science, Department of Physics, Faculty of sciences, Urmia University, Urmia, Iran
email:rezaeifard.urmia@gmail.com

Abstract
In this paper, we attempt to clarify the effects of noisy quantum system on the pattern classification. This classification uses the Grover's search algorithm. To do so, we first present the mixed state density matrix formalism for the Grover's algorithm then attempt to show how noises affect the probabilities of correct classification. In conclusion the results for three and four qubits systems with some noises have been presented.

Keyword: 03.67.ac Quantum algorithms, protocols and simulations-03.67.Lx Quantum computation architectures and implementations

Introduction

Pattern classification or recognition is a subdivision of artificial intelligence which deals with the classification of patterns in to classes such as character recognition computer aided diagnosis and mining for biomedical and DNA data analyses [1-3].

It is possible to classify patterns using quantum Grover's search algorithm in a quantum system [4]. In his study, the author established the three approaches to quantum pattern classification and presented simulation results. Grover's search algorithm is a quantum search algorithm that runs quadratic ally faster than any equivalent classical algorithm [5]. Computational speed-up of a quantum algorithms over classical ones for some important problems, such as database search problem. Makes great interest in the quantum computation area.

During the last decade the quantum pattern classification has been presented and applied to real quantum systems. In [6], the author proved that finding and identifying certain patterns in an unstructured picture (data set), using the quantum Fourier transform, can be accomplished efficiently by a quantum computer with an exponential speed-up relative to its classical counterpart. In [7], a model of quantum associative memory which the binary patterns of n bits are stored in the quantum superposition of n qubits is presented. That model provides an exponentialimprovement in capacity but with a probabilistic result. The accuracy of pattern recall can be tuned by adjusting a parameter playing the role of an effective temperature. In [8], the Hopfield model, which is realizable using quantum holography, is applied for quantum associative memory and pattern recognition. The authors propose that it is possible to give the quantum interpretation for all the elements of the Hopfield model, so that is experimentally implementable in real quantum system. The author in [9], have presented a quantum pattern recognition scheme. They have combined the idea of a classic Hopfield neural network with adiabatic quantum computation for this purpose. The algorithm can return a quantum superposition of multiple recognized patterns and for two qubits, the algorithm has been given using a liquid state NMR quantum computer.

The main purpose of this paper is to investigate quantum noise effects on a quantum pattern classification. To this end, first we develop the Grover's algorithm to the mixed quantum states, then we concentrate on the quantum noise on the Grover's algorithm process. Finally we apply these methods for quantum pattern classification. The structure of the article is as follows.

The second section reviews the Grover's algorithm for a quantum states and extend it for density matrix formulation of mixed states. Section 3 shows that how quantum noises can change the density matrix for n-qubits systems. In section 4, we attempt to clarify the formulation. To do this, we present two examples from three and four qubits systems with bit flip and amplitude damping noises respectively. The final section concludes with a discussion of consequences.

Mixed state pattern classification

In many of physical situations such as quantum computation, if an experiment is repeated with the same conditions, the outcome is observed with different values. This means that the state of the system is not completely knownor the system is in a mixed state. The conventional mathematical Formalism of quantum mechanics, including bracket notation, cannot be applied for mixed states. For such systems, the density operator formalism provides a convenient way for describing a mixed state system [10].

In this section, first we review the Grover's search algorithm for a system described by a ket (so pure state), then we generalize the Grover's algorithm for mixed states described by a density matrix.

Review of the Grover's search algorithm

Grover's algorithm is a quantum algorithm for searching an unsorted database with N entries [11]. This algorithm provides a quadratic speedup and has probabilistic nature and the probability of finding desired state can be increased by repeating the algorithm $\frac{\pi}{4}\sqrt{\frac{N}{k}}$ times where K is the number of matching entries.

Let $|\psi\rangle$ denote the uniform superposition over all quantum states

$$|\psi\rangle = \frac{1}{\sqrt{N}}\sum_{x=0}^{N-1}|x\rangle \qquad (1)$$

Where N is the number of elements in database and $|x\rangle$ is the index register. The oracle operator is defined as

$$\hat{O}|x\rangle = (-1)^{f(x)}|x\rangle \qquad (2)$$

where $f(x)=1$ if x is a solution to the search problem and $f(x)=0$ if x is not a solution to the search problem. The Grover's unitary iteration operator is defined as

$$\hat{G} = (2|\psi\rangle\langle\psi| - I)\hat{O} \qquad (3)$$

And the repeating the algorithm n times is denoted by

$$\hat{G}^n|\psi\rangle \qquad (4)$$

Finally the probability amplitude of finding the desired ket, $|s\rangle$ after n times iterations is equal with

$$\langle s|\hat{G}^n|\psi\rangle \qquad (5)$$

Density matrix formulation of pattern classification

Her we follow the quantum pattern classification in [4], but with density matrix formulation and noisy patterns improvements. Let $B=\{0,1\}$ and $A=\{(x_i,y_i)\}$

Be a set of m pairs of points x_i in B^n and labels y_i in B. We would like to construct a quantum classification system that approximates the function $f: B^n \rightarrow B$ from which the set A was drawn based on the Grover's search algorithm.

Consider the set of m given patterns

$$A = \{A_1, A_2, .., A_m\}$$

Where A_1 is the first pattern and so on. We can represent each of the labeled patterns not an A as a basis state in the superposition with a nonzero coefficient

$$|\varphi\rangle = \frac{1}{\sqrt{2^n - m}}\sum_{(x_i,y_i)}|x_i y_i\rangle \qquad (6)$$

Now we define the given patterns density matrix as

$$\rho_0 = |\varphi\rangle\langle\varphi| \qquad (7)$$

Suppose A_s be the desired pattern for classification. Define the desired pattern density matrix as

$$\rho_s = |A_s\rangle\langle A_s| \qquad (8)$$

The probability of correct classification after n iteration by the Grover's algorithm (5) is $\rho_{x,n} = (\hat{G}^n)^\dagger \rho_s \hat{G}^n$ where $\alpha = \langle A_s|G^n|\varphi\rangle$ so

$$P(n) = \langle\varphi|\hat{G}^n\rho_s\hat{G}^n|\varphi\rangle \qquad (9)$$

If we define the n-times iterated patterns density matrix as

$$\rho_{x,n} = \hat{G}^n\rho_s\hat{G}^n \qquad (10)$$

Then

$$\rho_{x,n} = \hat{G}^n\rho_s\hat{G}^n \qquad (11)$$

The pattern density matrix, (7), and the n-times iterated patterns density matrix, (10), have trace equal to one and are positive operators so they are density matrices. This allows us to interpret the equation (11) as the ensemble average of the observable ρ_s, [10]

$$\langle\rho_s\rangle = Tr(\rho_{x,n}\rho_0) \qquad (12)$$

Noise in patterns classification

In real world, there is no closed system (i.e., a system with no energy or particle exchange with the environment). In our pattern classification problem, noise arises from random exchange of energy between our system and its surrounding environment.

Suppose that our desired pattern initially have a statistical operator or density matrices. When the patterns system, called environment [12]. In quantum mechanics any change in the density matrix, such as interaction with environment, $\rho_s \rightarrow E(\rho_s)$ is called state transformation and the final density matrix can be written in the form

$$\tilde{\rho}_s = \varepsilon(\rho_s) = \sum_k E_k\rho_s E_k^\dagger \qquad (13)$$

Where the operator-sum, E_k representation,, satisfy

$$\sum_k E_k E_k^\dagger = 1 \qquad (14)$$

And are called the operation elements of the state transformation. For a single qubit these operators E_k, can be written as

$$E_k = \alpha_k I_2 + \sum_{i=1}^{3} a_{ki}\sigma_k \qquad (15)$$

Where α_i, a_{ki}, are parameters, I_2 is 2×2 identity matrix, $\sigma_1=\sigma_x$, $\sigma_2=\sigma_y$ and $\sigma_3=\sigma_z$ are Pauli matrices.

For a n-qubits system local interactions of the qubits with channels $C_1,...,C_n$ can be described by operators, which are constructed as tensor product of the single-qubitoperators $E_{i1},...,E_{in}$ .Therefore, if this system is affected by local noise, the ρ_{fin} can be obtained from its initial state, ρ_{ini} in the following manner

$$\tilde{\rho}_s = \sum_{i_1...i_n} U_{i_1...i_n}\rho_s U_{i_1...i_n}^\dagger \qquad (16)$$

Where

$$U_{i_1...i_n} = E_{i_1}\otimes ... \otimes E_{i_n} \qquad (17)$$

is the noise operator of the n-qubits system.

4 Examples

As first example, for the purpose of comparing results with

[4], here we investigate a three qubits system but here with noses on three qubits of the desired pattern, i.e. on the ρ_s. Let us consider a database with 8 entries so

$$|\psi\rangle = \frac{1}{\sqrt{8}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle)$$

First we use the exclusive method for learning the pattern classification. In this method each pattern which is not in set A, is included with a nonzero coefficient. This method is implemented for a quantum associative memory in [13]. Let us suppose the pattern set be

$A = \{|000\rangle, |111\rangle\}$

Where the desired pattern is $|000\rangle$. Using the exclusive method of [4], the exclusive superposition of database states is equal with

$$|\varphi\rangle = \frac{1}{\sqrt{6}}(|001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle)$$

The desired pattern density matrix is

$\rho_s = |A_s\rangle\langle A_s| = |000\rangle\langle 000|$

So the oracle operator becomes

$$\hat{O} = -(|000\rangle\langle 000| + |001\rangle\langle 001|) + |010\rangle\langle 010| + |011\rangle\langle 011| + |100\rangle\langle 100| + |101\rangle\langle 101| + |110\rangle\langle 110| + |111\rangle\langle 111|)$$

The bit flip channel flips the state of a qubitsfrom to (and vice versa) with probability, 1-p. It has operation elements $E_0^{(1)} = \sqrt{p_1} I_2$ and $E_1^{(1)} = \sqrt{1-p_1}\sigma_1$ and also for second and third qubits but with probabilities p_2 and p_3 respectively (clearly $0 \le p_1 p_2 p_3 \le 1$). So the noise operators is equal with

$$U_{i_1,i_2,i_3} = E_{i_1} \otimes E_{i_2} \otimes E_{i3} \tag{18}$$

Where the superscripts stands for the qubits index number. For example the first two components of the noise operator are $U_{0,0,0} = E_0 \otimes E_0 \otimes E_0 = \sqrt{p_1 p_2 p_3} I \otimes I \otimes I$ and $U_{0,0,1} = E_0 \otimes E_0 \otimes E_1 = \sqrt{p_1 p_2 (1-p_3)} I \otimes I \otimes \sigma_1$. The patterns density matrix after noise is

$$\tilde{\rho}_s = U_{i_1,i_2,i_3} \rho_s U^{\dagger}_{i_1,i_2,i_3} \tag{19}$$

Which is a diagonal density matrix with diagonal elements equal with

$$\begin{aligned}
\tilde{\rho}_{s11} &= p_1 p_2 p_3 \\
\tilde{\rho}_{s22} &= p_1 p_2 (1-p_3) \\
\tilde{\rho}_{s33} &= p_1 (1-p_2) p_3 \\
\tilde{\rho}_{s44} &= p_1 (1-p_2)(1-p_3) \\
\tilde{\rho}_{s55} &= (1-p_1) p_2 p_3 \\
\tilde{\rho}_{s66} &= (1-p_1) p_2 (1-p_3) \\
\tilde{\rho}_{s77} &= (1-p_1)(1-p_2) p_3 \\
\tilde{\rho}_{s88} &= (1-p_1)(1-p_2)(1-p_3)
\end{aligned} \tag{20}$$

The probability of finding noisy desired pattern, (12), is presented in Table 1. It is clear that the maximum probability of finding noisy desired pattern occurs after four iteration which is equal with

$$\frac{1}{6}[p_1(2p_2(p_3+1) + p_3 - 1) + (1-p_2)(1-p_3)] \tag{21}$$

TABLE 1. Correct state classification probability for the six iteration of the Grover's search algorithm. In the first column, n, is the number of iteration. The bit flip noise is present with the probability $(1-p_1)$ for the first and $(1-p_2)$ for the second qubit.

n	Probability	Maximum probability
1	$p_1 p_2/6$	$1/6$, for $p_1 = 1, p_2 = 1$
2	$1/24$	$1/24$, for $p_1 = 0, p_2 = 0$
3	$1/6$	$1/6$, for $p_1 = 0, p_2 = 0$
4	$2p_1 p_2/3$	$2/3$, for $p_1 = 0, p_2 = 0$
5	$(1+8p_1 p_2)/24$	$3/8$, for $p_1 = 1, p_2 = 1$
6	$(1-p_1 p_2)/6$	$1/6$, for $p_1 = 0, p_2 = 1/2$

This probability has a maximum value $\frac{2}{3}$, for $p_1 = p_2 = p_3 = 1$ (no error at all) and a minimum value 0 for $p_1 = p_2 = 0, p_3 = 1$. Figure 1 shows the probability of correct classification in terms of p_1 and p_2 where $p_3 = 1$. The probability is greater than $\frac{1}{2}$ for two sets: $\{p_1 = \frac{3}{4}, p_2 = 1\}$ and $\{\frac{3}{4} < p_1 \le 1, \frac{3}{4} \le p_1 p_2 \le 1\}$. For these two sets, the pattern is detectable (see table 2)

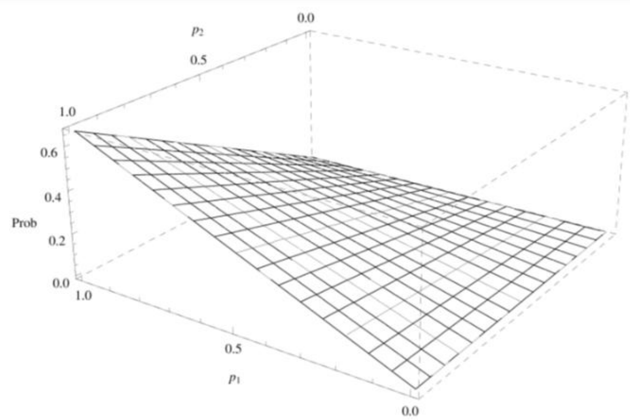

Fig 1. The probability of finding the correct state classification after four iteration for pattern $|000\rangle$, with the learning set $\{|000\rangle, |111\rangle\}$, in the presence of bit flip noise. The probability of the bit flip noise on the first and second qubits are $(1-p_1)$ and $(1-p_2)$ respictively. The third qubit is without noise.

TABLE 2.incorrect state classification probability for the six iteration of grover algorithm. In the first column, n, is the number of iteration. The bit flip noise is presented with the probability $(1-p_1)$ for the first and for $(1-p_2)$ the second qubits.

n	Probability	Maximum probability
1	$[1-p_2+p_1(5p_2-1)]/6$	$2/3$, for $p_1=1, p_2=1$
2	$[9-8p_2+8p_1(2p_2-1)]/24$	$3/8$, for $p_1=0, p_2=0$
3	$[p_1+p_2-2p_1p_2]/6$	$1/6$, for $p_1=0, p_2=1$
4	$[1-p_2+p_1(2p_2-1)]/6$	$1/6$, for $p_1=0, p_2=0$
5	$[9-8p_2+8p_1(2p_2-1)]/24$	$3/8$, for $p_1=0, p_2=0$
6	$[p_1+p_2-p_1p_2]/6$	$1/6$, for $p_1=0, p_2=1$

In the second example, we consider the four qubits system with amplitude damping noise. This noise cause energy dissipation from system (here qubits). The operators for this noise, for first qubit, can be represented as $E_0^{(1)}=\begin{pmatrix}1 & 0\\ 0 & \sqrt{1-\gamma_1}\end{pmatrix}$ and $E_1^{(1)}=\begin{pmatrix}0 & \sqrt{\gamma}\\ 0 & 0\end{pmatrix}$, where γ is the probability of amplitude damping noise on first qubit. Let us choose patterns set as

$A=\{|0110\rangle,|1000\rangle,|1010\rangle,|1100\rangle\}$

with the desired pattern. Now we have

$\rho_s=|A_s\rangle\langle A_s|=|1010\rangle\langle 1010|$

And

$$\begin{aligned}\tilde{\rho}_{s11}&=\gamma_2\gamma_3\\ \tilde{\rho}_{s33}&=\gamma_2(1-\gamma_3)\\ \tilde{\rho}_{s55}&=\gamma_3(1-\gamma_2)\\ \tilde{\rho}_{s77}&=(1-\gamma_2)(1-\gamma_3)\end{aligned} \tag{22}$$

and a maximum probability of finding noisy desired pattern occurs after iteration which is equal with

$$\frac{1}{196608}[112225-112056(\gamma_2+\gamma_3+\gamma_2\gamma_3)] \tag{23}$$

and allover others elements equal with zero. As seen there is no noise in the first and forth qubits. This probability is plotted in Fig. 2. The maximum of this probability is $\frac{112225}{196608}$ or 0.5708 when $\gamma_2=\gamma_3=0$.

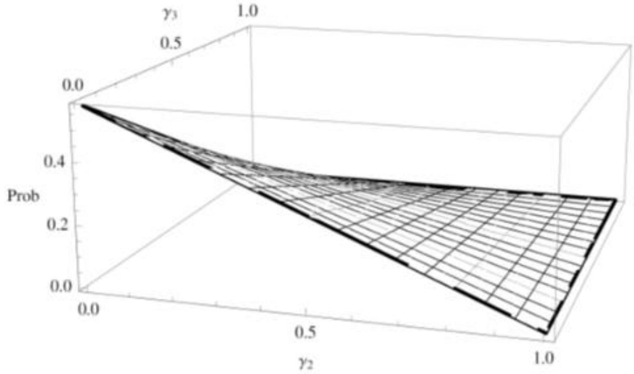

Fig 2.The probability of finding the correct state classification after six iteration for pattern $|0110\rangle$, with the learning set $\{|000\rangle,|111\rangle\}$, in the presence of amplitude damping noise.

Conclusion

Our results highlight the potential of the Grover's algorithm for the pattern classification of noisy mixed states. Particular examples was directed towards three and four qubits systems with bit flip noise and amplitude damping noise respectively. As we have seen, this noises affect (generally decrease) the classification probabilities. The quantum noise operation technique used in this paper is quite general and can be applied directly for similar problems. Specifically the method is useful for the mixed state pattern classification problems dealing with other quantum noises such as phase flip, bit-phase flip, phase damping and depolarizing channel. Previous studies showed that the quantum pattern classification could be realized. Further studies in the pattern classification will probe fundamental aspects of quantum advantages. Specially instead of using the Grover's algorithm, other quantum algorithms may improve the classification problems. In conclusion, it may be said that quantum classification will be one of the important techniques for practical recognition. Our expectation is that quantum classification will provide important improvement about neuroquant ology specially and to answer how human brain classifies patterns.

References

[1] Duda, R. O. Hart, P. E. and Stork, D.G., 2001, Pattern Classification, 2nd end, Wiley, New York.

[2] Bishop, C. M., 2006, Pattern Recognition and Machine Learning, Heidelberg, Germany, Springer.

[3] Benso, A., Di Carlo, S., Politano, G., and Sterpone, L., 2008, A graph-based representation of Gene Expression profiles in DNA microarrays, IEEE Symposium on Computational Intelligence in Bioinformatics and Computational Biology, pp. 1-6. http://dx.doi.org/10.1109/cibcb.2008.4675762

[4] Ventura D., 2002, proceeding of the gointernational Conference on Information Sciences March, 537-640.

[5] Grover, L. K, 1996, "A fast quantum mechanical algorithm for database search". In Proceedings of the twenty-eighth annual ACM symposium on Theory of computing (pp. 212-219). ACM.

[6] Schützhold, R., 2003, "Pattern recognition on a quantum computer". *Physical Review A*, 67(6), 062311.

[7] Trugenberger, C. A., 2002, "Quantum pattern recognition" *Quantum Information Processing*, 1(6), 471-493.

[8] Peruš, M., and Bischof, H., 2003, "Quantum-wave pattern recognition: from simulations towards implementation". In Proc. 7th Joint Conf. Information Sciences, JCIS/Association for Intelligent Machinery, Durham, NC.

[9] Neigovzen, R., Neves, J. L., Sollacher, R., and Glaser, S. J., 2009, "Quantum pattern recognition with liquid-state nuclear magnetic resonance", *Physical Review A*, 79(4), 042321.doi:10.1103/physreva.79.042321

[10] Pathria R. K., and Beale P. D., 2011, Statistical Mechanics, 3rdedn, Elsevier, Netherland.

[11] Nielsen M. A., and Chuang I. L., 2010, Quantum Computation and Quantum Information, Cambridge University Press, Cambridge.

[12] Petz D., 2008, Quantum Information Theory and Quantum Statistics, Theorical and Mathematical Physics, Springer, Berlin Heidelberg.

[13] Ezhov, A., Nifanova, A., and Ventura, D., 2000, "Distributed queries for quantum associative memory", *Information Sciences*, 128(3-4), 271-293.

[14] Siomau, M, 2012, "Entanglement dynamics of three-qubit states in local many-sided noisy channels", *Journal of Physics B: Atomic, Molecular and Optical Physics*, 45(3), 35501-35505.