

Equalization Suitability Estimation for 2x2 MIMO using QAM

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Abstract

Extensive advances have made innovatory changes in digital signal processing, computing and transmission. This change has accelerated wireless technology to jump from just a convenience to inevitability, earthling Generation changes from second to third, fourth and the count began. These technological advancements have also inlaid super highways for data transmission from single antenna to multi antenna called MIMO (Multiple Input Multiple Output) technology. As technology advances, user friendly applications are gushing to make an effort of inevitable need for every individual. And every need is associated with lots of data. Therefore, data transmission is the focusing prime factor for improving Interference Mitigation. The large-scale fading varies faster than path loss, attenuating 6-10dB. The mutual interference of concurrent transmissions between nodes also constrains the wireless network capacity. Therefore, a focused effort is needed to improve the performance of MIMO networks by improving path loss, ICI, ACI, and AFD. As a consequence there is an overall improvement in spectral efficiency- of large scale fading. The proposed methodology improves Spectral efficiency, Level Crossing Rate, and Average Fade Duration.

Keywords: AWGN, ISI, MIMO, Multipath, OFDM.

1. Introduction

Equalization is a process to eliminate intersymbol interference (ISI), phase and amplitude distortion. Equalizers require longer length filters for MIMO multipath correction of multimedia applications. It eliminates the inter symbol interference. Equalizers have become an important part in diversified multimedia and DSP applications. Superpositions of properly shaped transmit pulses (sinc function) results in no ISI[1, 2, 4, 5].

2. Algorithm

The algorithm for achieving the desired result is as in the flow diagram shown in figure 1:

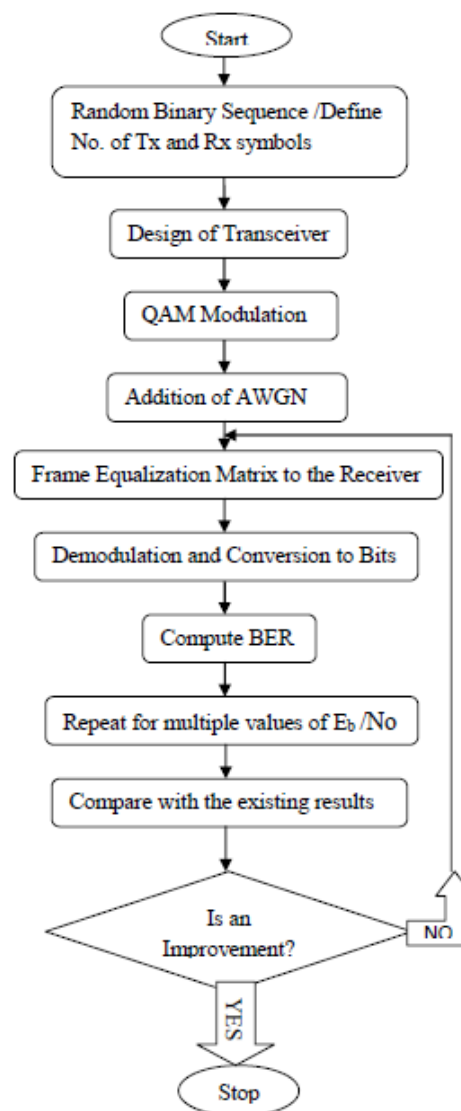


Figure1. Flow chart for the desired result

2. Pulse shaping

The transmitted rectangular pulse is shaped by cosine filter for no alias. Next at the equalizer input due to multipath reception and spectral shaping, the impulse is distorted. And also, the received signal is added with white noise and time domain distortion of multipath propagation. This distortion can be removed by an exact multiplicative inverse filter of the channel frequency response in spectral domain as in figure 2.

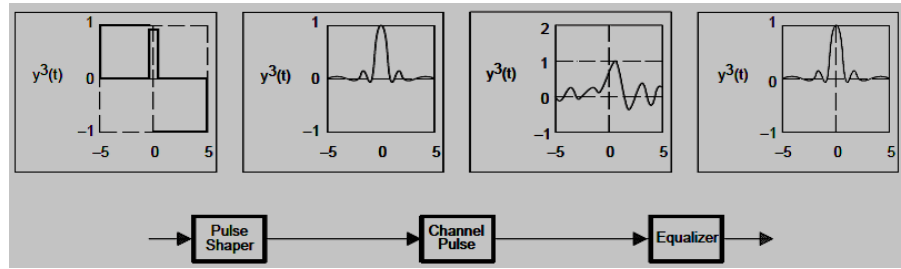


Figure2. Transmission Process with Example Pulse Responses

2.1 Zero Forcing Equalizer

Robert Lucky proposed a method called Zero Forcing Equalizer for restoring the message signal before reaching the receiver channel, by inverting channel frequency. Filters designed at base levels are easily tunable and low cost and hence, most equalizers are designed for baseband level. For 2x2 MIMO channel received symbol matrix Y is [2, 3, 6, 7]:

$$Y = HX + N \quad (1)$$

Where N is a noise matrix, X is a transmitted symbol matrix, and H is a full rank square matrix ($M_T = M_R$) of,

$$H = \begin{bmatrix} H_{1,1} & H_{1,2} & \dots & H_{1,M_T} \\ H_{2,1} & H_{2,2} & \dots & H_{2,M_T} \\ \vdots & \vdots & \ddots & \vdots \\ H_{M_R,1} & H_{M_R,2} & \dots & H_{M_R,M_T} \end{bmatrix}$$

Here received signals on the first and second receive antennas is a sum of the linear convolution and that in the received matrix notation is,

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = Y = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \quad (2)$$

Received symbols/impulses on the first and second antennae are respectively,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad (3)$$

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = \begin{bmatrix} h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad (4)$$

Where,

$h_{1,1}$ is the signal path from 1st transmit antenna to 1st receive antenna,
 $h_{1,2}$ is the signal path from 2nd transmit antenna to 1st receive antenna,
 $h_{2,1}$ is the signal path 1 from 1st transmit antenna to 2nd receive antenna,
 $h_{2,2}$ is the signal path from 2nd transmit antenna to 2nd receive antenna,
 x_1, x_2 are the transmitted symbols and
 n_1, n_2 are the noise on 1st and 2nd receive antennas.

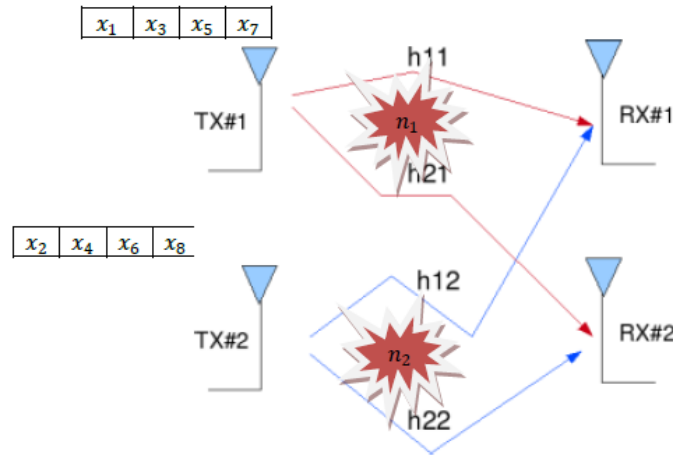


Figure3. MIMO-Transceiver system for 2x2

ZF equalizer tries for zeroing mutual interference between transmitted signals and the receiver can obtain an estimate of the two transmitted symbols x_1, x_2 , by

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = [H^H H + NoI]^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (5)$$

The received symbol power at both the antennae is respectively,

$$P\hat{x}_1 = [h_{1,1}]^2 + [h_{2,1}]^2 \quad (6)$$

$$P\hat{x}_2 = [h_{1,2}]^2 + [h_{2,2}]^2 \quad (7)$$

2.2 Minimum Mean Square Error (MMSE) Equalization

MMSE though unable to eliminate ISI completely, minimizes the mean square error (MSE), i.e., the total power of the noise and ISI components in the output as,

$$\text{MSE} = E \{ (X^{\wedge} - X^2) \} \quad (8)$$

This estimation has been realized with weighing the powers of the two symbols with the minimizing coefficient W by,

$$E \{ (X^{\wedge} - X^2) \} = E \{ [W_{y-x}] [W_{y-x}]^H \} \quad (9)$$

Now, an estimate of the symbols reduces to,

$$\begin{bmatrix} \widehat{x_1} \\ \widehat{x_2} \end{bmatrix} = [H^H H + N_0 I]^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (10)$$

2.3 Maximum Likelihood (ML) Receiver

In MIMO, channels are dispersive and the signal at each receive antenna is a combination of both the current and the past symbols sent from all transmit antennas corrupted by noise. The receiver do not know which $s_i(t)$ has been transmitted over the interval $0 \leq t < T$. So, the job of an efficient receiver is to make 'best estimate' of transmitted signal $[s_i(t)]$ upon receiving $r(t)$ and to repeat the same process during all successive symbol intervals. The receiver, depending on the modulation and transmission techniques usually knows about the signal constellation. Therefore, the received signal is a function of $r(t) = s_i(t) + w(t)$, $0 \leq t < T$. Where, $s_i(t)$ is the transmitted information-bearing symbol, $w(t)$ denotes a noise sample function over $0 \leq t < T$ [1, 2, 3, 4, 5].

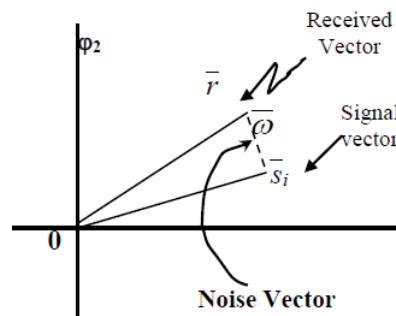


Figure4. Signal space showing a signal vector s_i and a received vector r .

The receiver obeying *maximum a posteriori probability* rule requires determining the probability of reception of a message from the received vector. Now, to take an optimum decision of a priori probability as per Bayer's rule, we have,

$$Pr(m_i | \bar{r}) Pr(\bar{r}) = Pr(\bar{r} | m_i) Pr(m_i) \quad (11)$$

... a posteriori probability of m_i given r . It is equivalent to

Where,

$Pr(m_i|\bar{r})$ is a posteriori prob. of m_i given \bar{r} ,

$Pr(\bar{r})$ is the joint prob. of \bar{r} ,

$Pr(\bar{r} | m_i)$ is a priori prob. of \bar{r} given m_i , and

$Pr(m_i) = 1/M$.

The 'likelihood function', an equivalent function of a priori and a posteriori probability, can be written as,

$$\hat{m} = m_i \text{ if } Pr(\bar{r} | m_i) \text{ is maximum for } k = i \quad (12)$$

By decision rule, it can be rewritten as,

$$\hat{m} = m_i \text{ if } \ln[Pr(\bar{r} | m_i)] \text{ is maximum for } k = i \quad (13)$$

When the signal space is divided into M decision regions like CDMA, a 'Maximum Likelihood Detector' realizes the above decision rule.

The received vector r from the-received signal $r(t)$ is determined from the Correlation Detector, shown in figure below:

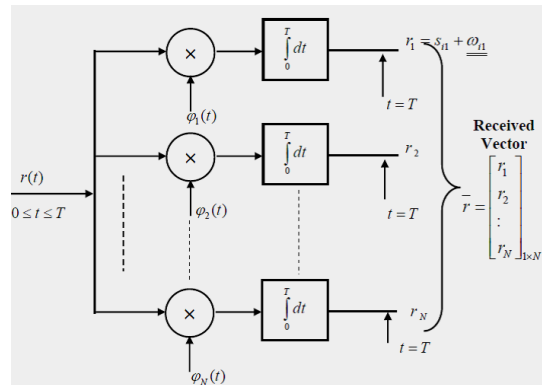
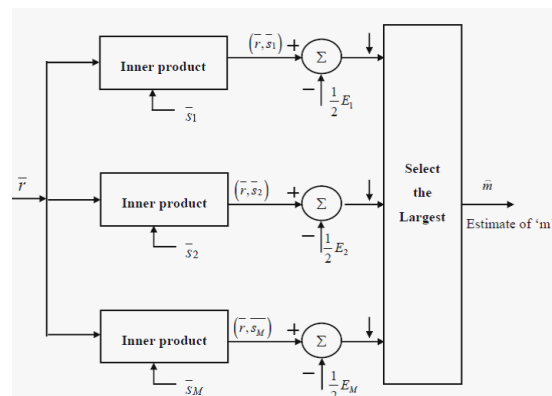


Figure5. Structure of a Correlation Detector



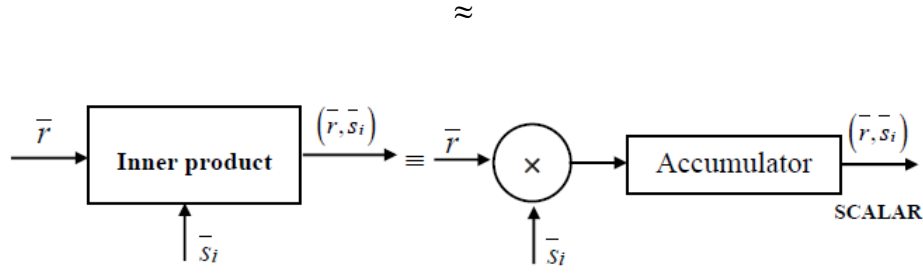


Figure6. Block schematic diagram for the Vector Receiver

3. MODULATION –QAM

Signals from the transmitting channel are modulated to a waveform to be compatible with the communication channel ‘according to the information in the message signal with the carrier signal’. QAM achieves high spectral efficiency/channel capacity because of its inherent nature, uses both amplitude as well as phase modulation to generate two carriers of $(90)^\circ$ phase of the same frequency. The resulting signal’s amplitude and phase will be the sum of I & Q signals as per,

$$M = \{s_i(t) = A_i p(t) \cos(2\pi f_c t + \varphi_i)\}_{i=1}^m \quad 0 \leq t \leq T_s, \quad (14)$$

Where

$$S_i(t) = (A_i \cos \varphi_i) p(t) \cos(2\pi f_c t) + (A_i \sin \varphi_i) p(t) \sin(2\pi f_c t) \quad (15)$$

Where, A_i is the amplitude, and φ_i is the phase of the i^{th} signal in the M-ary QAM signal set. We have,

$$I = A \cos(\Psi) = (A_i \cos \varphi_i) p(t) \cos(2\pi f_c t) \text{ and}$$

$$Q = A \sin(\Psi) = (A_i \sin \varphi_i) p(t) \sin(2\pi f_c t). \quad (16)$$

By their phase difference of 90° , the signal can be expressed as:

$$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta) \quad (17)$$

Using the expression $A \cos(2\pi f_c t + \Psi)$ for the carrier signal.

$$A \cos(2\pi f_c t + \Psi) = I \cos(2\pi f_c t) - Q \sin(2\pi f_c t) \quad (18)$$

Where, f_c is the carrier-frequency.

The M-ary square QAM signals can be denoted as,

$$S_i(t) = A_k \cos \theta_k(t) \cos(2\pi f_c t) - A_k \sin \theta_k(t) \sin(2\pi f_c t) \quad (19)$$

As a linear combination of two orthogonal functions,

$$S_i(t) = a_i \sqrt{\frac{2E_0}{T_s}} \cos(2\pi f_c t) + b_i \sqrt{\frac{2E_0}{T_s}} \sin(2\pi f_c t) \quad (20)$$

Therefore, the average energy of M-ary QAM signal corresponding to message points is,

$$E_{av} = \int_0^{T_s} E \{S_i^2(t)\} dt = \frac{1}{M} \sum_{i=1}^M (a_i^2 + b_i^2) E_0 \quad (21)$$

And the average power is,

$$P_{av} = \frac{1}{M} \sum_{i=1}^M (a_i^2 + b_i^2) \frac{E_0}{T_s} \quad (22)$$

Accordingly the coordinates of the i^{th} signal points derived from the elemental energy and power are represented in rectangular co-ordinate by $(a_i \sqrt{E_0}, b_i \sqrt{E_0})$ where (a_i, b_i) are symbols of the 64-QAM matrix represented as,

$$\{a_i, b_i\} = [I \& Q \text{ elements of } 64 \text{ QAM Constellations}]$$

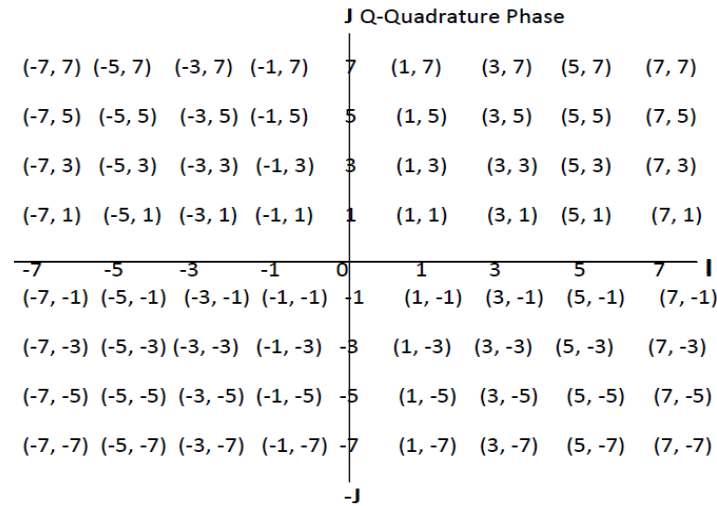


Figure7. 64 QAM Constellation diagram in the matrix form

Unique symbols of 64-QAM are the result of 16 I and 16 Q values with each symbol representing six bits ($2^6 = 64$). I and Q are amplitude and phase modulated (encoded) by the 6 bits per clock before being modulated to radio frequency (RF) [15, 16]. As the order of modulation increases, number of bits per symbol increases. But,

the link between bits in the symbol becomes more susceptible to noise. Figure 8 below shows the phase differences:

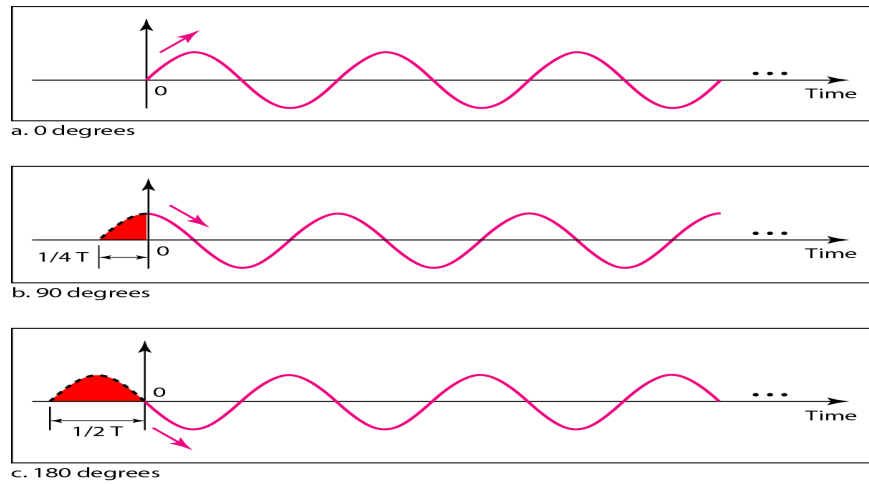


Figure8. Three sine waves with the same amplitude and frequency, but different phases

Higher order modulations are preferred when there is sufficiently high signal to noise ratio (SNR) or lower BER.

4. Simulation Results

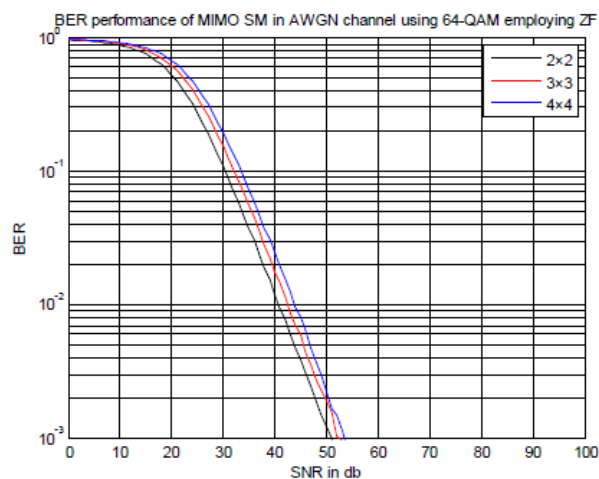


Figure9 (a): 64- QAM BER with ZF for AWGN

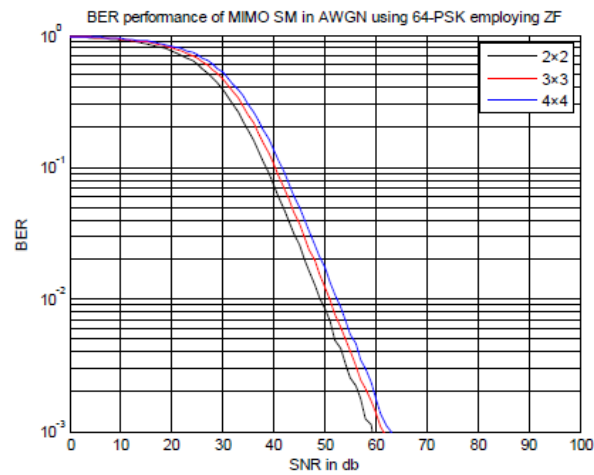


Figure9 (b): 64-PSK BER with ZF for AWGN

From the graphs of 9(a) & (b) we see that, SNR is 10 dB higher in 64-QAM than of 64-PSK for equally increased $m \times n$ antennae.

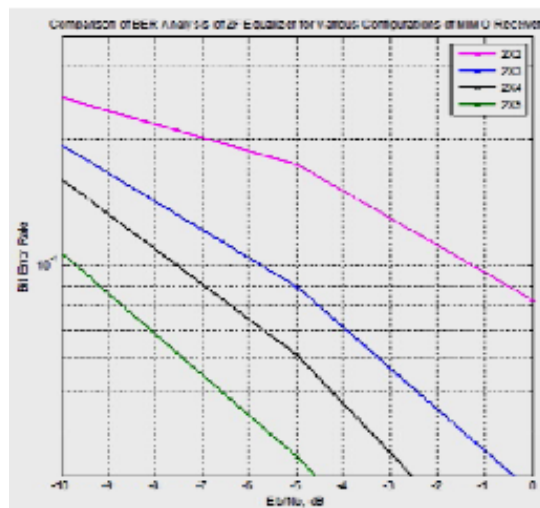


Figure9. (c)

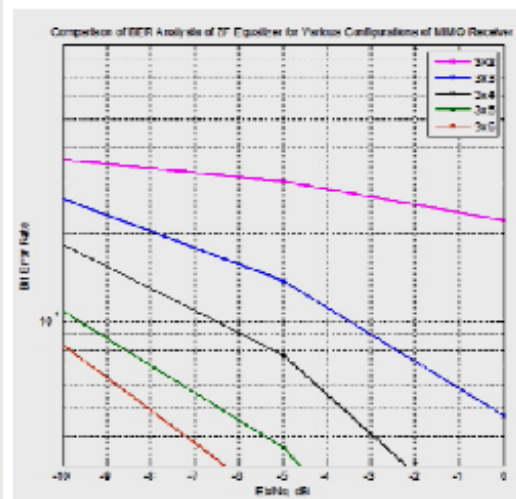


Figure9. (d)

Figure9. BER analysis of $m \times n$ Antenna configurations with ZF Equalizer

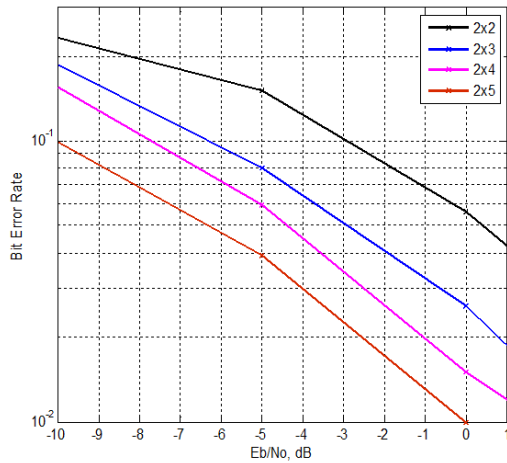
Similarly from figure 9(c) and 9(d) we see that the Bit Error Rate (BER) decreases with increase in the number of transmitters. Results observed are tabulated in table I below:

Table I. Bit Error Rate values for m x n antenna configurations of ZF Equalizer

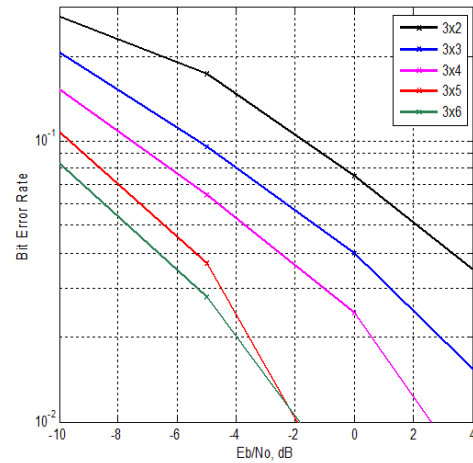
| Bit Error Rate value for = E_b / N_o - 10dB | | | | | | |
|---|-------|-------|-------|-------|-------|-------|
| Sl.No | m x n | Value | m x n | Value | m x n | Value |
| 1 | 2x2 | 0.253 | 3x2 | 0.364 | 4x2 | 0.433 |
| 2 | 2x3 | 0.194 | 3x3 | 0.256 | 4x3 | 0.297 |
| 3 | 2x4 | 0.157 | 3x4 | 0.193 | 4x4 | 0.227 |
| 4 | 2x5 | 0.113 | 3x5 | 0.112 | 4x5 | 0.163 |
| 5 | - | - | 3x6 | 0.086 | 4x6 | 0.098 |

Keeping m constant and increasing n continuously by 1 we see that BER decreases with n. The interesting point to be noted here is that for transmitting antenna 2 and incremental receiver antennae, we have better results than other varying mxn channels.

comparison of BER analysis of MMSE equalizer for various configurations of MIMO receiver

**Figure10. (a)**

comparison of BER analysis of MMSE equalizers for various configurations of MIMO receiver

**Figure10. (b)**

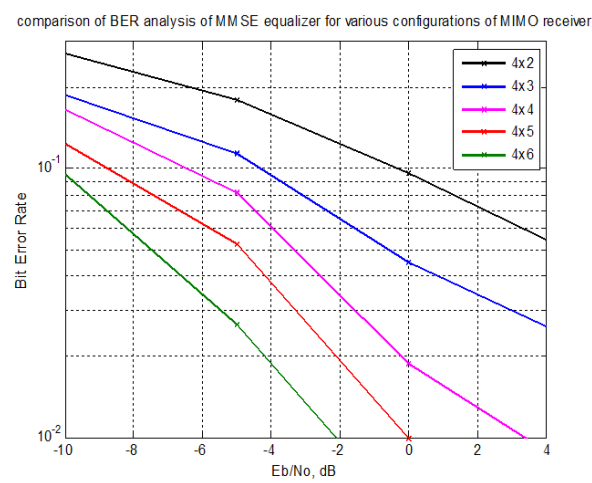


Figure10. ©

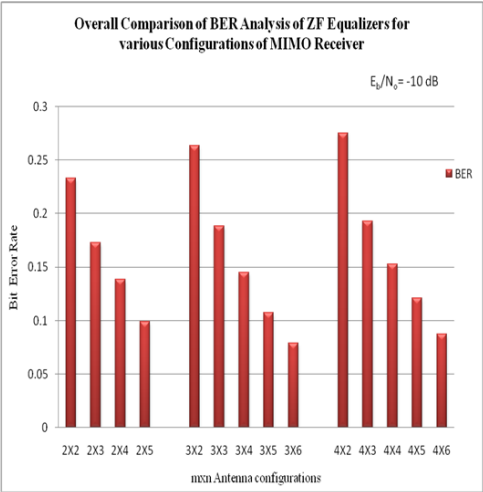


Figure10. (d)

Results observed from the graphs of 10 (a-d) are tabled as in table II below:

Table II. Bit Error Rate values for m x n antenna configurations of MMSE Equalizer

| Bit Error Rate value for = E_b / N_o - 10dB | | | | | | |
|---|-------|-------|-----|-------|-----|-------|
| Sl.No | m x n | Value | mxn | Value | mxn | Value |
| 1 | 2x2 | 0.234 | 3x2 | 0.264 | 4x2 | 0.276 |
| 2 | 2x3 | 0.174 | 3x3 | 0.189 | 4x3 | 0.194 |
| 3 | 2x4 | 0.137 | 3x4 | 0.146 | 4x4 | 0.153 |
| 4 | 2x5 | 0.098 | 3x5 | 0.121 | 4x5 | 0.123 |
| 5 | - | - | 3x6 | 0.079 | 4x6 | 0.089 |

The most important observation here to be noted is that MMSE gives better results for the same configuration than ZF equalizer.

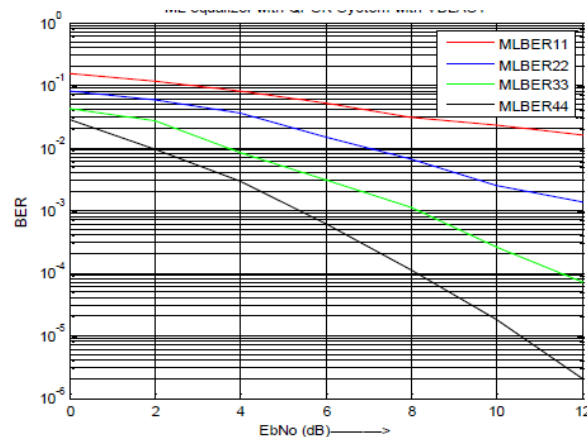


Figure11. ML BER for varying mxn configurations

From figure 11 it can be seen that BER is higher with ML for equally incremental values of mxn more than ZF and MMSE equalizers.

In totality comparing the results it can be expressed that MMSE is a linear balanced equalizer which minimizes the total noise power and ISI components (mean square error-MSE) but; does not eliminate ISI completely, and BER is less for MMSE compared to ML, and ZF for 2x2 MIMO.

Conclusion

The Zero Forcing Equalizer removes all ISI only when the channel is noiseless, otherwise, will amplify the noise greatly at higher frequencies and spoils the overall SNR in noisy channels. ML is suitable for Weibull and Log-normal fading under fixed $m \times n$, for varying it doesn't hold good. Especially for Rayleigh fading and variable $m \times n$ configurations, MMSE is a better choice than ZF & ML in terms of BER characteristics and under Noise performance. The novel methodology used is to improve Spectral efficiency, Level Crossing Rate, and Average Fade Duration. In turn, it helps to improve the performance of the received signal with lower SNR (28dBs) against existing 31dBs under favorable conditions.

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