

# Photoelastic Measurements for Mixed Mode Stress Intensity Factors of an Inclined Crack in a Tensile Plate

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## Abstract

This paper presents the measurement of mixed mode stress intensity factors of inclined cracks by use of photoelasticity. The distributions of isochromatics near a crack tip of specimen under uniaxial tensile load are used for analysis. Accuracy and reliability is enhanced by multiplying two-time and sharpening the measured isochromatics by digital image processing. The results of photoelastic experiment are compared with those obtained by finite element method and empirical equations. Good agreement between them shows that photoelastic measurements are reliable

**Keywords:** Mixed Mode, Stress Intensity Factor, Photoelasticity, Crack, Polariscope, Isochromatic Fringe

## Introduction

If various structures under unusual circumstances are destroyed in significantly lower than the material strength, geometric discontinuities such as a hole and/or a sharp notch of crack can be the main cause [1, 2]. Especially, it could cause brittle fracture in high-strength materials. Formulation of the concept of fracture mechanics has made a vigorous study for stress intensity factor through theoretical analysis, numerical analysis, and various experimental techniques [3]. Also, stress fields near circular and elliptical hole are analyzed with hybrid method that uses series-type conformal mapping function and isochromatic fringedata by photoelastic experiment [4].

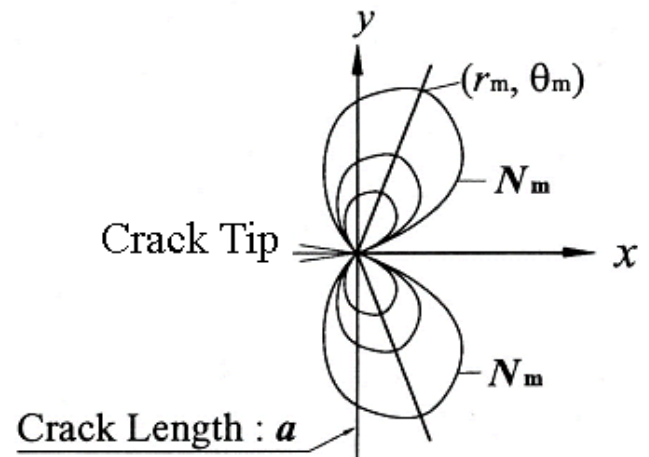
In this paper, the mixed mode stress intensity factors are measured by the isochromatics that are obtained through the photoelastic experiment and the image processing technique. The results are compared with those by empirical equations and finite element method (FEM).

## Photoelastic Experiment

The photoelastic fringes around a crack tip are represented in Figure 1. The stress intensity factors by photoelastic experiment are obtained with the following experimental equations by use of maximum radius ( $r_m$ ) from fringe loop, fringe order ( $N_m$ ), and angle ( $\theta_m$ ) of fringe loop in Figure 1. For the specimen with a inclined crack at the center under

uniaxial tensile load, the stress intensity factor,  $K_{Icenter} (=K_{Ic})$ , can be obtained with Equation (1) [5, 6].

$$K_{Ic} = \frac{N_m \sqrt{2\pi r_m} \left[ 1 + \frac{2 \tan(3\theta_m / 2)}{3 \tan \theta_m} \right]}{\alpha t \tan \theta_m \sqrt{1 + \left( \frac{2}{3 \tan \theta_m} \right)^2}} \quad (1)$$



**Figure 1:** Photoelastic fringes graphically drawn in front of a crack

For the specimen with inclined crack, the stress intensity factors and mixed mode stress intensity factors of  $K_I$  and  $K_{II}$  can be obtained with Equations (2) and (3).

$$K_I = \frac{N_m \sqrt{2\pi r_m}}{\alpha t \sqrt{(\sin \theta_m + 2A \cos \theta_m)^2 + A^2 \sin^2 \theta_m}} \quad (2)$$

$$K_{II} = \frac{AN_m \sqrt{2\pi r_m}}{\alpha t \sqrt{(\sin \theta_m + 2A \cos \theta_m)^2 + A^2 \sin^2 \theta_m}} \quad (3)$$

$$A = \frac{K_{II}}{K_I} = \frac{2}{3} \left( \cot \theta_m \pm \sqrt{\cot^2 \theta_m + \frac{3}{4}} \right) \quad (4)$$

where,

A = Ratio of stress intensity factors

$\alpha$  = Fringe constant

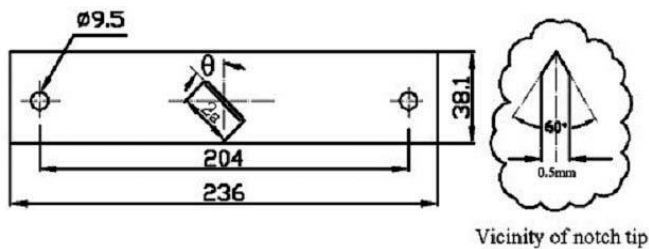
t = Specimen thickness

$N_m$  = Measured fringe order

The specimen is installed in the circular polariscope and the tensile load enough to observe the 3 ~ 5 fringes from the crack tip are applied to the specimen. Then, 2~3 high order fringes near the crack are selected and  $K_I$  and  $K_{II}$  are determined to the converged values.

### Machining the Specimen with Inclined Crack

The inclined crack is made at the center of specimen as in Figure 2 to measure the stress intensity factors by use of photoelastic experiment. The tensile load is applied to the specimen that is installed in loading device of the circular polariscope in order to get photoelastic fringes. Five different specimens are made for five inclined angles of crack ( $\beta$ ) that are 0°, 15°, 30°, 45°, and 60° [7].



**Figure 2:** Finite-width uniaxially loaded tensile plate containing an inclined central crack(unit : mm)



(a)



(b)

**Figure 3:** (a) TNV-40A machining center. (b) Cutting fluid was used to prevent from residual stress when machining a crack

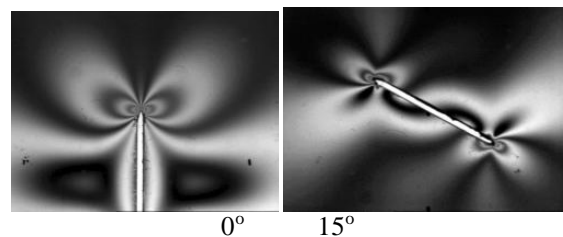
The material of specimen is PSM-1 that is excellent machinability, its material fringe constant ( $f_\sigma$ ) is 7, 500 N/m, its Young's elastic modulus ( $E$ ) is 2, 482 MPa, and Poisson's ratio( $\nu$ ) is 0.38 [8]. The cracks are machined with a machining center (TNV-40A) as in Figure 3(a). The diameter of end-mill tool in the machining center is 0.5mm. The crack tip is machined sharply as a natural crack by use of electrical discharge machining tool at 60°. Cutting fluid is used to prevent residual stress when machining a crack as in Figure 3(b).

### Photoelastic Experiment and Results

The specimen is installed in the circular polariscope with dark-field setup and the tensile load enough to observe the 3<sup>rd</sup> ~ 5<sup>th</sup> orders of fringe loop is applied to the specimen, depending on the crack angle. The tensile load is applied to the specimen with different crack angles as in Table 1 and the stress intensity factors are measured with the isochromatic fringes.

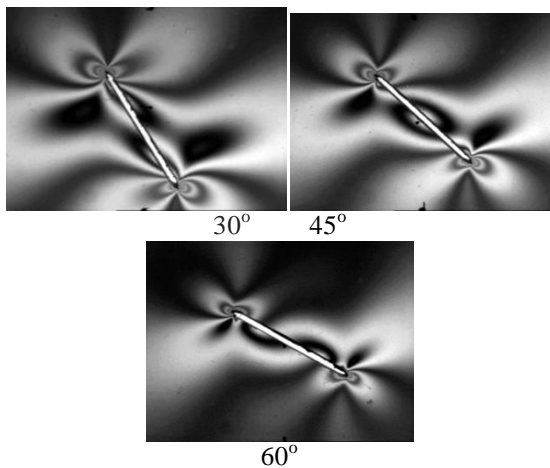
**Table 1:** Applied load to the specimen with different angles

| Crack angle | Applied load(N) | Stress (N/mm <sup>2</sup> ) |
|-------------|-----------------|-----------------------------|
| 0°          | 71.2            | 0.5886                      |
| 15°         | 89.0            | 0.7357                      |
| 30°         | 89.0            | 0.7357                      |
| 45°         | 89.0            | 0.7357                      |
| 60°         | 89.0            | 0.7357                      |



0°

15°

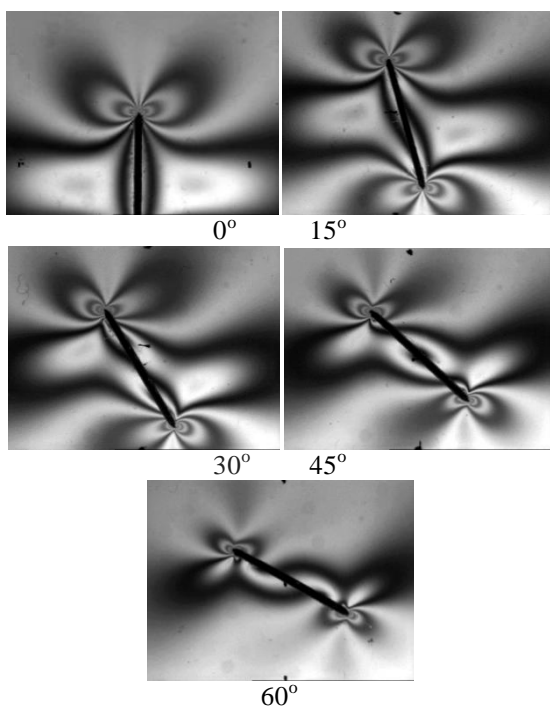


**Figure 4:** Fringes of specimen with inclined crack at center obtained from light-field setup of circular polariscope

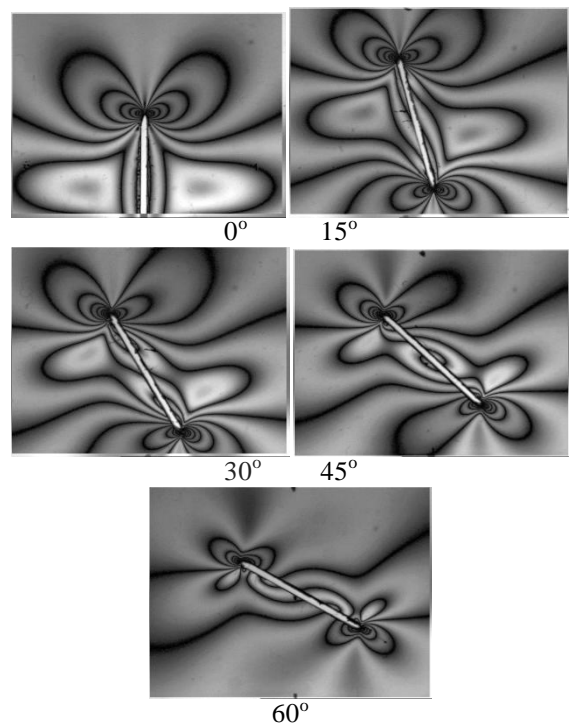
In other words, the stress intensity factors and mixed mode stress intensity factors of  $K_I$  and  $K_{II}$  are measured after the isochromatic fringes are two-times multiplied [9] and sharpened [10] by use of digital image processing techniques [11].

The fringes in Figure 4 are obtained by the polariscope in light field set-up and the fringes in Figure 5 are obtained by the polariscope in dark field set-up. The fringes of light field set-up or dark field set-up can be easily distinguished by the optical brightness of cracks at the center of specimen.

Image processing technique is applied to the fringes in Figures 4 and 5 for two-time multiplication [9]. Two-time multiplied fringes are shown in Figure 6. Using the fringe sharpening algorithm [10], the two-time multiplied fringes in Figure 6 are sharpened as shown in Figure 7.



**Figure 5:** Fringes of specimen with inclined crack at center obtained from dark-field setup of circular polariscope

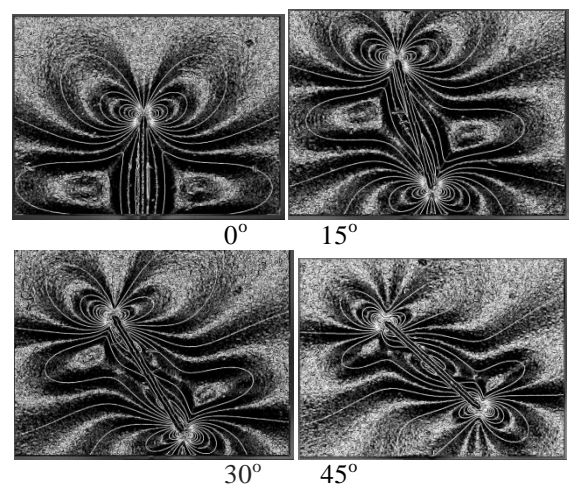


**Figure 6:** Two-times multiplied fringes by Figures 5 and 6

### Analysis of Results

Normalized non-dimensional stress intensity factor  $K_{In}$  of Mode I and the ratio of  $K_{II} / K_I$  are obtained by use of photoelastic experiment, empirical equation [1, 2], and FEM [12] as seen in Table 2. Variation of stress intensity factors with respect to inclined crack angle is shown in the graphs of Figures 8 and 9.

The stress intensity factors,  $K_{In}$  and  $K_{II} / K_I$ , obtained by the photoelastic experiment are close to those of empirical equation or FEM within experimental error. The errors in the results of the photoelastic experiment are 1.7% for  $K_I / \sigma \sqrt{\pi a}$  and 3.9% for  $K_{II} / K_I$ , based on the results of the finite element method.



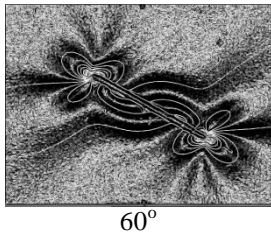


Figure 7: Sharpened fringes of Figure 6

Table 2: Comparison of stress intensity factors obtained from photoelasticity, empirical equation and FEM

| Crack angle | $K_{In}^*$ |          |                 | $K_{II} / K_I$ |          |                 |
|-------------|------------|----------|-----------------|----------------|----------|-----------------|
|             | FEM        | Equation | Photoelasticity | FEM            | Equation | Photoelasticity |
| 0°          | 1.074      | 1.065    | 1.086           | 0.000          | 0.000    | 0.000           |
| 15°         | 1.006      | 0.994    | 0.955           | 0.257          | 0.268    | 0.266           |
| 30°         | 0.814      | 0.799    | 0.848           | 0.554          | 0.577    | 0.569           |
| 45°         | 0.546      | 0.532    | 0.504           | 0.972          | 1.000    | 1.006           |
| 60°         | 0.277      | 0.267    | 0.261           | 1.670          | 1.732    | 1.746           |

$$K_{In}^* = K_I / \sigma \sqrt{\pi a}$$

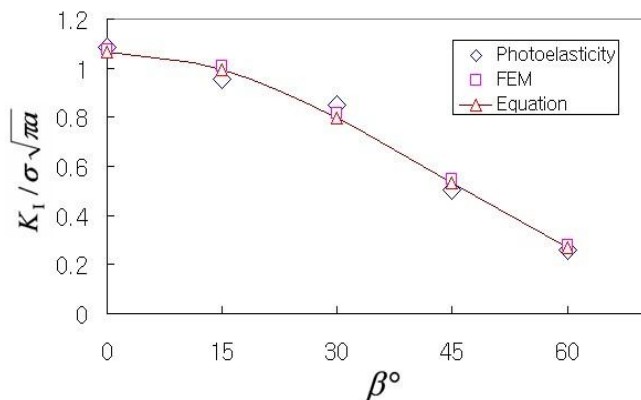


Figure 8: Variation of normalized stress intensity factor  $K_I / \sigma \sqrt{\pi a}$  with respect to inclined crack angle  $\beta$

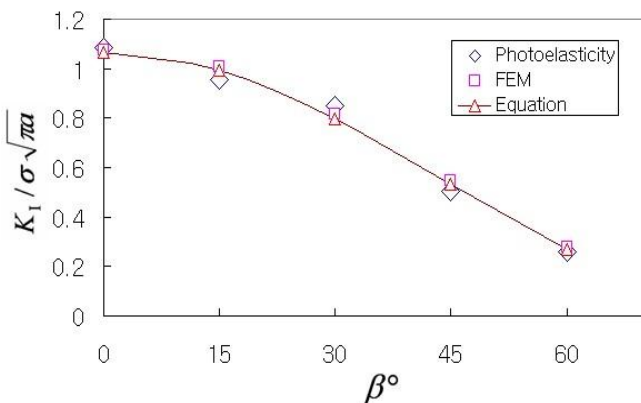


Figure 9: Variation of  $K_{II} / K_I$  with respect to inclined crack angle  $\beta$

## Conclusions

The stress intensity factors of specimen,  $K_I$  of Mode I and  $K_{II}$  of Mode II, are obtained with photoelastic experiment by use of the photoelastic fringes of specimens under tensile load that contain inclined crack of 0°, 15°, 30°, 45°, and 60° at the center. In this experiment, The crack in the specimen is machined artificially by the end-mill of which diameter is 0.5mm and the crack tip is machined sharply as a natural crack by use of electrical discharge machining tool at 60°.

For precise measurement, the two-time multiplied fringes are obtained by the image processing algorithm that uses the light-field isochromatic fringes and the dark-field isochromatic fringes from the circular polariscope. Then, the two-time multiplied fringes are image-processed into the sharpened fringes. The stress intensity factors,  $K_I$  and  $K_{II}$ , are obtained by measurement of fringe order ( $N_m$ ) near the crack tip, maximum radius ( $r_m$ ) from the origin of crack tip, and angle ( $\theta_m$ ) of fringe loop in the sharpened fringes. The conclusions are as follows:

- (1) The results of the photoelastic experiment are close to those of empirical equation or finite element method within 1.7% for  $K_I / \sigma \sqrt{\pi a}$  and 3.0% for  $K_I / K_{II}$ . These results suggest the possibility that the stress intensity factors of the real geometric shape and load condition can be measured with the photoelastic experiment.
- (2) From the data of photoelastic experiment, the ratio of maximum radius ( $r_m$ ) from fringe loop and crack length ( $a$ ) is  $r_m/a = 0.16$ , which is measured at a much wider range than 0.02 in Refs. [5] and [13].

There was a difficulty in machining of the specimen because the cracks in the specimen should be machined sharply as a natural crack. To overcome this difficulty, the cracks were machined in high speed with the end-mill of 0.5 mm even though many of the end-mills were broken because the cutting chips from the specimen clung to the end-mill. The scale factor used in conversion of the stress intensity factors must be closely reviewed because it is related to applied load, fringe constant of specimen, and deformation of specimen thickness from crack tip [14].

## Acknowledgments

This research was supported by 2014 Basic Science Research Program of the National Research Foundation of Korea (NRF) funded by the Ministry of Education (Grant number: 2014R1A1A2057220).

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