International Journal of Applied Engineering Research ISSN 0973-4562 Volume 10, Number 18 (2015) pp 39849-39852

A Fuzzy Approach To Three Stage Flow Shop Scheduling Model Including Job Block Criterion Using Branch And Bound Technique

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Abstract
This paper deals with determining the optimal solution to three stage fuzzy flow shop scheduling problem. The processing times which are associated with probabilities are considered as triangular fuzzy numbers. The triangular fuzzy numbers are then defuzzified into crisp number using Yager's Ranking method. Branch and Bound technique is adopted to find the optimal sequence. The determination of optimal sequence by branch and bound method and minimum elapsed time is illustrated with a numerical example.

Keywords: Fuzzy flow shop scheduling problem, Yager's ranking method, Branch and Bound, Job block criteria.

Introduction:

In our day to day life, there are inadequate and vague information, since adequate data are not available in advance. In such circumstances it is better to provide approximate solutions with most and least possible values than with exact values. In such cases, fuzzy systems provide the better way to find proper solution.

Flow shop scheduling problem is one of classical problems widely used in manufacturing, production, management and so on. This problem aims at finding the optimal sequence of processing the jobs so as to minimize the total elapsed time. Johnson [10] proposed the well-known Johnson's rule in the two stage flow shop makespan scheduling problem.

Further the work was developed by Ignall and Scharge [6]. Brown and Lomnicki [2] studied the concept of flow shop scheduling using branch and bound method.

McCahon and Lee [12] analysed the job sequencing with fuzzy processing time. Ishibuchi and Lee [7] formulated fuzzy flow shop scheduling problem with fuzzy processing time. Hong and Chuang [5] introduced a triangular Johnson algorithm. Martin and Roberto [11] applied fuzzy scheduling to real time system. A valid approach based on statistical data for constructing a fuzzy flow shop sequencing is given by Yao and Lin [9]. TemizIzzettin and SerpilErol [8] introduced fuzzy branch and bound algorithm for flow shop scheduling.

Singh and Gupta [15], worked to minimize the rental cost in two stage flow shop problem. A new approach to two machine flow shop problem with uncertain processing time is given by Sanuja and Xueyan [14]. An heuristic approach to fuzzy flow shop problem on two machines is given by Singh, Sunita and Allawalia [16]. Sakthi et al [13] adopted Yager's ranking method to transform triangular fuzzy number to a crisp one. Deepak Gupta [3] applied branch and bound technique for three stage flow shop scheduling in which processing time and set up time are both associated with probabilities including job block criteria.

D. Gupta, S. Sharma [4] studied specially structured two stage flow shop scheduling to minimize the rental cost. Ambika and Uthra [1] has worked with three stage flow shop scheduling problem using fuzzy processing times.

In this paper, triangular membership functions are used to represent fuzzy processing times associated with probabilities including job block criteria.

2. Definitions and preliminaries:

2. 1 Triangular fuzzy number:

A fuzzy number ã on R is said to be a triangular fuzzy number or linear fuzzy number if its membership functions ã: R→ [0, 1] has the following characteristics:

$$\tilde{a}(x) = \begin{cases} \frac{x-a}{b-c} & if\ a \le x \le b \\ \frac{c-x}{c-b} & if\ b \le x \le c \\ 0 & otherwise \end{cases}$$

The triangular fuzzy number is based on three value judgement: The minimum possible value a, the most possible value b and the maximum possible value c.

2. 2 α-cut and strong cut:

Given a fuzzy set A defined on X and any number $\alpha \in [0, 1]$, the α-cut α_A, and the strong α cut α_A^+, are the crisp sets,

α_A={x/A(x) $\geq \alpha$}

α_A^+={x/A(x)$> \alpha$}

2. 3 Yager's Ranking Formula

is defined by $Y(\tilde{c})=\int_0^1 0.5\ (c_\alpha^L + c_\alpha^U)$, where $(c_\alpha^L,\ c_\alpha^U)$ is the α-level cut of the fuzzy number $\tilde{c}$.

2. 4 Fuzzy arithmetic operations:

The following two operations are performed on triangular fuzzy numbers:

Let A=(a_1, a_2, a_3) and B =(b_1, b_2, b_3)

Addition: A+B = $(a_1 + b_1, a_2 + b_2, a_3 + b_3)$

Subtraction: A-B= $(a_1 - b_3, a_2 - b_2, a_3 - b_1)$

2. 5 Notations:

a_i: Processing time for i^{th} job on machine A

b_i: Processing time for i^{th} job on machine B

c_i: Processing time for i^{th} jsob on machine C

p_i: Probability associated with processing time a_i

q_i: Probability associated with processing time b_i

r_i: Probability associated with processing time c_i

A_i: Expected processing time for i^{th} job on machine A

B_i: Expected processing time for i^{th} job on machine B

C_i: Expected processing time for i^{th} job on machine C

C_{ij}: Completion time for i^{th} job on machines A, B and C.

S_0: Optimal sequence

J_r : Partial schedule of r scheduled jobs.

J'_r: The set of remaining (n-r) free jobs.

β: Equivalent job block.

2. 6 Assumptions:

O All the jobs are available for processing at time zero.
O Each job must be completed when started.
O To make job on a second machine, it must be completed on the first machine.
O Machines may be idle.
O Setup times are included in processing times along with their probabilities.

3. Mathematical development:

Consider n jobs say i= 1, 2, 3…. n are processed on three machines A, B & C in the order ABC. Each job i (i=1, 2, 3…n) has fuzzy processing time expressed by triangular fuzzy numbers a_i, b_i & c_i on each machine respectively, assuming their respective probabilities p_i , q_i and r_i such that $0 \leq p_i \leq 1, \sum p_i = 1, 0 \leq q_i \leq 1, \sum q_i = 1, 0 \leq r_i \leq 1, \sum r_i = 1.$

Let an equivalent job β is defined as (k, m) where k, m are any jobs among the given n jobs such that k occurs before job m in the order of job block (k, m). The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine A		Machine B		Machine C	
i	a_i	p_i	b_i	q_i	c_i	r_i
1	a_1	p_1	b_1	q_1	c_1	r_1
2	a_2	p_2	b_2	q_2	c_2	r_2
3	a_3	p_3	b_3	q_3	c_3	r_3
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.						
N	a_n	p_n	b_n	q_n	c_n	r_n

Our objective is to obtain the optimal schedule of all jobs which minimize the total elapsed time using branch and bound technique.

3. 1 Algorithm:

Step 1:

Calculate expected processing time A_i, B_i, and C_i on machines A, B and C respectively as follows:

(i) $A_i = a_i \times p_i$ (ii) $B_i = b_i \times q_i$

(iii) $C_i = c_i \times r_i$

Step 2:

For triangular fuzzy numbers, using Yager's Ranking formula find expected processing times for machines A_i, B_i and C_i

Step 3:

Find the equivalent processing time of job block β = (k, m) for machines A, B and C denoted by A_β, B_β and C_β

$A_\beta = A_k + A_m$ - min (A_m, B_k)

$B_\beta = (B_1 + B_2)$ /2

Where $B_1 = B_k + B_m$ - min (A_m, B_k) and

$B_2 = B_k + B_m$ - min (B_m, C_k)

$C_\beta = C_k + C_m$ - min (B_m, C_k)

Step 4:

Replace the processing time of jobs k and m by a single equivalent job β with processing time A_β, B_β and C_β.

Step 5:

Calculate a lower bound for the 3 machine make span problem where A_i, B_i, C_i are the processing times of the i^{th} job on machines A, B and C using the formulae

(1) $l_1 = t(J_r, 1) + \sum_{i \in J'_r} A_i + \min(B_i + C_i)$

(2) $l_2 = t(J_r, 2) + \sum_{i \in J'_r} B_i + \min(C_i)$

(3) $l_3 = t(J_r, 3) + \sum_{i \in J'_r} C_i$

Step 6:

Calculate L= max {l_1, l_2, l_3 } and evaluate L first for the n classes of permutations,
i. e. for these starting with 1, 2, 3… n respectively having labeled the appropriate vertices of the scheduling tree by these values.

Step 7:

Now explore the vertex with lowest label. Evaluate L for the (n-1) subclasses starting with this vertex and again concentrate on the lowest label vertex. Proceeding until the end of the tree represented by two single permutations is reached, the total work duration is evaluated. Thus the optimal schedule of the jobs is obtained.

Step 8:
Prepare in-out table for the optimal sequence obtained in step 3 and get the minimum total elapsed time.

4. Numerical illustration.

Consider 6 jobs 3 machine flow shop problem where processing time of the jobs described by triangular fuzzy numbers in which jobs 2, 4 are to be processed as a group job (2, 4). Our objective is to obtain an optimal schedule using branch and bound technique and finding the total elapsed time.

Table 1: Processing time represented as triangular fuzzy numbers

Jobs	Machine A		Machine B		Machine C	
	a_i	p_i	b_i	q_i	c_i	r_i
1	(1, 3, 7)	0. 2	(5, 7, 9)	0. 2	(18, 20, 22)	0. 2
2	(3, 5, 8)	0. 2	(27, 29, 32)	0. 2	(2, 4, 7)	0. 2
3	(27, 29, 31)	0. 3	(1, 5, 6)	0. 2	(1, 3, 7)	0. 1
4	(1, 2, 5)	0. 1	(1, 3, 7)	0. 1	(1, 4, 6)	0. 1
5	(1, 4, 6)	0. 1	(8, 11, 12)	0. 2	(1, 5, 6)	0. 2
6	(1, 5, 6)	0. 1	(1, 3, 5)	0. 1	(1, 5, 6)	0. 2

Table 2: Processing times multiplied with probabilities

Jobs	A	B	C
1	(0. 2, 0. 6, 1. 4)	(1, 1. 4, 1. 8)	(3. 6, 4. 0, 4. 4)
2	(0. 6, 1. 0, 1. 6)	(5. 4, 5. 8, 6. 4)	(0. 4, 0. 8, 1. 4)
3	(5. 1, 8. 7, 9. 3)	(0. 2, 1. 0, 1. 2)	(0. 1, 0. 3, 0. 7)
4	(0. 1, 0. 2, 0. 5)	(0. 1, 0. 3, 0. 7)	(0. 1, 0. 4, 0. 6)
5	(0. 1, 0. 4, 0. 6)	(1. 6, 2. 2, 2. 4)	(0. 2, 1. 0, 1. 2)
6	(0. 1, 0. 5, 0. 6)	(0. 1, 0. 3, 0. 5)	(0. 2, 1. 0, 1. 2)

Table 3: Defuzzification by Yager's ranking method

Jobs	A	B	C
1	0. 8	1. 4	4. 0
2	1. 05	5. 85	0. 85
3	7. 95	0. 85	0. 35
4	0. 25	0. 35	0. 375
5	0. 375	2. 1	0. 85
6	0. 425	0. 3	0. 85

Table 4: Blocking the jobs 2, 4 into a single job β by job block

Jobs	A	B	C
1	0. 8	1. 4	4
β	1. 05	5. 9	0. 87
3	7. 95	0. 85	0. 35
5	0. 375	2. 1	0. 85
6	0. 425	0. 3	0. 85

Table 5: Calculation of Lower bounds

J_r	Lb(J_r)
1	11. 75
β	13. 87
3	19. 35
5	11. 75
6	11. 8
1β	11. 75
13	18. 75
15	11. 75
16	11. 8
1β3	13. 9
1β5	12. 25
1β6	11. 8
1β63	14. 02
1β65	11. 8

Table 6: The in-out table showing the minimum elapsed time

Jobs	A		B		C	
	In	Out	In	Out	In	Out
1	0	0. 8	0. 8	2. 2	2. 2	6. 2
β	0. 8	1. 85	2. 2	8. 1	8. 1	8. 98
6	1. 85	2. 275	8. 1	8. 4	8. 98	9. 83
5	2. 275	2. 65	8. 4	10. 5	10. 5	11. 35
3	2. 65	10. 6	10. 6	11. 45	11. 45	11. 8

Solution:

Step 1:
The processing times are represented as triangular fuzzy numbers with their probabilities as shown in table 1.

Step 2:
The processing times are multiplied with their probabilities as shown in table 2.

(i) $A_i = a_i \times p_i$ (ii) $B_i = b_i \times q_i$

(iii) $C_i = c_i \times r_i$

Step 3:
The processing times are defuzzified into crisp ones by using Yager's ranking formula

Y $(\tilde{c})=\int_0^1 0.5\ (c_\alpha^L + c_\alpha^U)$, where $(c_\alpha^L,\ c_\alpha^U)$ is the α-level cut of the fuzzy number $\tilde{c}$ as shown in Table 3.

Step 4:
Find an equivalent job β for the job block (2, 4) by calculating their processing times as follows.

$A_\beta = A_k + A_m$-min (A_m, B_k)

$B_\beta = (B_1 + B_2)\,/2$

where $B_1 = B_k + B_m$ - min ($A_{\mathrm{m}}, B_{\mathrm{k}}$)

And $B_2 = B_k + B_m$ - min ($B_{\mathrm{m}}, C_{\mathrm{k}}$)

$C_\beta = C_k + C_m$ - min ($B_{\mathrm{m}}, C_{\mathrm{k}}$)

The reduced problem is as shown in table 4.

Step 5:
Calculation of lower bounds follows from steps 5, 6 and 7 of Algorithm 3. 1 and is shown in table 5.

Step 6:
The in-out table is prepared for the optimal sequence showing the minimum elapsed time. The optimal Sequence is S_0: 1β653. The minimum total elapsed time is 11. 8 and is shown in table 6.

5. Conclusion.

In this paper, the processing times are considered as triangular fuzzy numbers. The fuzzy scheduling problem is converted into crisp one by Yager's ranking formula. The optimal sequence is determined using Branch and Bound techniques including job block criterion. Thus the scheduling problem for determining the minimum elapsed time in fuzzy environment is very effective under uncertainty.

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