International Journal of Applied Engineering Research ISSN 0973-4562 Volume 10, Number 18 (2015) pp 39803-39807

Sylvester Matrix Theorem an Ideal Tool in MIMO Channel Estimation

Vikash Sharma

Department of Electronics & Communication Engineering
Waljat College of Applied Sciences, Muscat, Oman vikash@waljat.net

Dr. Sanjay Kumar

Associate Professor Department of Electronics & Communication Engineering
Birla Institute of Technology, Mesra, Ranchi, India skumar@bitmesra.ac.in

Abstract:
In recent past MIMO has evolved as a prominent mode of technique in the situation involving more than one transmitter and more than one receiver. The question of privacy between a pair of transmitter and receiver is also a point of interest amongst the researchers. A number of algorithms were proposed by the reserchers in finding solutions for such situations. In our present paper we have proposed a nobel technique to apply the solutions for Sylvester matrix theorem in handling the situation that differentiates between authorised and unauthorised receivers.

Key words: Sylvester Matrix, MIMO, authorised receiver, unautorised receiver.

Introduction

Increase in channel capacity, promising higher data transmission rate with low error probability in wireless medium has been a topic of interest among researchers for last few decades. Multi-antenna communication in wireless environment has provided enormous potential in this regard. Application of UWB signals in MIMO medium has further contributed in enhancing the channel capacity and hence improved data transmission rate. A number of theoretical models as well as experimental findings are available in literatures.

The MIMO channel models, proposed by researchers, can be broadly classified into three categories namely stochastic models, deterministic models and geometry based stochastic models. In general, two approaches are followed to understand the relationship between channel parameters with time and frequency: estimation of equivalent discrete channel taps and estimation of physical propagation parameters like multipath propagation delays and multipath complex gains. Choice of the length of the training signal and hence the duration also affects the channel capacity. Training symbol based techniques suffers from limitations like wastage of bandwidth. Semi – blind and Blind channel estimation techniques were developed for channels having deterministic model. [1, 2]

Sylvester matrix theorem has found wide applications in optimising matrices. A number of direct methods and iterative algorithms has been proposed by researchers in this area. The research work includes theoretical, practical and applied mathematics. The areas of applications is not limited to control theory, model reduction, signal processing, image restoration, decoupling techniques for ordinary and partial differential equations, block diagonalization of matrices, filtering and system identification.[3, 4, 5]

Methods adopted include direct general purpose solutions and iterative merhods. In the direct method the coefficient matrices are transformed to Schur or Hessenberg form and then the resultant linear equations is solved by a backward substitution process. On the other side the iteravive methods focus on large sparse systems. But these methods lack in providing explicit formula for the solutions. [5, 6]

Algorithms proposed to solve the Sylvester matrix also include the ones proposed by Bartel and Stewart, Golub, Nash and van Loan, Hessenberg and Schur, and many other researchers. Kagstrom and Wiberg studied and compared the methods for transforming a matrix into a Jordan – Schur form and a matrix pencil into a Weiestrass – Schur form, Jones et. al. proposed a method to transform coupled Sylvester equation into standard Sylvester equation and used standard techniques to solve it.Wu et. al and E. Souza and S.P. Bhattacharya propsed a closed form finite series representation of the unique solution. Zhou. S., Caiqin et. al, in their research proposed gradient based iterative solution method for Sylvester - conjugate matrix that does not need transformation of the coefficient matrices into canonical form. Bischof, Datta and Purkayastha noted that the Hessenberg-Schur method has a limitation that it is not suitable for parallelism and they proposed a method which can be applied to parallel machines. [6, 7, 8]

Development of System Model

The general UWB-MIMO channel model is expressed as:

$$y(n) = Hx(n) + v(n) \tag{1}$$

The above equation can be represented in equivalent matrix form as:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ \vdots \\ y_{N_r} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N_t} \\ h_{21} & h_{22} & \cdots & h_{2N_t} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_r1} & h_{N_r2} & \cdots & h_{N_rN_t} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_{N_t} \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ \vdots \\ v_{N_t} \end{bmatrix} \tag{2}$$

The elements of transmitted signal vector $x(n)$ are assumed independent identically distributed (i.i.d), having Gaussian random variables with zero mean and σ^2 as *variance*($i.e., \mathcal{C}N(0,\sigma^2)$).
The role of the channel estimation algorithm is to estimate the value of H depending upon the values of $y(n)$ and $x(n)$.
Privacy of the communication between the two entities is the top most objective for any researcher. It leads to the concept of reception of transmitted signal by authorised receiver (AR).

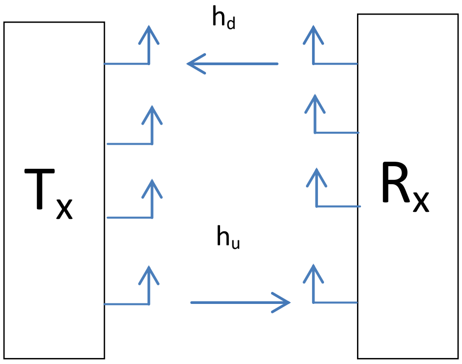

Fig. 1 Generalised MIMO Network

Transmission of training signals are used in both blind and semi – blind channel estimation techniques to understand the behaviour of the signal. Chang et. al. elaborated the discriminatory channel estimation (DCE) scheme, as a possible approach leading to isolating the unauthorised receivers (UR). Under this scheme an artificial noise is inserted in the training signal. The artificial noise is so carefully placed that it jams the UR and the interference caused at AR is minimum. [1, 9, 10, 11]
The length of the training signal is a question of interest. It is understood that; if the length of the training signal is short we may not have enough time to estimate the channel characteristics but we have ample time for data signal. On the other hand if the length of the training signal is too large we have the advantage that the channel characteristics are properly judged but at the same time we have less time left for communication of data signals.
If *T* is the duration required for training and data transmission, M is the number of transmitting antennas, then Hassibi and Hochwald in [3] proposed the following three situations (i) when *T*> 2M –more training power is required, (ii) if *T*< 2M –more power for data transmission is required and (iii) when *T* = 2M we need same power.
We have asumed that the channel is reciprocal by nature and the channel characteristics are constant for some discrete time interval T, after which it changes to an independent value that holds for another time interval T (i.e. it obeys discrete – time block – fading law). The forward link and the reverse link use TDD – type training protocol over the same frequency band. The CSI is obtained by first applying reverse link followed by the forward link.[1, 2, 3, 12].

The Training Protocol
It is understood that at the beginning of transmission, the channel characteristics H is not known to the receiver. The training signals help in estimating the channel properties and it is followed by the data transmission phase.

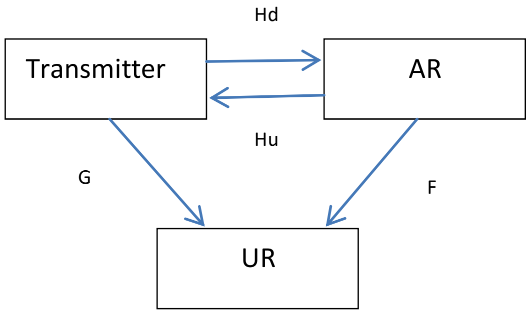

Fig. 2 Wireless MIMO Network with Authorised and Unauthorised Receivers.

In figure 2 the number of antennas at transmitter, authorised receiver and unauthorised receivers are N_t, N_L and N_U respectively. It is assumed that there is sufficient spacing between them. It leads to i.i.d. channel behavior between the three.
In the two way training scheme, both transmitter and receiver send pilot signals. Two options are available for a DCE channel model, namely reciprocal and non-reciprocal channel models. The literatures propose two as well as four phases of training protocol in a reciprocal channel. We have referred the two phases of training for our study.
In the reciprocal channel varience is expressed as $\sigma_h^2 = \sigma_{hu}^2 = \sigma_{hd}^2$. The transmitter can have an estimate of the channel characteristic by obtaining the transpose of the estimated channel matrix through reverse training.[1, 10, 12]

First Phase –
The reverse training phase –In this phase the transmitter obtains a reliable estimate of the downlink channel to AR without benefitting the channel estimation process in UR.
The process starts with the AR sending a training signal to the transmitter given as:

$$X_L = \sqrt{\frac{P_R \tau_R}{N_L}} C_L \qquad (3)$$

Here C_L is the pilot matrix, P_R is the transmission power for reverse training, τ_R is the length of the reverse training signal. The transmitter with the help of the reverse training signal from AR estimates the channel.Using LMMSE the uplink channel can be estimated as

$$\widehat{H} = \left(\sigma_h^2 X_L^H \left(\sigma_h^2 X_L X_L^H + \sigma_\omega^2 I_{\tau H}\right)^{-1} Y_t\right)^T \qquad (4)$$

Second Phase –

Forward Training with AN – After obtaining the downlink channel estimate the $\hat{H}$ from first phase, the transmitter sends a forward training signal to enable the channel estimation at AR. An artificial noise is carefully inserted in this phase that affects the channel estimation process with UR but not affecting it at AR.

The forward training signal is given as:

$$X_t = \sqrt{\frac{P_F \tau_F}{N_t}} C_t + AK_{\hat{H}}^H \qquad (5)$$

In this equation, τ_F is the length of training signal, P_F is the power of pilot signal, A is the artificial noise added, K_H is a matrix whose column vectors form an orthonormal basis for the left hand null space of $\widehat{H}$.

The received signal at AR and UR are as given below:

$$Y_L = \sqrt{\frac{\varepsilon_F}{N_t}} C_t H + AK_{\hat{H}}^H H + W \qquad (6)$$

$$Y_U = \sqrt{\frac{\varepsilon_F}{N_t}} C_t G + AK_{\hat{H}}^H G + V \qquad (7)$$

$\varepsilon_F \cong P_F \tau_F$ is the forward pilot signal energy.

W and V are additive i.i.d. white gaussian noise matrices in AR and UR respectively.

Using the above relations the AR channel is estimated as:

$$\hat{H}_L = R_H \widetilde{C}^H \left(\widetilde{C} R_H \widetilde{C}^H + R_{\overline{W}}\right)^{-1} Y_L \qquad (8)$$

Here $R_H = \mathrm{E}\{HH^H\} = N_L \sigma_h^2 I_{Nt}$

and $R_W = \mathrm{E}\{\widetilde{W}\widetilde{W}^H\}$ (9)

The Proposed Algorithm:

The generalised Sylvester equation is expressed as:

$$AX - XB = C \text{ or } AX - YB = C \qquad (10)$$

Where A, B and C are $m \times m, n \times n$ and $m \times n$ matrices and the unknown matrix X is of dimension $m \times n$.

The above equation is a special case of the linear equation

$$\sum_{i=1}^{N} A_i \times B_i = E \qquad (11)$$

The above equation has a unique solution if A and B has no common eigenvalues i.e.

$$\sigma(A) \cap \sigma(B) = \phi\,. \qquad (12)$$

Any one of the matrices A, B, C and D may be singular without causing the linear operator to be singular. [9]

Lancaster and Tismenetsky proposed to express the Sylvester Equation as:

$$[I_n \otimes A - B^T \otimes I_m]x = c \qquad (13)$$

Here $\otimes$ is a Kronecker product of matrices, $x = V_c(X)$ and $c = V_c(C)$.

$$V_c(X) = (x_{11}, x_{21}, x_{31}, \ldots\ldots x_{1n}, x_{2n}, \ldots x_{mn})^T \qquad (14)$$

The coefficient matrix is non-singular iff A and B have no common eigen values.

Perturbation analysis of generalised eigen spaces of matrix pencils from equation (1) leads to a pairs of generalised Sylvester equations of the form:

$$\begin{aligned} A_1 X - YB_1 &= C_1, \\ A_2 X - YB_2 &= C_2 \end{aligned} \qquad (15)$$

Theorem 1:

Let F be a field, and let $A_i, B_i, C_i; i = 1,2$ be matrices over F of respective sizes $m \times n; p \times k; m \times k$ then equation (15) has a simultaneous solution $X \in F^{n \times k}, Y \in F^{m \times p}$ if and only if there exists non singular matrices $R \in F^{(n+k)\times(n+k)}$ and $S \in F^{(m+p)\times(m+p)}$ such that:

$$S\left[\begin{pmatrix} A_1 & C_1 \\ 0 & B_1 \end{pmatrix} - s\begin{pmatrix} A_2 & C_2 \\ 0 & B_2 \end{pmatrix}\right]$$

$$= \left[\begin{pmatrix} A_1 & 0 \\ 0 & B_1 \end{pmatrix} - s\begin{pmatrix} A_2 & 0 \\ 0 & B_2 \end{pmatrix}\right] R \qquad (16)$$

The above equation (15) can be combined into a single equation over F[s] of the form

$$(A_1 - sA_2)X - Y(B_1 - sB_2) = C_1 - sC_2 \qquad (17)$$

It is an extension of Roth's equvalence theorem to a pair of Sylvester equations. [20]

Lemma 1:
Let $\widetilde{A} \in F^{m\times n}[s], \widetilde{B} \in F^{p\times k}[s], C \in F^{m\times k}[s]$ be given polynomial matrices. Then the equation

$$\widetilde{A}X - Y\widetilde{B} = \widetilde{C} \tag{18}$$

has a constant solution
$X \in F^{n\times k}, Y \in F^{m\times p}$ if and olny if there exists non singular constant matrices
$R \in F^{(n+k)\times(n+k)}, S \in F^{(m+p)\times(m+p)}$ such that

$$\begin{pmatrix} \widetilde{A} & 0 \\ 0 & \widetilde{B} \end{pmatrix} R = S \begin{pmatrix} \widetilde{A} & \widetilde{C} \\ 0 & B \end{pmatrix} \tag{19}$$

(For detailed proof please refer [15]). [5, 6, 7, 8, 15]
The floating – point operations (flops) involved required is an important criteria while selecting the method and hence the algorithm. From the literatures available we note that the flops required (also known as work count) for different algorithms are as follows:

a) Hessenberg – Schur method

$$8.8m^3 + 33n^3 + (5+5.5t)m^2n + 3mn^2 \tag{20}$$

b) Bartels – Stewart method

$$-\ 33m^3 + 33n^3 + 3m^2n + 3mn^2 \tag{21}$$

c) On the other hand the cost of iterative methods range between $2(m+n)^3$ flops for Newton Iteration method to $2(n^3 + nm(n+m) + m^3)$ flops for the one suggested by Roberts et. al. [14]

Amir Shahzad and his team [16] proposed an efficient algorithm that need only $2p^2q + 2pq^2$ flops.
The salient features of the algorithm are as follows:

a) It does not need an inverse of the matrix.
b) Does not need matrix – by – matrix multiplication. And
c) No need to solve standard Sylvester equation.

The steps involved in Sylvester Matrix optimization are:

a) Initilize the input parameters.
b) Define the output parameters.
c) for i=1 to q **do**

using backward subsitution solve $E_1 r_1 = -e_1^2 - \sum_{k=1}^{i-1} e_k^3 l_k$ for r_i

compute $l_i = -\frac{1}{f_{ii}^3}\left(f_i^{\,2} + F_1 r_1 + \sum_{k=1}^{i-1} f_{ki}^3 l_k \right)$

d) end for

Results and Discussion

We have considered a 4 X 4 MIMO channel. The parameters choosen for our objective are as follows: no. of transmiting antennas =4, no. of authorised receivers = 2, no. of unauthorised receivers =2, the channel parameters H, H_u, H_d, G are i.i.d. complex Gaussian distributed with zero mean and unit variance. The length of reverse training signal $\tau_R = N_L = 2$, length of forward training signal $\tau_F = N_L = 4$. We have adopted 64 QAM modulation with STBC coding.
The following parameters were compared a) Variation in NMSE vs. Average Signal Power, b) Variation in Symbol Error Rate vs. Average Signal Power, c) Total execution time.We have noted that with increase in AN power there is an increase in the NMSE at both the receivers, the effect at the AR can be reduced by increasing the value of ε_R.Thus we have to carefully choose the values of power content of the training signal and AN.Power content of all the training signals and AN power increases with increase in total power between 15 dB and 25 dB.The curve becomes flat after 25dB.More power should be allocated to AN and less power to the training signals as our objective to degrade the performance at UR.No change in the values of NMSE with variation in Average Power with the values of γ considered at the UR.No change in the values of SER with variation in Average Power with the values of γ considered at the UR. It is noted that the signal transmitted from a transmitter is successfully detected by AR and it improves as average power budget increases while the symbol errors at UR is more than 0.2 indicating poor channel estimation.

Conclusion

In our present work we have combined the advantages of UWB signal with those of discriminatory channel estimation (DCE) scheme. We have adopted the algorithm proposed by Shehzad in solving the channel matrix. This combination not only helped us in establishing a secured communication over the MIMO channel and finding the channel matrix parameters but also optimising the channel estimation time.

References

1. C.W. Huang, T.H. Chang, X.Zhou and Y.W. Peter Hong, "Two-Way Training for Discriminatory Channel Estimation in Wireless MIMO Systems"; IEEE Transactions on Signal Processing, Vol.61, No. 10, pp 2724-2738, 2013.
2. V.Sharma and S. Kumar, "Recent Developments in MIMO Channel Estimation Techniques", IEEE Proceedings ofThe Second International Conference on Digital Information and Communication Technology and its Applications (DICTAP2012), pp1-6, 2012.

3. Z. Chen, L. Lu, "A Gradient based Iterative Solutions for Sylvester Tensor Equations", Mathematical Problems in Engineering, pp1-7, 2013.
4. N. Truhar and R.C. Li., "On ADI Method for Sylvester Equation", Journal of Computational and Applied Mathematics, Volume 233, Issue 4, pp 1035-1045, 2009.
5. Qingxi Hu, D. Cheng, "The polynomial solution to the Sylvester matrix equation", Applied Mathematics Letters, vol. 19, pp 859-864, 2006.
6. Kagastrom B. and Westin L."Extracting Partial Canonical Structure for Large Scale Eigen Value Problems", Numerical Algorithms, Volume 24, No. 3, 195-237, 2000.
7. S. Caiqin, F. Jun-e, Z. Jianli and M. Min, " A New Technique for Solving Continuous Sylvester-conjugate Matrix Equation"; Proc. Of 33rd Chinese Control Conference, pp 6545-6548, 2014.
8. C.H. Bischof, B.N. Datta and A. Purkayastha, "A Parallel Algorithm for the Multi – input Sylvester – Observer Equation"; pp 152-158, 1992.
9. T.L. Marzetta, "How Much Training is Required for Multiuser MIMO?" pp 359-363.
10. B.Hassibi and B.M. Hochwald, " How Much Training is Needed in Multiple –Antenna Wireless Links?"; IEEE Transactions on Information Theory, Vol. 49, No. 4, pp 951-963, 2003.
11. T.H. Chang, W.C. Chiang, Y.W. Hong and C.Y. Chi, "Training sequence design for discriminatory channel estimation in wireless MIMO systems"; IEEE Transactions on Signal Processing, Vol. 58, No. 12, pp 6223-6237, 2010.
12. K.S. Gomadam, H.C. Papadopoulos and C.E.W. Sundberg, "Techniques for Multi-user MIMO with Two-way Training", Proceedings of IEEE Communications Society, pp 3360-3366, 2008.
13. J.D. Gardiner and A.J. Laub;"Solution of the Sylvester Matrix Equation $A \times B^T + C \times D^T = E$"; ACM Transactions in Mathematics Software, Volume 18, No. 2, pp 223-231, 1982.
14. Peter Benner, "Factorized Solution of Sylvester Equations with Applications in Control"; Proceedings of the 16th Intl. Symp. on Mathematical Theory of Networks and Systems, 2004.
15. H.K. Wimmer, " Consistency of a Pair of Generalized Sylvester Equations"; IEEE Transactions on Automatic Control, Vol. 39, No. 5, pp 1014-1016, 1994.
16. A. Shahzad, Bryn Li Jones, Eric C.Kerrigan and G.A. Constantinides, "An Efficient Algorithm for the Solution of a Coupled Sylvester Equation appearing in Descriptor Systems"; Automatica., 2010.