

Weight minimization of Spring design problem using PSO, SA, PS, GODLIKE, CUCKOO, FF, FP, ALO, GSA and MVO

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Abstract

The objective functions used in Engineering Optimization are complex in nature with many variables and constraints. Conventional optimization tools sometimes fail to give global optima point. Very popular methods like Genetic Algorithm, Pattern Search, Simulated Annealing, and Gradient Search are useful methods to find global optima related to engineering problems. This paper attempts to use new non-traditional optimization algorithms which are used to find the minimum weight of designing a coil spring to obtain global optimum solutions. The weight, number of iterations and the total elapsed time to complete the problems are all compared using these ten non-traditional optimization methods.

Keywords: Designing a coil spring, optimization, non-traditional, Pattern search, Simulated annealing, Pattern search, GODLIKE, Cuckoo search, Firefly algorithm, Flower pollination, Ant lion optimizer, Gravitational search algorithm, Multi-verse optimizer

Introduction

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are to

- (1) apply forces, as in brakes and clutches and spring loaded valves
- (2) measure forces, as in spring balances
- (3) store energy, as in watch springs
- (4) absorb shock and vibration as in car springs and railway buffers
- (5) measure force, as in spring balances and engine indicators.

The different types of springs are Helical springs, Conical and volute springs, Torsion springs, Laminated or leaf springs and Disc or belleville springs (Bhandari.V.B.). The helical springs are made up of a wire coiled in the form of helix and are primarily intended for compressive or tensile loads. The helical springs are said to be closely coiled when the spring is so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. That is in a closely coiled helical spring the helix spring is very small. The major stresses produced in the helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring. In open coiled helical spring, the spring wire is coiled in such a way that there is a gap between two consecutive turns, as a result

of which helix angle is large. Applications of open coiled helical springs are limited. The advantages of helical springs are

- (1) easy to manufacture
- (2) available in wide range
- (3) reliable
- (4) constant spring rate
- (5) can be predicted more accurately
- (6) characteristics can be varied by changing dimensions.

The springs are mostly made up of oil-tempered carbon steel wires containing 0.6 to 0.7 percent carbon and 0.6 to 1.0 percent manganese. The helical springs are either cold formed or hot formed depending on the size of the wire.

Nomenclature

g	gravitational constant
γ	weight density of spring material
G	shear modulus
ρ	mass density of material
τ_a	allowable shear stress
Q	number of inactive coils
P	applied load
Δ	minimum spring deflection
ω_0	lower limit on surge wave frequency
D_0	limit on outer diameter
δ	deflection along the axis of spring
D	mean coil diameter
d	wire diameter
N	number of active coils

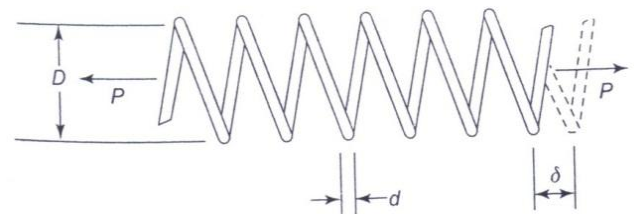


Fig.1 Schematic of the helical spring to be designed

2. PROBLEM DEFINITION

Coil springs are used in numerous practical applications. Detailed methods for analyzing and designing such

mechanical components have been developed over the years (e.g., Spotts, 1953; Shigley, Mischke and Budynas, 2004; Haug and Arora, 1979). The purpose of this project is to design a minimum weight of the tension/compression spring subject to constraints on shear stress, surge frequency and minimum deflection.

2.1.PARAMETERS:

(Spring Design Optimization)

To formulate the problem of designing coil springs, the following notation and data are defined:

Gravitational constant (in s)

$$g = 386 \text{ in/s}^2$$

Weight density of spring material (lb in)

$$\gamma = 0.285 \text{ lb/in}^3$$

Shear Modulus (lb in

$$G = (1.15 * 10^7) \text{ lb/in}^2$$

Mass density of material (lb s in)

$$\rho = (7.38342 * 10^{-4}) \text{ lb-s}^2/\text{in}^4$$

Allowable shear stress (lb in)

$$\tau_a = 80000 \text{ lb/in}^2$$

Number of inactive coils

$$Q = 2$$

Applied load (lb)

$$P = 10 \text{ lb}$$

Minimum spring deflection (in)

$$\Delta = 0.5 \text{ in}$$

Lower limit on surge wave frequency (Hz)

$$\omega_0 = 100 \text{ Hz}$$

Limit on outer diameter of coil (in)

$$D_0 = 1.5 \text{ in}$$

Deflection along the axis of spring δ , inch

2.2. DESIGN VARIABLES

The three design variables for the problem are defined as

Mean coil diameter D, inch

Wire diameter d, inch

Number of active coils N

2.3. DESIGN CONSTRAINTS:

The four important constraints under consideration are

2.3.1.Deflection constraint:

It is often requirement that deflection under a load P be at least Δ . Therefore, the constraint is that the calculated deflection δ must be greater than or equal to Δ . Such a constraint is common to spring design. The function of the spring in many applications is to provide a modest restoring force as parts undergo large displacement in carrying out kinematic functions. Mathematically, this performance requirement ($\delta \geq \Delta$) is stated in an inequality form using load deflection equation

$$P = K\delta; \frac{P}{K} \geq \Delta$$

$$\frac{D^3 N}{71785 d^4} \geq 1 \quad (1)$$

2.3.2.Shear stress constraint:

To prevent overstressing, shear stress in the wire must be no greater than τ , which is expressed in mathematical form a

$$\frac{4D^2 - Dd}{12566(Dd^3 - d^4)} + \frac{1}{5108 d^2} \leq 1 \quad (2)$$

2.3.3.Constrain on the frequency of surge waves:

We want to avoid resonance in dynamic applications by making the frequency of surge wave (along the spring) as great as possible. For the present problem, we require the frequency of surge waves for the spring to be at least (Hz). The constrain is expressed in mathematical form as

$$\frac{140.45 d}{N D^2} \geq 1 \quad (3)$$

2.3.4.Diameter constraint:

The outer diameter of the spring should not be greater than D_0

Table 1	No. of coils (N=x ₁)		Mean coil diameter(D=x ₂)		Wire diameter(d=x ₃)	
	No unit	No unit	inch	mm	inch	mm
Upper Bound	15	-----	1.5	38.1	2	50.8
Lower Bound	2	-----	0.25	6.35	0.05	1.27
Optimum	7.307391	-----	0.492709	12.5148	0.056491	1.4348714

$$D + d \leq D_0$$

$$D + d \leq 1.5 \quad (4)$$

2.3.5. Variables bounds: (Harish Garg,, 2014)

To avoid the fabrication and other practical difficulties, we put minimum and maximum size limits on the wire diameter, coil diameter and number of turns:

$$d_{\min} \leq d \leq d_{\max}; 2 \leq d \leq 15$$

$$D_{\min} \leq D \leq D_{\max} \quad 0.25 \leq D \leq 1.5$$

$$N_{\min} \leq N \leq N_{\max} \quad 0.05 \leq N \leq 2$$

2.4..Optimization criterion:- cost function

The problem is to minimize the mass of the spring (volume*mass density) which is given as

$$\text{Minimize } f = \frac{1}{4} (N + Q) D d^2 \pi^2 \rho$$

2.5. Engineering Relationship

(Yang, X-S., Deb.S, 2010), (Kavesh. A, S.Talatahari, 2009)

A design under tension or compression, the wire experiences twisting. Therefore, the shear stress constrain should be imposed. We have the following design expressions for the spring:

Load deflection equation: $P = K \delta$

Spring constant

$$K = \frac{d^2 G}{8 D^3 N}$$

Shear stress

$$\tau = \frac{8 k P D}{\pi d^3}$$

Wahl stress concentration factor

$$k = \frac{(4D - d)}{4(D - d)} + \frac{0.615d}{D}$$

Frequency of surge wave

$$\omega = \frac{d}{2 \pi N D^2} \sqrt{\frac{G}{2 \rho}}$$

The expression for the Wahl stress concentration factor k has been determined experimentally to account for unusually high stresses of certain points of the spring. The expression can be used to define constraints for the problem.

2.6. MATHEMATICAL FORMULATION

(Harish Garg, 2014), (Afondo C.C.Lemonge, Helio I.C.barbosa Carlos C.H.Borges, Francilene B.S.Silva, 2010)

The mathematical formulation of the objective function $f(X)$ which is the weight of the tension/compression string mainly comprised of shear stress, surge frequency and minimum deflection is as follows

The design variables are the mean coil diameter $D (=x_2)$, the wire diameter $d (=x_3)$ and the number of active coils $N (=x_1)$. The objective is to minimize the weight of the spring design problem. The problem can be state as

$$\text{Minimize } f(X) = (x_1 + 2)x_2 x_3^2$$

Subject to

$$\frac{x_2^3 x_1}{71785 x_3^4} \geq 1$$

$$\frac{4x_2^2 - x_3 x_2}{12566(x_2 x_3^3 - x_3^4)} + \frac{1}{5108 x_3^2} \leq 1$$

$$\frac{140.45 x_3}{x_2^2 x_1} \geq 1 \quad \frac{x_2 + x_3}{1.5} \leq 1$$

Variable region is

$$2 \leq x_1 \leq 15$$

$$0.25 \leq x_2 \leq 1.5$$

$$0.05 \leq x_3 \leq 2$$

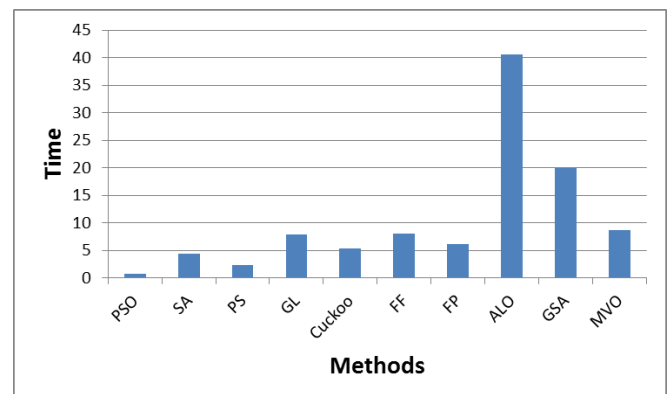
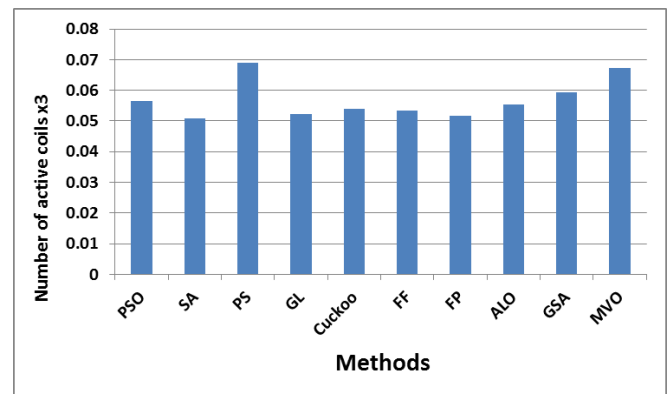
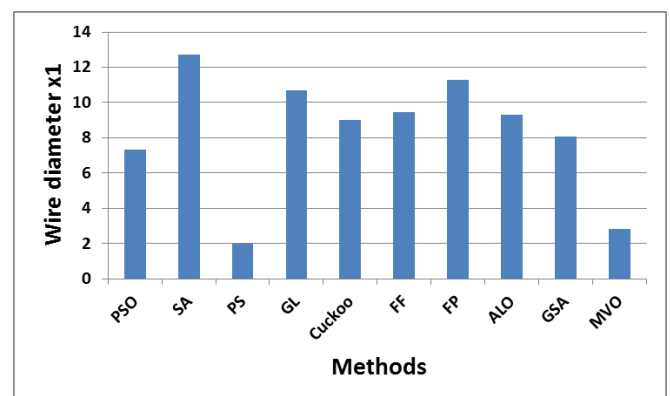
3. Comparative Results

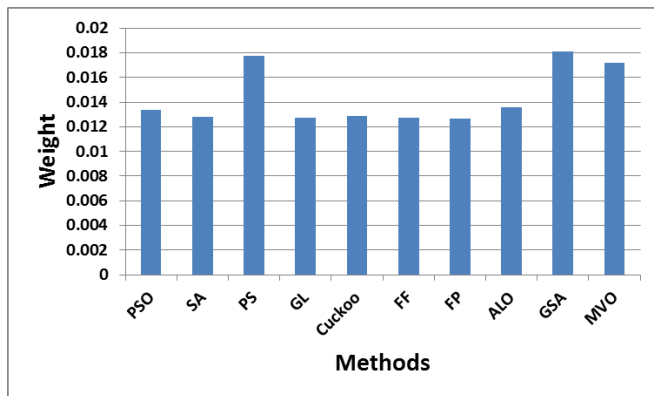
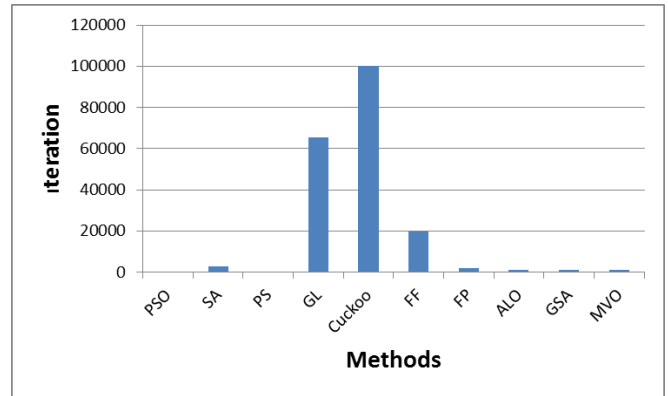
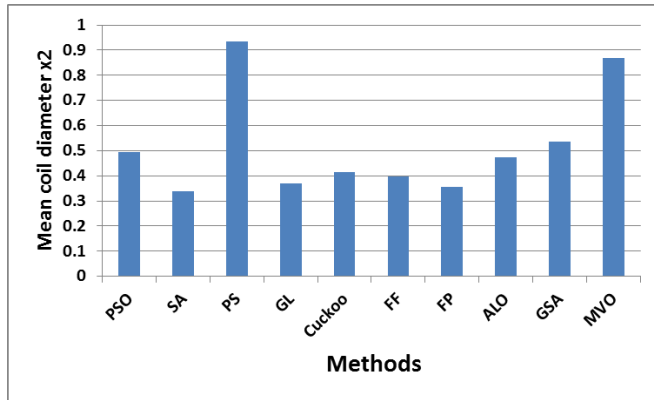
The ten methods are run 20 trails and the average is taken and the results were compared.

The number of active coils $N (=x_1)$.

The design variables are the mean coil diameter $D (=x_2)$

The wire diameter $d (=x_3)$





From the above graphs we know that the weight, the number of iteration and the elapsed time is minimum in PSO and PS but PS is high in other two parameters x_1 , x_2 ,

4. Results and Discussion:

With the two extreme values of the parameters the optimization is carried out with different solvers. As they are stochastic type the results may vary from trial to trial. So the problem is made to run for 20 trials. (Elbeltagi.E., Tarek Hegazy.I., Grierson D., 2005) And an average of all trials is taken as a final value of the parameter by the solver. The solvers are compared with three different criteria.

4.1. Consistency

The weight is consistent in Pattern Search (0.017773)

4.2. Minimum run time:

For minimum run time of the problem we have PS (2.357923 seconds), PSO (0.794744 seconds).

4.3. Minimum Evaluation:

This Criterion will determine the effectiveness of the algorithm. From the table we see that the PS and PSO algorithm have minimum evaluation of 5 and 125 respectively.

Table 3.Comparative table

Trial No	PSO	SA	PS	GL	Cuckoo	FF	FP	ALO	GSA	MVO
X_1	7.307391	12.73351	2	10.70877	9.008896	9.464149	11.27815	9.309473	8.083863	2.808628
X_2	0.492709	0.338024	0.933441	0.370175	0.413817	0.397588	0.356917	0.474617	0.534752	0.868862
X_3	0.056491	0.050927	0.068994	0.052239	0.053904	0.053292	0.051697	0.055414	0.059218	0.067245
Weight	0.013346	0.012828	0.017773	0.01273	0.012866	0.012751	0.012666	0.013587	0.018114	0.017178
Time	0.794744	4.349136	2.357923	7.962026	5.399062	8.089031	6.239744	40.6075	19.98511	8.649393
Iteration	125	2972	5	65563	100000	20000	2000	1000	1000	1000

Table.4.Tables for option set and Stopping criteria for the ten methods

methods	PSO	SA	PS	GL	CUCKOO	FF	FP	ALO	GSA	MVO
Option set	Max.Generation=200 Max.Time Limit=∞ Average change in fitness value=10 ⁻⁶ Function Tolerance:10 ⁻⁶ Cognitive Attraction=0.5 Population Size=40 Social Attraction=1.25	Initial Temperature:100 Annealing Function: Fast Reannealing Interval:100 Time limit:∞ Max.Function Evaluation:3000*No.of variables. Max.Iteration:∞ Function Tolerance:10 ⁻⁶ Objective Limit:10 ⁻⁶	Poll Method:GPS positive basis 2N Initial Mesh Size:1 Expansion Function:2 Contraction Factor:0.5 Mesh Tolerance:10 ⁻⁶ Max.Function Evaluation:2000*No.of variables. Max.Iteration:100*No.of variables. Max.Time Limit:∞ Function Tolerance:10 ⁻⁶	Max.Fun Evaluations=10 ⁻⁵ Max.Iterations=20 Min.Iterations=2 Total Iterations=15 Functions Tolerance=10 ⁻⁴	Max.Fun Evaluations=10 ⁻⁵ Max.Iterations=20 Functions Tolerance=10 ⁻⁶ Max.Time Limit=∞	Max.Fun Evaluations=10 ⁻⁵ Max.Iterations=20 Functions Tolerance=10 ⁻⁶ Max.Time Limit=∞	Max.Fun Evaluations=10 ⁻⁵ Max.Iterations=20 Functions Tolerance=10 ⁻⁶ Max.Time Limit=∞	Max.Fun Evaluations=10 ⁻⁵ Max.Iterations=20 Functions Tolerance=10 ⁻⁶ Max.Time Limit=∞	Max.Fun Evaluations=10 ⁻⁵ Max.Iterations=20 Functions Tolerance=10 ⁻⁶ Max.Time Limit=∞	Max.Fun Evaluations=10 ⁻⁵ Max.Iterations=20 Functions Tolerance=10 ⁻⁶ Max.Time Limit=∞
Stopping criteria	Max.Generation=200 Max.Time Limit=∞ Average change in fitness value=10 ⁻⁶ Function Tolerance:10 ⁻⁶	Max.Time reached The average change in value of the objective function is < 10 ⁻⁶ max.iterations are reached if the number of functions evaluations reached. If the best objective function value is less than or equal to the value of objective limit.	Mesh Tolerance:10 ⁻⁶ Max.Iteration: 100*No.of variables. Evaluation:2000*No.of variables Max.Time Limit:∞ Function Tolerance:10 ⁻⁶	Max.Fun Evaluations= 10 ⁻⁵ Max.Iterations=20 Min.Iterations=2 Total Iterations=15 Functions Tolerance= 10 ⁻⁴	Max.Fun Evaluations=10 ⁻⁶ Max.number of Iterations= 100000 Functions Tolerance= 10 ⁻⁶ Max.Time Limit=∞	Max.Fun Evaluations=10 ⁻⁶ Max.number of Iterations= 100000 Functions Tolerance= 10 ⁻⁶ Max.Time Limit=∞	Max.Fun Evaluations=10 ⁻⁶ Max.number of Iterations= 100000 Functions Tolerance=10 ⁻⁶ Max.Time Limit=∞	Max.Fun Evaluations= 10 ⁻⁶ Max.number of Iterations= 100000 Functions Tolerance=10 ⁻⁶ Max.Time Limit=∞	Max.Fun Evaluations=10 ⁻⁶ Max.number of Iterations= 100000 Functions Tolerance=10 ⁻⁶ Max.Time Limit=∞	Max.Fun Evaluations=10 ⁻⁶ Max.number of Iterations= 100000 Functions Tolerance= 10 ⁻⁶ Max.Time Limit=∞

4.4. The Simplicity of Algorithm:

Of all the algorithms, Pattern Search algorithm is the most simplest followed by Particle Swarm Optimization.

Thus it is seen that the PS solver satisfies all the criteria. Even though the pattern search satisfies all the above criteria, the weight becomes maximum whereas the weight in PSO is 0.013346. Therefore the particle swarm optimization has the minimum weight with time 0.794744 seconds and 175 iteration so the appropriate algorithm for pressure vessel design is suggested as Particle Swarm Optimization. It is apparent from the results that PSO algorithm is able to provide promising solutions with less objective function evaluations. This desirable characteristic of PSO algorithm would be more significant in one engineering problems which entail higher computational effort.

5. Conclusion:

In the present study the PSO algorithm is proposed as a simple and efficient optimization technique for handling spring design problem. PSO algorithm is a population based technique which follows a stochastic iterative procedure to locate the optimum or a reasonably near- optimum solution for the spring design optimization. Performance evaluation of the PSO algorithm through spring design optimization reveals the efficiency of this technique in solving practical optimization problems. Although in the present study the PSO algorithm is utilized only for solving spring design optimization problem, it can be easily employed for solving other types of optimization problems as well

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