

Optimization the Volume of Business Services for Companies-residents in Linear Model of Technology Park

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Abstract

For determining the optimal volume of services of technology park an algorithm maximizing revenue of innovative companies in terms of multiproduct production and general limitations on resources is proposed in the article. Discussed in the article the main difference of the model is outsourcing the part of business services to service companies provided by the management company of the technology park to residents as well as the usage of indirect criterion of effectiveness in the form of comprehensive income from the sale of products (services) of innovative companies due to the need to fulfill the mission of technology park which is securing the most effective conditions for developing of innovative businesses on its territory.

Keywords: Technology Park, Management Company, Service Company, Innovative Company, Optimization, Business Services, Linear Model, Differential Property, Quasigradient Rise

Problem statement

In the conditions of improving the efficiency and increasing competition between economic agents, providing support for innovation activity enterprises, the role of technology parks greatly increases. In the technology parks the concentration of all key elements of information and organizational infrastructure on the same area is provided.

The main business process of technology parks as subjects of the economy is providing business services by residents at all stages of the life cycle of an innovative company that is a source of income of management company of technology park structure and significantly reduces the costs of residents on the execution of business processes which are complementary to them.

In the modern conditions due to the diversification of activities of technology parks the composition and the content of business services considerably varies.

Analysis of the data of the International Association of Science Parks (IASP) has shown that the most common types of business services in modern science and technology parks are rent of premises (90%); access to computer networks (more than 80%); services for developing of business and access to investments (about 80%); arrangements for feeding

(more than 70%); security enforcement (about 70%); consulting services (more than 60%). Certain types of services that contribute to the rational organization of life (hotel, kindergarten, gym, etc.) are less common. These services are provided only by highly developed technology park structures which are usually a relatively dedicated area away from the city.

An analysis of different approaches to the classification of business services of modern technology parks was conducted in the paper [1]. Thus, the list of business services offered by the Association of technology parks in the sphere of high technologies contains 46 items; the Ministry of Communications of Mass Communications of Russian Federation allocated 23 of their species.

It is obvious that not all of the possible types of business services will be in demand within a specific technology park. At the same time those services with high demand from the residents must be submitted in the pool of business services of technology park due to its mission which is to create the most effective environment for development of residents' business. Meeting the needs of residents in full as well as extension of a pool of business services is possible both by the management company and with the involvement of service companies in the terms of outsourcing.

In fact, the management company has a choice: to provide the service themselves or engage a service company as outsourcer. The different criteria may be allocated and on the base of these criteria such decision will be taken but they will not always lead to the optimal solution.

As was showed in the papers of the author the technology park is the subject both of micro-economy, i. e. must maximum efficiently operate as a business, and meso-economy that determines its mission at the regional level as a center of innovation support.

It is obvious that the implementation by the technology park of its mission is the most important criterion of its effectiveness; and thus will become the basis for making choices in terms of providing of business services to residents. Currently in the scientific literature the optimization problem (in the classical sense of the word) of a pool of business services was not set and was not solved. At the same time it's very important for an increasing number of advanced and developing technology parks.

Analysis of the latest publications on the problem

In this article the authors consider the problem of determining the optimal volume of sales of services of technology park maximizing its revenue from the sale of products in the conditions of multiproduct innovative production and general resource limits considered as a multiproduct monopoly studied in the paper [2]. The difference is the separation of cash flow from operating activities of technology park in accordance with the recommendations of the paper [3] to the flow from the implementation of innovative products of innovative companies; the flow from the sale of services of service companies and the flow from rents coming from management company of technology park. The planning horizon is one period. The services sales volume is selected according to the general restrictions on production resources. The authors propose an algorithm for determining the optimal volume of sales of services and rental rates maximizing the annual income of technology park from the sale of innovative products.

This article is based on the paper [4] in a part of study the stability of residents of technology park from possible changes in the volume of sales of services in the sense of science work [5]. The main difference between the model discussed in the article is the transfer the part of the services of technology park (TP) to service companies (SC) with the replacement of direct income criteria of the management company (MC) by an indirect criteria of efficiency in the form of income from sales of innovative companies (IC). This replacement required the proof of additional properties of the model. Considered problem is formally derivative problem of resource management in the classical linear production problem. Using the duality theorem it comes down to the decision of the maximin problem. This task can be accomplished using standard algorithms of subgradient optimization [6].

In the methodological sense the study follows the general development paradigm of systems theory and its application in corporate and strategic management developed in the fundamental work [7] in relation to the development of technology park structures. As in the linear model of multiproduct monopoly studied in the paper [2] the solution of the problem is carried out in two stages: optimization of the volume of production of ICs at fixed volumes of services sales of SCs and optimization by volumes of services sales of service companies. The criterion on the second level is obtained by solving the problem of linear programming of optimization criterion at the first level in the dual form with the resources that depend from external variables which leads eventually to maximin form of problem.

The relevance of the general development paradigm of systems theory and its application in corporate and strategic management confirms many studies an overview of which can be found in the fundamental work which was mentioned above [7]. In this article the limitations of production capacity is considered but it is assumed that the change in output does not affect the optimal use of production factors. In this regard, considered problem of services volumes of management and service companies may be regarded as a problem of resource management in an auxiliary linear programming problem. The main result of work is that it reduces to the solution of

maximin problem for which solution there are proven algorithms of subgradient type.

The goal of research is to develop a model to determine the optimal allocation of a pool of business services to residents of technology park between the management company and service companies using the methodological fundamentals of linear programming.

The main results of research

1. AGGREGATED LINEAR MODEL OF TECHNOLOGY PARK.

1. 1. Formulation of the aggregated problem of optimization of volumes production of products and services of technology park. There is a technology park producing the innovative products of n species and using m types of resources.

Lets introduce the notations $b_i = S_i b_i^0 > 0; i = 1, 2, \dots, m$; - resource stocks understood in the simplest case as a useful area of i - homogeneous type of buildings of administration, office and warehouse types, where b_i^0 - the number of buildings of i - type, S_i - useful area of one building of i - type; $s_{ij}^{xb} \geq 0; i = 1, 2, \dots, m; j = 1, 2, \dots, n$; - useful area of the i - type of building needs during the production of the product unit of j - innovative products; $s_j = p_j - c_j; j = 1, 2, \dots, n$; - marginal income in the realization of unit of j - products; $p_j \geq 0$ - the price of the unit of j - products; $c_j > 0$ - demand costs per unit of j - products. Technological matrix $s^{xb} = \|s_{ij}^{xb}\|$ sets specific time expenditures of technology park resources, managed by management company during the manufacture of innovative products of innovative company. Let's suppose that the condition of production profitability of innovative products was fulfilled:

$$s_j = p_j - c_j \geq 0, j = 1, 2, \dots, n. \quad (1)$$

MC is considering the possibility of transmission of most services of technology park to service company. The volume of services will be denoted for the differences from the innovative production volumes by letters $y_k, k = 1, 2, \dots, l$.

Technological matrix $s^{yb} = \|s_{kj}^{yb}\|$ specifies the unit cost of resources of technology park during the production of services of service company.

Thus, let's assume that the majority of services of IC provide SCs that need to be included in the model at the next level of description of TP. Let's

$a_{kj} \geq 0; k = 1, 2, \dots, l; j = 1, 2, \dots, n$; - the unit costs of k - type of services during the production of the unit of j - products. General restrictions on the services of SC can be written as:

$$a^{xy} x \leq y,$$

where $a^{xy} = \|a_{kj}^{xy}\|$ - technological matrix, y - column vector of production services volumes $y_k, k=1,2,\dots,l$.

Technological matrix $a = a^{xy}$ sets the unit costs of services of SC, during the production of innovative products of IC.

Now, the general resource constraints can be written as:

$$s^{xb}x + s^{yb}y \leq b,$$

Technological matrix $t = t^{yb} = \|t_{ik}^{yb}\|$ specifies the unit costs of TP resources during the production of IC services. Thus, the technological matrix in this case is:

$$A = \begin{pmatrix} a^{xy} & 0 \\ t^{xb} & t^{yb} \end{pmatrix}, \quad (2)$$

i. e. it is a generalization of decomposable [8] on non-square matrix.

In the scalar form the constraints of resource have the form:

$$\sum_{j=1}^n a_{kj}^{xy} x_j = \langle a_{xy}^k, x \rangle \leq y_k, k=1,2,\dots,l, \quad (3)$$

And

$$\sum_{j=1}^n s_{ij}^{xb} x_j + \sum_{k=1}^l s_{ik}^{yb} y_k = \langle s_{xb}^i, x \rangle + \langle s_{yb}^i, y \rangle \leq S_i b_i^0, i=1,2,\dots,m. \quad (4)$$

Here a_{xy}^k - it is k - row of a matrix a^{xy} , $s_{xb}^i (s_{yb}^i)$ - it is i - row of a matrix $s^{xb} (s^{yb})$.

As an indirect criterion of MC' effectiveness the marginal revenue of IC can be taken:

$$w = \sum_{j=1}^n s_j x_j \rightarrow \max_{x_j, y_k}. \quad (5)$$

Let's called the problem (3)-(5) of finding the non-negative vectors of production of innovative products $x \geq 0$ and services $y \geq 0$ a linear model of technology park.

Note 1. It is expected that the MC has monopoly power and can influence on the volume of innovative products of IC and services of SC by signing or not signing the related leases. This is due to the fact that in TP there are many relatively small ICs and SCs each of which separately has a minor impact on the structure of TP.

2. DECOMPOSITION OF LINEAR MODEL OF TECHNOLOGY PARK

Under these assumptions, the task of optimizing the volume of innovative TP products can be set as the maximin problem which is the essence of the decomposition of the original problem of high dimensionality in two consecutive problems of smaller dimensionality. This opportunity arises from the fact that the internal task of optimizing of the volume of innovative IC' products allows dual formulation. Differential properties of internal minimum function and method of quasi gradient rise for solution of obtained maximin problem are investigated under the scheme proposed in some papers [6], [9], which allowing to close the proposed model and to move the issues of optimization of TP services in the practice subspace.

2. 1. 1. Maximization by the volume of MC' services in terms of fixed volume of SC' services. Let's exclude

parameters y_k from the constraints (4):

$$\sum_{j=1}^n s_{ij}^{xb} x_j \leq S_i b_i^0 - \sum_{k=1}^l s_{ik}^{yb} y_k, i=1,2,\dots,m. \quad (6)$$

Let's suppose that the right-hand side of inequations (6) is non-negative:

$$\sum_{k=1}^l s_{ik}^{yb} y_k \leq S_i b_i^0, i=1,2,\dots,m. \quad (7)$$

Criterion (5) has the form:

$$w = \sum_{j=1}^n (p_j - c_j) x_j \rightarrow \max_{x_j}. \quad (8)$$

Let's call the problem (4), (6), (8) of finding of non-negative vectors of production of innovation IC' products $x \geq 0$ the internal auxiliary problem of optimization of volume of innovative IC' products.

2. 1. 2. Maximizing the volume of SC services. Let's now consider the problem of maximizing the optimal value of the criterion $w^* = w^*(y)$ of the internal auxiliary problem (4), (6), (8). Restrictions of the task consists of constraints (7) by y and the conditions of non-negativity of the variables $y_k \geq 0, k=1,2,\dots,l$.

Restrictions (7) by y take the form:

$$D^i(y) = S_i b_i^0 - \sum_{k=1}^l s_{ik}^{yb} y_k \geq 0, i=1,2,\dots,m, \quad (9)$$

Let's denote by $Y = \{y \in E^l | D(y) \geq 0\}$ the set of nonnegative vectors y satisfying the constraints (9). Here $D(y) = \min_{i=1,\dots,m} D^i(y)$. Then the function $D(y)$ will be concave and its subdifferential defined by the formula [6]:

$$\partial D(y) = \text{conv} \left\{ s \in E^l \mid s = D_y^i(y), i \in \text{Arg min}_{i=1,\dots,m} D^i(y) \right\}. \quad (10)$$

Here conv - closure of the convex hull of the set.

Moving from problem (3), (6), (8) to the dual problem we will obtain the problem:

$$F^0(y, q) = \sum_{i=1}^m q_i \{S_i b_i^0 - \sum_{k=1}^l s_{ik}^{yb} y_k\} + \sum_{k=1}^l r_k y_k \rightarrow \min_{q_i, r_k}. \quad (11)$$

With restrictions:

$$F^j(q, r) = \sum_{i=1}^m s_{ij}^{xb} q_i + \sum_{k=1}^l a_{kj}^{xy} r_k - p_j + c_j \geq 0, j=1,2,\dots,n \quad (12)$$

and the conditions of non-negativity of the dual variables:

$$F^{n+i}(q, r) = q_i \geq 0, i=1,2,\dots,m; \quad (13)$$

$$F^{n+m+k}(q, r) = r_k \geq 0, k=1,2,\dots,l.$$

Let's denote:

$$V = \{q, r \in E^{n+l} \mid F^j(q) \geq 0; j=1,2,\dots,n+m+l\}. \quad (14)$$

Let's consider the maximin problem:

$$f(y) = \min_{(q,r) \in V} F^0(y, q, r) \rightarrow \max_{y \in Y}. \quad (15)$$

Let's call the maximin problem (15) the external subsidiary problem of optimization of SC' services.

Note 3. The problem (15) is a decomposition of the linear problem (3) - (5). Although this problem is not linear, but it admits, as we will see below, solution algorithm of type of quasi gradient descent, which has a simple software implementation.

Note 4. As is well-known the dual variables $q_i(r_k)$ can be regarded as estimates of resource scarcity (services) [8]. In the sense that if at least for one optimal solution of the direct problem some $i - (k -)$ restriction is disabled, then for any solution of the dual problem the corresponding dual variable $q_i(r_k)$ is equal to zero. This resource (service) is not deficient for any other optimal solution.

2. 2. Differential properties of the internal function of minimum. Let's assume that the set Y is limited. Let's the sets V is also limited. Then the functions of F^j are continuous with F_y^j, F_q^j, F_r^j at

$Y' \times E^{m+l}, j=0,1,...,n$, where $Y' \supset Y$ - a constrained open set. This can be seen directly from the definition of functions F^j in (17) - (19).

Also let's the regularity condition will be executed [9]:

$$F_{(q,r)}^j(q,r), j \in \tilde{J}_0(q,r),$$

- linearly independent at all $(q,r) \in \tilde{V}_0(y)$. (16)

Here is indicated:

$$\tilde{J}_r(q,r) = \{j \in J | F^j(q,r) \leq \tau; j = 1,2,...,n\}$$

$$\tilde{V}_r(y) = \left\{ q \in V | F^0(y,q,r) \leq \min_{q \in V} F^0(y,q,r) + \gamma; \gamma \geq 0 \right\}$$

Under these conditions from the results of the work [5] and its further generalizations (see. precise references in the work [9]) arise from the existence of the derivative in any direction $\nu \in E^l$ of maximum function f on Y' , wherein:

$$\frac{\partial f(y)}{\partial \nu} = \min_{r \in M(y)} \langle g, \nu \rangle, \quad (17)$$

where

$$M(y) = \text{conv} \{ g \in E^m | g = F_y^0(y,q,r), (q,r) \in \tilde{V}_0(y) \}. \quad (18)$$

In the paper [9] is set in addition to (17) that under the assumptions which were made (18) an exact expression for quasi differential $\partial f(y)$ (sets of it quasi gradients) of weakly upwards function $f(y)$ of minimum (15) with decomposing variables.

Quasi gradient g in (18) in this case can be found by the formula:

$$g = F_y^0(y,q) = \left(- \sum_{i=1}^m q_i s_{ik}^{yb} + r_k, k=1,2,...,l \right)$$

2. 3. Method of quasi gradient rise for the solution of the problem (15). Having quasi differentials (10), (18) for the solution of the problem (15) now we can use the combined method using quasi gradients of criteria and aggregate functions in limit proposed in the work [6]:

$$y_k(t+1) = \begin{cases} P(y_k(t) + a_t g_k^t), D(y(t)) \geq 0; \\ P(y_k(t) + a_t d_k^t); D(y(t)) < 0; \end{cases}; k=1,...,l, \quad (19)$$

where

$$g^t \in \partial f(y(t)), d^t \in \partial D(y(t)), t=1,2,...; \quad (20)$$

Where t - step number; $a_t = D_Y t^{-\sigma}$ - program step of the method, $1/2 < \sigma \leq 1$, - parametric variable, for example $\sigma = 3/4$; D_Y - the characteristic size of the set Y of admissible decision of the problem (15), for example estimate of the diameter;

$$P(y_k) = \begin{cases} 0, y_k \leq 0 \\ y_k, y_k > 0 \end{cases} = \max(y_k, 0)$$

- the projection operator on the beam $E_+^1 = [0, \infty)$.

Then, by the results of the work [9] under the assumptions which were made the quasi gradient rise method (19) - (20) converges by derivative to the stationary set of the problem (15).

3. DISAGGREGATION OF THE LINEAR MODEL OF TECHNOLOGY PARK

3. 1. The derivative problem of determining the internal prices for the SC' services. Criterion (5) can be considered as the aggregate criterion of general welfare which is a marginal amount of income of all TP' members [10]. The criterion in this case comes down to profit of IC as all other expenses and revenues are cancel on. In the sense of ICs pay for the services to service companies but at the same time this fee is the income of SK. Finally, ICs and SCs pay the rent to MC but at the same time this fee is the income of MC which is the owner and do not have its own interests different from the owner which created this structure to provide the most favorable conditions to ICs. It is assumed that the conditions of profitability for all participants of TP were fulfilled. These conditions generalize (1) taking into account the additional costs of the services. At the same time, internal prices for services of SC remain outside the study but we can come back to them after the solving of the aggregate problem (3) - (5). From the solution of the aggregate problem one can find volumes of delivery of SC' services and know these volumes one can use the well-known economic interpretation of

the game Γ_1 (see paper [11]) for the prices of these services where SC reported the price of their services to IC which selected output.

3. 2. Stackelberg's Equilibrium. It is expected that MC has the monopoly power and can indirectly influence on the services prices of SC by their prior approval. Under these assumptions one can take equilibrium Stackelberg's prices as appropriate domestic prices. These prices suggest the possibility of a benevolent cooperation between the TP' members the definition of which was given by Helly [12] which exists by the lemma 10. 1 in the work [13]. Practical finding the

equilibrium prices by the method of penalty function can be reduced to the calculation of the maximin with decomposing variables of higher dimensionality. This is done in the same way as during finding of the maximin prices directly in the game Γ_1 in the section 6 in the work [14].

3. 3. The derivative problem of determining the volume of SC' services. In paragraph 23 in the work [14] proposed a method of branches and borders to solve linear games with the transfer of information and forbidden situations which is obtained in our case if prices for the SC's services will be fixed as well as prices for innovative IC products and rental of SC will be fixed. A similar method is proposed to use for finding Stackelberg's equilibrium in this linear game. Moreover, equilibrium value of criterion of majority revenue of IC obtained at that can be compared to its value obtained in the aggregate problem. Their non-positive difference can be interpreted as a fee for the disaggregation of the problem, i. e. for the accounting of the independence of IC and SC of technology park.

A similar algorithm is designed in the work [15] to solve some maximin stochastic problems in the case of linear criteria for players and binding constraints. There is a desire to use the procedure of gradual improvement of prices for SC' services and volumes of production of innovative products and services of TP occurs when you use the principle of contracting mapping [16] but there is no confidence in the compressibility of the corresponding implicit mappings in our case. Therefore, these questions remain open.

Conclusions

Studies of the linear model of technology park allows to suggest that the task of optimizing the volume of innovative products of TP can be set as a linear programming problem. The variables of the problem are the volumes of innovative products of IC. Restrictions of the problem are given by technological matrix with elements that are linearly dependent from the parameters of TP which include the volume of SC' services. It is expected that MC may indirectly affect on the volume of SC' services by signing or not signing the related leases. This is due to the fact that in TP there are many comparatively small SCs. Under these assumptions the task of optimizing of the volume of SC' services can be set as the maximin problem which is the essence of the decomposition of the original problem of high dimensionality in two consecutive problems of smaller dimensionality. This opportunity arises from the fact that the internal problem of optimizing of volume of innovative IC' products allows dual formulation. Differential properties of internal minimum function and method of quasi gradient rise to solutions obtained maximin problem are investigated under the scheme proposed in the works [6], [9] which allowing to close the proposed model and to translate the issues of optimization of TP' services into practical space.

It has been shown that the criterion of the optimization of volume of innovative TP' products can be considered as the

aggregate welfare criterion which is a sum of marginal income of all TP' members which in this case comes down to profits of IC as all other expenses and revenues are cancel on. At the same time, internal prices for services of SC remains outside the study but we can come back to them after the solving of the aggregate problem if one can use the well-known economic interpretation of the game Γ_1 where SC reported the price of their services to IC which selected output. It is expected that MC has monopoly power and can indirectly influence on the prices of SC' services by their prior approval. Under these assumptions, one can take equilibrium Stackelberg's prices as appropriate domestic prices. These prices suggest the possibility of a benevolent cooperation between the members of TP which always exists and practical finding of which by the method of penalty function can be reduced to the calculation of the maximin with decomposing variables of higher dimensionality. These issues are beyond of this article and we'll expect to devote them another paper.

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References

- [1] Maltseva, A. A., 2013, Management of technology park structure at the micro level: effective approaches and solutions (monograph), Kursk, South-West State University, 244 p.
- [2] Perevozchikov, A. G., and Lesik, I. A. 2015, "Determination of the optimal volumes and sale prices in the linear model of multiproduct monopoly", Economics and Mathematical Methods. (in press).
- [3] Maltseva, A. A., 2012, "Financial Management Technopark structure using cash flow model", Financial Management. - M. : Finpress, 5, pp. 23-33.
- [4] Mishchenko, A. V., and Artemenko, O. A., 2012, "Management models of production and financial activities of the company in the conditions of borrowed capital", Financial analytics, 42 (132), pp. 2-13.
- [5] Makarov, V. L., Rubinov, F. M., 1973, Mathematical Theory of Economic Dynamics and equilibrium, Moscow, Nauka.
- [6] Polyak, B. T., 1983, Introduction to optimization, Moscow, Nauka.
- [7] Kleiner, G. B., 2010, The development of systems theory and its application in corporate and strategic management: a preprint, Moscow, Central Economic and Mathematical Institute of RAS, 59 p.
- [8] Ashmanov, S. A., 1984, Introduction to Mathematical economy, Moscow, Nauka.

- [9] Zavriev, S. K., and Perevozchikov, A. G., 1990, "Stochastic generalized gradient for solving minimax problems related variables", Computational Mathematics and Mathematical Physics, 4(30), pp. 491-500.
- [10] Vasin, A. A., 2005, Non-cooperative games in nature and in society, Moscow, MAX Press.
- [11] Vatel, I. A., and Ereshko F. I., 1974, Mathematics of conflict and cooperation. Moscow, Znanie.
- [12] Helly, E., 1936, "On the set of convex bodies with common points", UMN, 2, pp. 80-81.
- [13] Vasin, A. A., and Morozov, V. V., 2005, Game theory and mathematical models of the economy, Moscow, MAX Press.
- [14] Fedorov, V. V., 1979, Numerical Methods of maximin, Moscow, Nauka.
- [15] Vladimirov, A. A., 1977, "On the solution of linear stochastic minimax problems", Vestnik of MGU, Ser. Computational Mathematics and Cybernetics, 4, pp. 63-71.
- [16] Kolmogorov, A. N., and Fomin, S. V., 1972, Elements of the theory of functions, functional analysis, Moscow, Nauka.
- [17] Maltseva, A. A., 2011, "Background management organization Technopark using the methodology of corporate governance", Theory and practice of institutional transformations in Russia: Sat. scientific papers. Vol. 20. – Moscow, Central Economic and Mathematical Institute of RAS, pp. 138-141.