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Electromagnetic Wave Transmission Issue In Cylindrical Waveguide

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Abstract

In this article we consider the diffraction and transmission issue for an electromagnetic waves in a cylindrical waveguide on a perfectly conducting thin screen. A waveguide is limited with an perfectly conducting surface, the screen is located in a cross section of the waveguide. The issue of transmission: there is an electromagnetic field on one side of the screen, the field is searched on the other side of it. The problem of transmission is a kind of an inverse problem to the problem of diffraction.

First of all the solution of diffraction problem is presented, which is obtained by the method of an overdetermined boundary problem. It is shown how the problem of an electromagnetic wave on a thin screen can be reduced to the pair summatory functional equation concerning the coefficients of a required wave in a series according to eigenfunctions of a waveguide. The pair equation with the use an integral-summation identity is reduced to an infinite system of linear algebraic equations.

Then, on the basis of diffraction problem solution to the transmission problem is solved. The task of transmission may also be reduced to an infinite system of linear algebraic equations for the expansion coefficients of a desired wave in a series according to eigenfunctions. At an approximate solution of linear algebraic equations infinite system it is proposed to use the method of truncation.

Keywords: electromagnetic waves, transmission problem, cylindrical waveguide, inverse problem of diffraction.

1. INTRODUCTION

There are many publications concerning the problems of diffraction at the heterogeneities in the waveguides with metal walls. The classical electrodynamics problem is the problem of diffraction on a perfectly conducting thin screen [1]. The problem of diffraction on a thin conducting screen in a cylindrical waveguide is considered in the paper [2]. The problem of diffraction concerning heterogeneity has its own waves of a waveguide. It is required to find an electromagnetic field that occurs as the result of diffraction.

The problem of transmission is the inverse one for diffraction problem. Usually in inverse problems concerning a falling and a reflected measured field the waveguide parameters or heterogeneity parameters should be restored [3], [4]. The problem of transmission has the waveguide parameters, the heterogeneity is set, it is known that the field on one side of inhomogeneities must restore the field on its other side. The expression "transmission coefficients" is often in English literature. It is used to obtain the coefficients of a passed wave in diffraction problems. Also, the phrase "transmission condition" occurs.

The problem of transmission in a planar waveguide and an open space was considered in [5], [6].

In this paper we consider the problem of transmission in a cylindrical waveguide with a thin conducting screen inside.

2. PROBLEM DETERMINATION

Let's consider the waveguide structure (Figure 1) - a cylindrical area with a generatrix along the axis z, bounded by a perfectly conducting surface R. Let's S – a cross-section structure with the plane $z = 0$, C is the contour bounding this section. Let's assume that the contour is a closed plane curve without intersection points.

Let's an infinitely thin conductive plate (screen) is placed in the cross section S of the waveguide. Let's denote the screen part S via M, $N = S \setminus M$. We assume that the inhomogeneity splits the waveguide into two parts, filled by homogeneous isotropic media with different electromagnetic characteristics. The dielectric conductivity to the left and to the right of the screen makes ε^{-} и ε^{+} respectively. The magnetic permeability in both media is equal to μ.

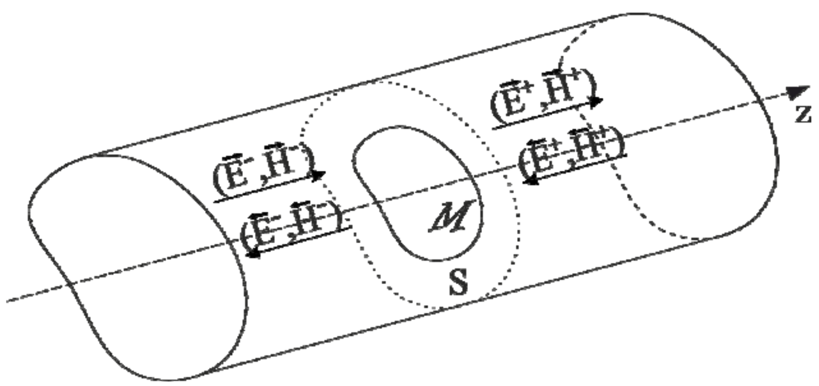

Fig. (1). Cylindrical waveguide

Generally, any electromagnetic wave in a waveguide may be represented as the sum of two waves with opposite orientation. We say that in the case of a medium without loss a wave is oriented positively (forward wave) when it transfers energy or attenuates in the direction of the waveguide axis. The wave of negative orientation (reverse wave) carries energy, or fades in the opposite direction. We will mark the waves of positive orientation by the symbol $\rightarrow$ and the waves of negative orientation by the symbol $\leftarrow$. The selection of the wave orientation is equal to the choice of a condition at infinity (radiation condition).

The problem of transmission involves four waves: one positively and one negatively oriented wave from one side and the two waves of different orientation from the other side of the screen.

An electromagnetic field harmonically time-dependent ($e^{i\omega t}$) is described by the following Maxwell equation system for the complex amplitudes inside a waveguide:

$$rotH = i\omega\varepsilon_0\varepsilon E, \quad rotE = -i\omega\mu_0\mu H. \tag{1}$$

Here ε_0, μ_0 is an electric and a magnetic constant. The tangential components of the electric field must be equal to zero on the side surface R, i.e.

$$[v, E]|_R = 0, \tag{2}$$

where v is the outward normal to the surface R.

Let's an electromagnetic field is from the right of the screen at $z > 0$: a positively oriented wave $(\overleftarrow{E}^+, \overleftarrow{H}^+)$ and a negatively oriented wave $(\overrightarrow{E}^+, \overrightarrow{H}^+)$. These functions can not be set arbitrarily, the following condition should be performed:

$$[n, \overleftarrow{E}^+] + [n, \overrightarrow{E}^+] = 0, \ (x, y) \in M,$$

where n is the normal to the section S.

The problem of transmission requires to find a positively oriented $(\overleftarrow{E}^-, \overleftarrow{H}^-)$ and negatively oriented wave $(\overrightarrow{E}^-, \overrightarrow{H}^-)$, satisfying the system of Maxwell's equations (1) at $z < 0$, boundary conditions (2), boundary conditions and conjugation conditions on the section S

$$[n, \overleftarrow{E}^-] + [n, \overrightarrow{E}^-] = 0, \quad (x, y) \in M, \tag{3}$$

$$[n, \overleftarrow{E}^-] + [n, \overrightarrow{E}^-] = [n, \overleftarrow{E}^+] + [n, \overrightarrow{E}^+], \ (x, y) \in N, \tag{4}$$

$$[n,\overleftarrow{H}^{-}]+[n,\overrightarrow{H}^{-}]=[n,\overleftarrow{H}^{+}]+[n,\overrightarrow{H}^{+}],\ \ (x,y)\in N. \qquad (5)$$

3. ELECTROMAGNETIC FIELD IN CYLINDRICAL WAVEGUIDE

In 1947-1948 the works of A.N. Tikhonov and A.A. Samarsky [7], [8] developed the solution of Maxwell equations (1) with the boundary condition (2) and the possibility of an electromagnetic field expansion in a waveguide is substantiated as the sum of TE fields and TM waves. We present the main result here.

The component $E_z = 0$ for TE-waves, and $H_z = 0$ for TM-waves. The problem of TE waves determination is reduced to the solution of the Neumann boundary issue on eigenvalues for Laplace operator in the area S :

$$\Delta\varphi+\lambda\varphi=0,\ (x,y)\in S,\ \ \frac{\partial\varphi}{\partial\nu}|_C=0. \qquad (6)$$

Let the pairs $\lambda_m,\varphi_m(x,y),m=0,1,\ldots,$ – are the solutions of the issue (6). We assume $\varphi_m(x,y)$ functions as orthonormal ones. TE-wave fields have the following formulae:

$$E_m^{\varphi}=-i\omega\mu_0\mu rot\Pi_m^{\varphi},\ \ H_m^{\varphi}=grad\,div\Pi_m^{\varphi}+k^2\Pi_m^{\varphi},$$

where $\Pi_m^{\varphi}=(0,0,e^{i\gamma_m z}\varphi_m(x,y))$, k is a wave number, $\gamma_m=\sqrt{k^2-\lambda_m}$.

The problem of TM waves determination is reduced to the solution of Dirichlet problem on eigenvalues for Laplace operator in the S area:

$$\Delta\psi+\chi\psi=0,\ (x,y)\in S,\ \ \psi|_C=0. \qquad (7)$$

Let the pairs $\chi_m,\psi_m(x,y),m=0,1,\ldots,$ are the solutions of the task (7). We assume the functions $\psi_m(x,y)$ as orthonormal ones. TM wave fields are presented as follows

$$E_m^{\psi}=grad\,div\Pi_m^{\psi}+k^2\Pi_m^{\psi},\ \ H_m^{\psi}=i\omega\varepsilon_0\varepsilon rot\Pi_m^{\psi},$$

where $\Pi_m^{\psi}=(0,0,e^{i\delta_m z}\psi_m(x,y))$, $\delta_m=\sqrt{k^2-\chi_m}$.

Let's express the components of E,H fields via potential functions $\varphi_m(x,y),\psi_n(x,y)$:

$$E_m^{\varphi} = i\omega\mu_0\mu\left(-\frac{\partial\varphi_m}{\partial y}, \frac{\partial\varphi_m}{\partial x}, 0\right)e^{i\gamma_m z},$$
$$H_m^{\varphi} = \left(i\gamma_m\frac{\partial\varphi_m}{\partial x}, i\gamma_m\frac{\partial\varphi_m}{\partial y}, (k^2-\gamma_m^2)\varphi_m\right)e^{i\gamma_m z}, \tag{8}$$

$$E_m^{\psi} = \left(i\delta_m\frac{\partial\psi_m}{\partial x}, i\delta_m\frac{\partial\psi_m}{\partial y}, (k^2-\delta_n^2)\psi_m\right)e^{i\delta_m z},$$
$$H_m^{\psi} = i\omega\varepsilon_0\varepsilon\left(\frac{\partial\psi_m}{\partial y}, -\frac{\partial\psi_m}{\partial x}, 0\right)e^{i\delta_m z}. \tag{9}$$

We agree that $Re\gamma_m > 0$ or $Im\gamma_m < 0$. The real values γ_m are represented by the waves, that carry the energy along a waveguide, the imaginary values γ_m are represented by the damped waves. Then, with the selected dependence on time the components with the factor $e^{i\gamma_m z}$ consistent with the negative orientation waves, and with the factor $e^{-i\gamma_m z}$ these components consistent with the positive orientation waves.

Similarly let's agree to select the values δ_m: $Re\delta_m > 0$ or $Im\delta_m < 0$. Then, the formulae (8) and (9) define the negative orientation waves and it is necessary to put a sign for positive orientation waves in the formulas (8) and (9) before the permanent values γ_m and δ_m.

The book [9] proves that any solution of Maxwell equations (1), satisfying the boundary conditions (2), may be represented as a superposition of TE and TM waves with positive and negative orientation. Thus, the solution of the problem (1), (2) may be represented as the following series according to its own TE and TM waves of the type (8) and (9).

$$\begin{pmatrix} E \\ H \end{pmatrix} = \sum_{m=0}^{+\infty} \overleftarrow{A}_m \begin{pmatrix} \overleftarrow{E}_m^{\varphi} \\ \overleftarrow{H}_m^{\varphi} \end{pmatrix} + \sum_{m=0}^{+\infty} \overleftarrow{B}_m \begin{pmatrix} \overleftarrow{E}_m^{\psi} \\ \overleftarrow{H}_m^{\psi} \end{pmatrix} + \sum_{m=0}^{+\infty} \overrightarrow{A}_m \begin{pmatrix} \overrightarrow{E}_m^{\varphi} \\ \overrightarrow{H}_m^{\varphi} \end{pmatrix} + \sum_{m=0}^{+\infty} \overrightarrow{B}_m \begin{pmatrix} \overrightarrow{E}_m^{\psi} \\ \overrightarrow{H}_m^{\psi} \end{pmatrix}. \tag{10}$$

4. ELECTROMAGNETIC WAVE DIFFRACTION IN A CYLINDRICAL WAVEGUIDE ON A CONDUCTIVE SCREEN

Let's consider the solution of diffraction issue. According to this solution the problem of transmission will be solved.

In [2] the solution is obtained by the method of overdetermined boundary diffraction problem in a cylindrical waveguide on a conducting screen. Here are the necessary results of this work.

Let the screen on the left is covered with a positively oriented electromagnetic wave $(\overleftarrow{E}^-, \overleftarrow{H}^-)$. At its diffraction the waves reflected to the left $(\overrightarrow{E}^-, \overrightarrow{H}^-)$ and passed to the right $(\overleftarrow{E}^+, \overleftarrow{H}^+)$ appear.

The problem of diffraction in a cylindrical waveguide is formulated as follows: it is required to find a positively oriented component at $z > 0$ and a negatively oriented component at $z < 0$ the solution of Maxwell equations (1), satisfying the boundary conditions (2), the boundary conditions and conjugation conditions on the section S

$$[n, \overleftarrow{E}^- + \overrightarrow{E}^-] = 0, \ \ [n, \overleftarrow{E}^+] = 0, \ \ (x,y) \in M, \qquad (11)$$

$$[n, \overleftarrow{E}^- + \overrightarrow{E}^-] = [n, \overleftarrow{E}^+], \ \ [n, \overleftarrow{H}^- + \overrightarrow{H}^-] = [n, \overleftarrow{H}^+], \ \ (x,y) \in N. \qquad (12)$$

It is convenient to divide the diffraction problem into two sub-tasks: first the problem of an electromagnetic wave reflection and refraction at the boundary of media separation, and then solve the problem of diffraction.

The problem of an electromagnetic wave reflection and refraction on the junction of two dielectrics in a waveguide is as follows. The wave $(\overleftarrow{E}^-, \overleftarrow{H}^-)$ on a joint is set. We have to obtain $(\underline{\overrightarrow{E}}^-, \underline{\overrightarrow{H}}^-)$ and $(\underline{\overleftarrow{E}}^+, \underline{\overleftarrow{H}}^+)$ functions of a scattered field, satisfying the conjugation conditions at S

$$[n, \overleftarrow{E}^- + \underline{\overrightarrow{E}}^-] = [n, \underline{\overleftarrow{E}}^+], \ \ [n, \overleftarrow{H}^- + \underline{\overrightarrow{H}}^-] = [n, \underline{\overleftarrow{H}}^+], \ \ (x,y) \in S. \qquad (13)$$

Let $\overleftarrow{A}_m^-, \overleftarrow{B}_m^-$ – are the expansion coefficients of a falling wave in a series of its own waves, $\underline{A}_m^+, \underline{B}_m^+$ are the expansion coefficients for $(\underline{\overleftarrow{E}}^+, \underline{\overleftarrow{H}}^+)$ wave in a series of its own waves, $\underline{A}_m^-, \underline{B}_m^-$ -- the expansion coefficients for $(\underline{\overrightarrow{E}}^-, \underline{\overrightarrow{H}}^-)$ wave.

Lemma 3.11 The solution of the problem concerning an electromagnetic wave reflection and refraction in a cylindrical waveguide is the following

$$\underline{A}_n^+ = \frac{2\gamma_n^-}{\gamma_n^- + \gamma_n^+}\overleftarrow{A}_n^-,\ \underline{B}_n^+ = \frac{2\varepsilon^-\delta_n^-}{\varepsilon^+\delta_n^- + \varepsilon^-\delta_n^+}\overleftarrow{B}_n^-,$$
$$\underline{A}_n^- = \frac{\gamma_n^- - \gamma_n^+}{\gamma_n^- + \gamma_n^+}\overleftarrow{A}_n^-,\ \underline{B}_n^- = \frac{\varepsilon^+\delta_n^- - \varepsilon^-\delta_n^+}{\varepsilon^+\delta_n^- + \varepsilon^-\delta_n^+}\overleftarrow{B}_n^-, \qquad n = 0,1,\ldots$$

The lemma statement is derived directly from the terms of (13), using the representation (10). The term of eigenfunctions $\varphi(x,y)$ and $\psi(x,y)$ is used during the proof.

Now suppose that the interface of media division in the plane S has an infinitely thin perfectly conducting screen M. Let's seek the diffraction problem solution as the sum of the waves resulting from the reflection and refraction at the interface of media and waves that appeared as screen fluctuations: $(\overleftarrow{E}^+,\overleftarrow{H}^+) = (\underline{\overleftarrow{E}}^+,\underline{\overleftarrow{H}}^+) + (\overleftarrow{E},\overleftarrow{H})$, $(\overrightarrow{E}^-,\overrightarrow{H}^-) = (\underline{\overrightarrow{E}}^-,\underline{\overrightarrow{H}}^-) + (\overrightarrow{E},\overrightarrow{H})$, where $(\underline{\overleftarrow{E}}^+,\underline{\overleftarrow{H}}^+)$, $(\underline{\overrightarrow{E}}^-,\underline{\overrightarrow{H}}^-)$ – the reflection and refraction problem solution; $(\overleftarrow{E},\overleftarrow{H})$, $(\overrightarrow{E},\overrightarrow{H})$ – field fluctuations from a screen.

Lemma 3.22 The problem of an electromagnetic wave diffraction on a metal screen in a cylindrical waveguide is reduced to two independent pair summation functional equations (PSFE)

$$\begin{gathered}\sum_{m=0}^{+\infty}\overleftarrow{A}_m\lambda_m\varphi_m(x,y) = -\sum_{m=0}^{+\infty}\underline{A}_m^+\lambda_m\varphi_m(x,y),\ (x,y)\in M,\\ \sum_{m=0}^{+\infty}\overleftarrow{A}_m(\gamma_m^+ + \gamma_m^-)\lambda_m\varphi_m(x,y) = 0, \qquad (x,y)\in N\end{gathered} \tag{14}$$

and

$$\begin{gathered}\sum_{m=0}^{+\infty}\overleftarrow{B}_m\delta_m^+\chi_m\psi_m(x,y) = -\sum_{m=0}^{+\infty}\underline{B}_m^+\delta_m^+\chi_m\psi_m(x,y),\ (x,y)\in M,\\ \sum_{m=0}^{+\infty}\overleftarrow{B}_m\left(\varepsilon^-\frac{\delta_m^+}{\delta_m^-} + \varepsilon^+\right)\chi_m\psi_m(x,y) = 0, \qquad (x,y)\in N.\end{gathered} \tag{15}$$

Proof. Let's $e = [n,E], h = [n,H]$. According to the boundary conditions and the conditions of the conjugation (11) and (12) on the section S of the waveguide with the conditions (13) it follows that $\vec{e}(x,y)=\overleftarrow{e}(x,y)$ everywhere on S and $\vec{h}(x,y)=\overleftarrow{h}(x,y)$ at $(x,y)\in N$.

Let's put down these conditions, using the representation of the component functions e, h as a series.

$$\sum_{m=0}^{+\infty}\left(-\vec{A}_m i\omega\mu_0\mu\frac{\partial\varphi_m}{\partial x}(x,y)-\vec{B}_m i\delta_m^-\frac{\partial\psi_m}{\partial y}(x,y)\right)$$
$$=\sum_{m=0}^{+\infty}\left(-\overleftarrow{A}_m i\omega\mu_0\mu\frac{\partial\varphi_m}{\partial x}(x,y)+\overleftarrow{B}_m i\delta_m^+\frac{\partial\psi_m}{\partial y}(x,y)\right),(x,y)\in S,$$
$$\sum_{m=0}^{+\infty}\left(-\vec{A}_m i\omega\mu_0\mu\frac{\partial\varphi_m}{\partial y}(x,y)+\vec{B}_m i\delta_m^-\frac{\partial\psi_m}{\partial x}(x,y)\right)$$
$$=\sum_{m=0}^{+\infty}\left(-\overleftarrow{A}_m i\omega\mu_0\mu\frac{\partial\varphi_m}{\partial y}(x,y)-\overleftarrow{B}_m i\delta_m^+\frac{\partial\psi_m}{\partial x}(x,y)\right),(x,y)\in S, \tag{16}$$

$$\sum_{m=0}^{+\infty}\left(-\vec{A}_m i\gamma_m^-\frac{\partial\varphi_m}{\partial y}(x,y)+\vec{B}_m i\omega\varepsilon_0\varepsilon^-\frac{\partial\psi_m}{\partial x}(x,y)\right)$$
$$=\sum_{m=0}^{+\infty}\left(\overleftarrow{A}_m i\gamma_m^+\frac{\partial\varphi_m}{\partial y}(x,y)+\overleftarrow{B}_m i\omega\varepsilon_0\varepsilon^+\frac{\partial\psi_m}{\partial x}(x,y)\right),(x,y)\in N,$$
$$\sum_{m=0}^{+\infty}\left(\vec{A}_m i\gamma_m^-\frac{\partial\varphi_m}{\partial x}(x,y)+\vec{B}_m i\omega\varepsilon_0\varepsilon^-\frac{\partial\psi_m}{\partial y}(x,y)\right)$$
$$=\sum_{m=0}^{+\infty}\left(-\overleftarrow{A}_m i\gamma_m^+\frac{\partial\varphi_m}{\partial x}(x,y)+\overleftarrow{B}_m i\omega\varepsilon_0\varepsilon^+\frac{\partial\psi_m}{\partial y}(x,y)\right),(x,y)\in N. \tag{17}$$

According to the equations (16) it follows that $\vec{A}_m=\overleftarrow{A}_m$, $m=0,1,\ldots$. It is enough to differentiate the first equation in (16) by x, the second equation in (16) by y and combine them. Let's differentiate the first equation in (16) by y, the second equation in (16) by x and substract one from the other. We shall obtain the following formula $\vec{B}_m\delta_m^-=-\overleftarrow{B}_m\delta_m^+$, $m=0,1,\ldots$

Similarly, according to (17) it turns out that the following conditions must be satisfied on N

$$\sum_{m=0}^{+\infty} \overrightarrow{A}_m (\gamma_m^- + \gamma_m^+) \lambda_m \varphi_m (x, y) = 0$$

or

$$\sum_{m=0}^{+\infty} \overleftarrow{A}_m (\gamma_m^- + \gamma_m^+) \lambda_m \varphi_m (x, y) = 0;$$

and

$$\sum_{m=0}^{+\infty} \overrightarrow{B}_m \left(\varepsilon^- + \varepsilon^+ \frac{\delta_m^-}{\delta_m^+} \right) \chi_m \psi_m (x, y) = 0$$

or

$$\sum_{m=0}^{+\infty} \overleftarrow{B}_m \left(\varepsilon^- \frac{\delta_m^+}{\delta_m^-} + \varepsilon^+ \right) \chi_m \psi_m (x, y) = 0.$$

The first PSFE equations are derived from the boundary conditions on M according to the same pattern as the second equations.

Similarly, you can get two independent PSFE for the wave expansion coefficients $(\overrightarrow{E}, \overrightarrow{H})$, to which the problems of diffraction are reduced.

Lemma 3.33 The diffraction problem of an electromagnetic wave on a metal screen in a cylindrical waveguide is reduced to two independent PSFE

$$\begin{aligned} \sum_{m=0}^{+\infty} \overrightarrow{A}_m \lambda_m \varphi_m (x, y) &= -\sum_{m=0}^{+\infty} (A_m^0 + \underline{A}_m^-) \lambda_m \varphi_m (x, y), \ (x, y) \in M, \\ \sum_{m=0}^{+\infty} \overrightarrow{A}_m (\gamma_m^+ + \gamma_m^-) \lambda_m \varphi_m (x, y) &= 0, \qquad (x, y) \in N \end{aligned} \tag{18}$$

and

$$\sum_{m=0}^{+\infty} \overrightarrow{B}_m \delta_m^- \chi_m \psi_m(x,y) = \sum_{m=0}^{+\infty} (B_m^0 - \underline{B}_m^-) \delta_m^- \chi_m \psi_m(x,y), \ (x,y) \in M,$$
$$\sum_{m=0}^{+\infty} \overrightarrow{B}_m \left(\varepsilon^- + \varepsilon^+ \frac{\delta_m^-}{\delta_m^+} \right) \chi_m \psi_m(x,y) = 0, \qquad (x,y) \in N. \tag{19}$$

From PSFE (14), (15) by the method of integral summation identities one may obtain two independent infinite systems of linear algebraic equations (ISLAE), to which the problem of diffraction is reduced.

Lemma 3.44 The problem of an electromagnetic wave diffraction on a metal screen in a cylindrical waveguide is reduced to two independent ISLAE

$$-\overleftarrow{A}_k \lambda_k + \sum_{n=0}^{+\infty} \overleftarrow{A}_n (\gamma_n^+ + \gamma_n^-) \lambda_n \sum_{m=0}^{+\infty} \frac{1}{\gamma_m^+ + \gamma_m^-} I_{n,m}^{\varphi} J_{m,k}^{\varphi}$$
$$= \sum_{n=0}^{+\infty} \overleftarrow{A}_n^{-} \frac{2\gamma_n^-}{\gamma_n^+ + \gamma_n^-} \lambda_n I_{n,k}^{\varphi}, \ k = 0,1,...,$$
$$-\overleftarrow{B}_k \chi_k \left(\varepsilon^- \frac{\delta_k^+}{\delta_k^-} + \varepsilon^+ \right) + \sum_{n=0}^{+\infty} \overleftarrow{B}_n \delta_n^+ \chi_n \sum_{m=0}^{+\infty} \frac{1}{\delta_m^+} \left(\varepsilon^- \frac{\delta_m^+}{\delta_m^-} + \varepsilon^+ \right) J_{n,m}^{\psi} I_{m,k}^{\psi}$$
$$= \sum_{n=0}^{+\infty} \overleftarrow{B}_n^{-} \chi_n \frac{2\varepsilon^- \delta_n^- \delta_n^+}{\varepsilon^+ \delta_n^- + \varepsilon^- \delta_n^+} \sum_{m=0}^{+\infty} \frac{1}{\delta_m^+} \left(\varepsilon^- \frac{\delta_m^+}{\delta_m^-} + \varepsilon^+ \right) I_{n,m}^{\psi} I_{m,k}^{\psi}, \ k = 0,1,...,$$

where

$$I_{n,m}^{\varphi} = \iint_M \varphi_n(\xi,\eta) \varphi_m(\xi,\eta) d\xi d\eta, \quad J_{n,m}^{\varphi} = \iint_N \varphi_n(\xi,\eta) \varphi_m(\xi,\eta) d\xi d\eta,$$
$$I_{n,m}^{\psi} = \iint_M \psi_n(\xi,\eta) \psi_m(\xi,\eta) d\xi d\eta, \quad J_{n,m}^{\psi} = \iint_N \psi_n(\xi,\eta) \psi_m(\xi,\eta) d\xi d\eta.$$

Similarly, one may obtain the following ISLAE from PSFE (18) and (19) in respect of the following ratios $\overrightarrow{A}_m$, $\overrightarrow{B}_m$:

$$-\vec{A}_k\lambda_k+\sum_{n=0}^{+\infty}\vec{A}_n(\gamma_n^+ +\gamma_n^-)\lambda_n\sum_{m=0}^{+\infty}\frac{1}{\gamma_m^+ +\gamma_m^-}I^{\varphi}_{n,m}J^{\varphi}_{m,k}$$
$$=\sum_{n=0}^{+\infty}\overleftarrow{A}^{-}_n\frac{2\gamma_n^-}{\gamma_n^+ +\gamma_n^-}\lambda_n I^{\varphi}_{n,k},\ k=0,1,..., \tag{20}$$

$$\vec{B}_k\chi_k\left(\varepsilon^- +\varepsilon^+\frac{\delta_k^-}{\delta_k^+}\right)-\sum_{n=0}^{+\infty}\vec{B}_n\delta_n^-\chi_n\sum_{m=0}^{+\infty}\frac{1}{\delta_m^-}\left(\varepsilon^- +\varepsilon^+\frac{\delta_m^-}{\delta_m^+}\right)J^{\psi}_{n,m}I^{\psi}_{m,k}$$
$$=\sum_{n=0}^{+\infty}\overleftarrow{B}^{-}_n\chi_n\frac{2\varepsilon^-\delta_n^-\delta_n^+}{\varepsilon^+\delta_n^- +\varepsilon^-\delta_n^+}\sum_{m=0}^{+\infty}\frac{1}{\delta_m^-}\left(\varepsilon^- +\varepsilon^+\frac{\delta_m^-}{\delta_m^+}\right)I^{\psi}_{n,m}I^{\psi}_{m,k},\ k=0,1,..., \tag{21}$$

5. ELECTROMAGNETIC WAVE TRANSMISSION ISSUE IN A CYLINDRICAL WAVEGUIDE

It is convenient to divide the transmission task into two subtasks. First you need to solve the diffraction problem - by the wave falling on the screen $(\vec{E}^{+},\vec{H}^{+})$ one may find the previous wave $(\vec{E}',\vec{H}')$ and the reflected wave $(\overleftarrow{E}',\overleftarrow{H}')$. Then, by the difference

$$(\overleftarrow{E},\overleftarrow{H})=(\overleftarrow{E}^{+},\overleftarrow{H}^{+})-(\overleftarrow{E}',\overleftarrow{H}')$$

you may recover a positively oriented wave $(\overleftarrow{E}^{-},\overleftarrow{H}^{-})$ and the second component $(\vec{E},\vec{H})$ of a negatively oriented wave in the left half of the waveguide. Thus, we obtain

$$(\vec{E}^{-},\vec{H}^{-})=(\vec{E}',\vec{H}')+(\vec{E},\vec{H}).$$

Thus, in addressing the simplified problem of transmission by the functions $(\overleftarrow{E},\overleftarrow{H})$ it is necessary to find $(\overleftarrow{E}^{-},\overleftarrow{H}^{-})$ and $(\vec{E},\vec{H})$.

In the problem of transmission according to a given set of coefficients $\overleftarrow{A}_n$, $\overleftarrow{B}_n$ one has to find the following coefficients $\overleftarrow{A}^{-}_n$, $\overleftarrow{B}^{-}_n$ and $\vec{A}_n$, $\vec{B}_n$. One may use ISLAE (20), (21) obtained during the solution of a diffraction problem to find these coefficients. Let's note that

$$\frac{2\gamma_n^-}{\gamma_n^+ + \gamma_n^-}\overleftarrow{\overrightarrow{A}}_n = \overleftarrow{A}_n - \overrightarrow{A}_n,$$
$$\frac{2\varepsilon^-\delta_n^-\delta_n^+}{\varepsilon^+\delta_n^- + \varepsilon^-\delta_n^+}\overleftarrow{\overrightarrow{B}}_n = \delta_n^+\overleftarrow{B}_n + \delta_n^-\overrightarrow{B}_n.$$

Then ISLAE (20), (21) may be rewritten as follows:

$$-\overrightarrow{A}_k\lambda_k + \sum_{n=0}^{+\infty}\overleftarrow{\overrightarrow{A}}_n(\gamma_n^+ + \gamma_n^-)\lambda_n\sum_{m=0}^{+\infty}\frac{1}{\gamma_m^+ + \gamma_m^-}I_{n,m}^{\varphi}J_{m,k}^{\varphi} + \sum_{n=0}^{+\infty}\overleftarrow{\overrightarrow{A}}_n\lambda_n I_{n,k}^{\varphi}$$
$$= \sum_{n=0}^{+\infty}\overleftarrow{A}_n\lambda_n I_{n,k}^{\varphi},\ k = 0,1,..., \tag{22}$$

$$\overrightarrow{B}_k\chi_k\left(\varepsilon^- + \varepsilon^+\frac{\delta_k^-}{\delta_k^+}\right) - \sum_{n=0}^{+\infty}\overleftarrow{\overrightarrow{B}}_n\delta_n^-\chi_n\sum_{m=0}^{+\infty}\frac{1}{\delta_m^-}\left(\varepsilon^- + \varepsilon^+\frac{\delta_m^-}{\delta_m^+}\right)J_{n,m}^{\psi}I_{m,k}^{\psi}$$
$$-\sum_{n=0}^{+\infty}\overleftarrow{\overrightarrow{B}}_n\delta_n^-\chi_n\sum_{m=0}^{+\infty}\frac{1}{\delta_m^-}\left(\varepsilon^- + \varepsilon^+\frac{\delta_m^-}{\delta_m^+}\right)I_{n,m}^{\psi}I_{m,k}^{\psi} \tag{23}$$
$$= \sum_{n=0}^{+\infty}\overleftarrow{B}_n\chi_n\delta_n^+\sum_{m=0}^{+\infty}\frac{1}{\delta_m^-}\left(\varepsilon^- + \varepsilon^+\frac{\delta_m^-}{\delta_m^+}\right)I_{n,m}^{\psi}I_{m,k}^{\psi},\ k = 0,1,...$$

We simplify (23), and obtain the following

$$\overrightarrow{B}_k\chi_k\left(\varepsilon^- + \varepsilon^+\frac{\delta_k^-}{\delta_k^+}\right) - \sum_{n=0}^{+\infty}\overleftarrow{\overrightarrow{B}}_n\chi_n\left(\varepsilon^- + \varepsilon^+\frac{\delta_n^-}{\delta_n^+}\right)I_{n,k}^{\psi}$$
$$= \sum_{n=0}^{+\infty}\overleftarrow{B}_n\chi_n\delta_n^+\sum_{m=0}^{+\infty}\frac{1}{\delta_m^-}\left(\varepsilon^- + \varepsilon^+\frac{\delta_m^-}{\delta_m^+}\right)I_{n,m}^{\psi}I_{m,k}^{\psi},\ k = 0,1,... \tag{24}$$

Theorem 5.1 The transmission target of electromagnetic waves on the screen in a cylindrical waveguide is reduced to two independent ISLAE (22), (24).

It should be noted that the problem of transmission may be solved in many ways.

The method of truncation is used at approximate solution of ISLAE (22), (24). If we move from ISLAE to finite SLAE, leaving only the N of unknown coefficients, and the rest are presumed to be equal to zero, so we select only the approximate solution. N here must match the number of specified ratios $\overleftarrow{A}_n$, $\overleftarrow{B}_n$, $n = 0,..N$.

The proposed numerical method for solving the problem of transmission is well parallelized. Most of the time during the solution of obtained SLAE the matrix elements are taken into account, in particular the elements $I^{\varphi}_{n,m}, J^{\varphi}_{n,m}, I^{\psi}_{n,m}, J^{\psi}_{n,m}$. These elements are independent of each other (from the aspect of data) and can be computed in a parallel mode.

Here are the simple reasoning, clarifying the connection between the problem of diffraction and the problem of transmission. If the diffraction problem (direct problem) is reduced to two operator equations of the following form

$$C_{\varphi}\overrightarrow{A} = D_{\varphi}\overleftarrow{A}^{-}, \quad C_{\psi}\overrightarrow{B} = D_{\psi}\overleftarrow{B}^{-},$$

then during the solution of the transmission problem (inverse problem) one needs to look for the vector $\overrightarrow{A}$ to obtain $D_{\varphi}\overleftarrow{A}^{-} + D'_{\varphi}\overrightarrow{A} = D'_{\varphi}\overleftarrow{A}$, and the vector $\overrightarrow{B}$ to obtain $D_{\psi}\overleftarrow{B}^{-} - D'_{\psi}\overrightarrow{B} = D''_{\psi}\overleftarrow{B}$. Then the new equations for $\overrightarrow{A}$ and $\overrightarrow{B}$ have the following form

$$(C_{\varphi} + D'_{\varphi})\overrightarrow{A} = D'_{\varphi}\overleftarrow{A}, \quad (C_{\psi} - D'_{\psi})\overrightarrow{B} = D''_{\psi}\overleftarrow{B}.$$

Consequently, the task of transmission may be solved numerically with about the same accuracy as the diffraction problem, if during the perturbation of an operator C_{φ} by the operator D'_{φ}, and the operator C_{ψ} by the operator D'_{ψ} their properties are preserved.

An approximate solution of the transmission problem may also be obtained by the method that is often used during the solution of inverse problems. Let's choose a set of linearly independent vectors $\overleftarrow{A}^{-,j}, \overleftarrow{B}^{-,j}$, $j = 1..N$, and find the corresponding solutions $\overleftarrow{A}^{j}, \overleftarrow{B}^{j}$ of the diffraction problem. Если векторы $\overleftarrow{A}^{j}$ тоже линейно независимы, то любой заданный вектор $\overleftarrow{A}$ можно разложить по этому базису. Аналогично, если векторы $\overleftarrow{B}^{j}$ линейно независимы, то любой заданный вектор $\overleftarrow{B}$ можно разложить по этому базису. Если $\overleftarrow{A} = \alpha_1\overleftarrow{A}^{1} + \ldots + \alpha_N\overleftarrow{A}^{N}$ и $\overleftarrow{B} = \beta_1\overleftarrow{B}^{1} + \ldots + \beta_N\overleftarrow{B}^{N}$, то решение задачи трансмиссии $\overleftarrow{A}^{-} = \alpha_1\overleftarrow{A}^{-,1} + \ldots + \alpha_N\overleftarrow{A}^{-,N}$ и $\overleftarrow{B}^{-} = \beta_1\overleftarrow{B}^{-,1} + \ldots + \beta_N\overleftarrow{B}^{-,N}$.

6. CONCLUSIONS

The proposed method of transmission problem solution is applicable when you may develop your own waves of a waveguide, i.e., when it is possible to solve the boundary Neumann problem and Dirichlet problem on eigenvalues for the Laplace operator in the field of a waveguide cross-section.

7. SUMMARY

The method of the transmission problem solution electromagnetic waves in a cylindrical waveguide. The problem of transmission is reduced to ISLAE concerning the expansion coefficients of the desired wave in a series by eigenfunctions. A numerical method to solve the resulting ISLAE is proposed.

CONFLICT OF INTERESTS

The author confirms that the presented data do not contain any conflict of interests.

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