

Convergence Analysis of Underwater Moving Object Tracking Based On Bearing and Elevation Measurements

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Abstract

Autonomous Underwater Vehicles (AUVs), provided with underwater sensors enabled the exploration of natural submerged resources, acquiring of accurate data in monitoring missions, assisted navigation and tactical surveillance applications. To make these applications viable there is a need to development of various solutions which aid the AUV for its safe navigation in the dense underwater environment. It is important to note that the convergence time plays very important role in aiding the AUV for its safe navigation as the AUV requires the sufficient time to maneuvering itself for avoiding the objects that comes into its path. Towards this various methods / techniques such as Least Squares (LS), Kalman Filter (KF) and Extended Kalman Filter (EKF) methods have been explored, however all these methods have their own drawbacks. In this paper a new method has been developed wherein tracking algorithm using EKF has been extended to the Bearing and Elevation only Tracking (BEOT) method. The performance of this algorithm has been evaluated using Monte Carlo Method. Subsequently the convergence time has been calculated and the results have been accordingly plotted.

KeyWords—AUV, Tracking, EKF, BEOT.

I. INTRODUCTION

In order to operate in underwater environment automatic detection and tracking are extremely important for AUV for its safe navigation. In many civil and military

applications including sonar based robotic navigation, infrared seekers and underwater weapon systems based tracking, Bearing only tracking (BOT) is used. For under water weapon guidance system, passive tracking sensors are used in BOT applications. For aerospace and naval applications object tracking is generally performed using sonars or seekers. The available information from the sensors are either bearing only data or both range and bearing data or bearing and elevation data. The passive target tracking of maneuvering objects using angle only measurements is paramount field of scientific research in the areas of Autonomous robotics and mobile systems, aircraft surveillance and submarine tracking [1-5].

Due to inherent property of environment the sensor data such as range, bearing and elevation are often noisy, which also results in nonlinear relation between the states and measurements. These inaccuracies of the measurements have a direct impact on the accomplishment of the tracking algorithm.

For target tracking in the ocean environment, two approaches are commonly used. The first approach is a linear Kalman filter (KF) [6], designed by R.E. Kalman in 1960, wherein the state measurements are linear and designed for prediction and estimation problem. KF is simply an optimal recursive data processing algorithm and is described as an accurate estimation of state variables under noisy environments. It is suitable for navigation, industrial and real time applications. The KF algorithm is developed in two steps which act on; prediction and updating. The second approach is an Extended Kalman Filter (EKF) [7], wherein the measurements are nonlinear state functions. It is well established fact that in EKF the initial covariance is based on the initial converted measurement and the gain is based on the accuracies of the subsequent linearization and therefore the overall performance depends on these accuracies.

In the early research on the bearing-only filtering problem discrete-time EKF with relative Cartesian coordinates [8] is used in 2D. The filter which was implemented based on the nearly constant velocity model (NCVM) [9] and nonlinear measurement model for bearing. The EKF is implemented for only 2D tracking problems in [10] and in [11], the EKF, UKF, GHKF and CKF are implemented for only 2D tracking problems.

In [12], the EKF is implemented for both the predicted state estimate and covariance using a discretized linear approximation. All of the approaches mentioned use a two Dimension state estimation. The performance of the EKF, UKF, and particle filter (PF) of angle-only filtering problem in 3D using bearing and elevation measurements from a single maneuvering sensor are compared in [13]. To estimate the kinematics, such as position and velocity of an object, with the help of noisy-corrupted data of corresponding object from a moving observation platform alone is a nonlinear function.

In early research algorithms, based on the EKF which linearizes the nonlinear measurement model, often results in unstable performances, including poor track accuracy and divergences [7,8]. In this paper a new method of EKF tracking algorithm has been proposed to solve the problems of underwater environment. By examining the illustration of single maneuvering sensor bearing and elevation only target tracking

exhibits good accuracy and efficiency as the inaccuracies can be handled effectively in this method.

The target tracking basics is covered in [14]. Most aspects of tracking covered in [15]. Comparison of different tracking methods presents in [16] and [17] derives a tracking filter that is well suited for angle-only target tracking. The approach that has been followed in this paper is that the nonlinearities are modified prior subjecting to the tracking algorithm with angles only measurements. At same time the modified gains of bearing and elevation problem have also been derived in a simplistic manner. The results have been promising and have shown enhanced performance beyond the discrete time EKF.

Song T.L. and Speyer LL [18] derived a Modified Gain Extended Kalman Filter (MGEKF) for nonlinear estimation problems. This algorithm was then developed by Galkowski P.J. and Islam M.A. [19] based on the pseudo measurements. Mo, Longbin, Liu, Qi, Zhou, Yiyu, Sun, Zhongkang [20] [21] presented the improved modified gain functions for 3D angles-only tracking. In this paper the nonlinearities are modified and then applied to a tracker with bearing and elevation only measurements. It shows a better performance over the standard Extended Kalman Filter (EKF). In this paper, modified gains in a simpler manner for the bearings and elevations problem have been derived.

II. TRACKING ALGORITHM

The estimation of object/target trajectory is the fundamental problem in bearing and elevation only tracking using noisy corrupted sensor data [22]. In this scenario single observer tracks with single sensor, which measures the bearings and elevations of the moving target with respect to positions of the sensor. In this scenario single observer tracks with single sensor, which measures the bearings and elevations of the moving target with respect to positions of the sensor. The state of the target motion model (TMM) is characterized by a Nearly Constant Velocity Model (NCVM) and at $(k+1)^{\text{th}}$ time step it consists of three dimensional (3D) position in Cartesian coordinates x, y and z and the velocity towards those coordinate axes v_x, v_y and v_z . Thus, the dynamics of the target is modeled as a state space model. The state of the target is defined in the tracker coordinate frame (T frame) for which the X, Y, and Z axes are along the local east, north, and upward directions, respectively as shown in Fig.1. The target and own ship/observer states in Cartesian coordinates are defined [13].

$$X_t = [v_{xt} v_{yt} v_{zt} x_t y_t z_t] \quad (1)$$

and

$$X_o = [v_{xo} v_{yo} v_{zo} x_o y_o z_o] \quad (2)$$

The relative state vector in the T frame is defined by

$$X = X_t - X_o \quad (3)$$

Let the relative state vector in the T frame is

$$X = [v_x v_y v_z x y z]' \quad (4)$$

Then, $x = x_t - x_o, v_x = v_{xt} - v_{xo}$, etc.

The range vector of the target from the observer (or sensor) in the T frame is

$$r_t = [x y z]' = [x_t - x_o \ y_t - y_o \ z_t - z_o]' \quad (5)$$

Then the range is defined as

$$r = \sqrt{x^2 + y^2 + z^2} \quad (6)$$

The range vector can be expressed in terms of range, bearing (φ) and elevation (θ), as defined in Figure 1, by

$$r_t = r \begin{bmatrix} \cos \varphi * \sin \theta \\ \sin \varphi * \sin \theta \\ \cos \theta \end{bmatrix} \quad (7)$$

Target motion model (TMM) is described in the Cartesian coordinate system by linear discrete-time difference equation with some additive noise as

$$X_t(k+1) = F(k) X_t(k) + w(k) \quad (8)$$

Where the state vector (X_t) includes the position and velocity components of the object moving in the 3-D space.

$$\text{i.e. } X_t(k) = [v_x(k) v_y(k) v_z(k) x(k) y(k) z(k)] \quad (9)$$

Where $F(k)$ and $w(k)$ are the state transition matrix and integrated process noise, respectively, for the time interval $[t(k+1), t(k)]$ and process noise is assumed to be zero mean white gaussian noise.

$$dt = t(k+1) - t(k) \quad (10)$$

$$F(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ dt & 0 & 0 & 1 & 0 & 0 \\ 0 & dt & 0 & 0 & 1 & 0 \\ 0 & 0 & dt & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

The target is tracked by a sonar in the underwater and provides measurements of bearing (φ_m) and elevation (θ_m).

The measurement model is given as

$$Z_m(k+1) = h X_t(k+1) + v(k) \quad (12)$$

$$h X_t(k+1) = \begin{bmatrix} \varphi_m(k+1) \\ \theta_m(k+1) \end{bmatrix} = \begin{bmatrix} \tan^{-1} \frac{y(k+1)}{x(k+1)} \\ \tan^{-1} \frac{\sqrt{x^2(k+1) + y^2(k+1)}}{z(k+1)} \end{bmatrix} \quad (13)$$

Where $v(k)$ is uncorrelated, zero-mean white Gaussian noise with variances σ_φ^2 , σ_θ^2 in the bearing (φ) and elevation (θ) measurements respectively.

The measured range, bearing, and elevation from sonar are converted to target positions in Cartesian coordinates with respect to own ship as origin using the following relations:

$$\begin{aligned} x(k) &= r_m * \cos \varphi_m * \sin \theta_m \\ y(k) &= r_m * \sin \varphi_m * \sin \theta_m \\ z(k) &= r_m * \cos \theta_m \\ Y_m &= [x(k) \ y(k) \ z(k)] \end{aligned} \quad (14)$$

Due to nonlinear measurement model we replace the KF algorithm with EKF. The dynamic model using NCVM in 3D is linear and the measurement model for bearing and elevation is nonlinear for this problem. In general EKF is based on linearized approximations to nonlinear dynamic and/or measurement models [23], [24] and is widely used. For this problem, the linearized approximation is performed in the updated measurement step as described in [23], [24].

The algorithm of the new method of modified gain EKF is as follows.

$$X_{(k+1)}^- = F_{(k)} X_{(k)} \quad (15)$$

$$P_{(k+1)}^- = F_{(k)} P_{(k)} F_{(k)}^T + Q_{(k)} \quad (16)$$

$$X_{(k+1)} = X_{(k+1)}^- + K_{(k+1)} (Z_{(k+1)} - H_{(k+1)} X_{(k+1)}^-) \quad (17)$$

$$K_{(k+1)} = P_{(k+1)}^- H_{(k+1)}^T (H_{(k+1)} P_{(k+1)}^- H_{(k+1)}^T + R)^{-1} \quad (18)$$

$$P_{(k+1)}^- = \left(I - K_{(k+1)} g \left(Z_{(k+1)}, X_{(k+1)}^- \right) \right) P_{(k+1)}^- \left(I - K_{(k+1)} g \left(Z_{(k+1)}, X_{(k+1)}^- \right) \right)^T + K_{(k+1)} R K_{(k+1)}^T \quad (19)$$

Where,

$F_{(k)}$ = State transition matrix at time k.

$X_{(k)}^-$ = State estimate at time k before update.

$X_{(k)}^+$ = State estimate at time k after update.

$K_{(k)}$ = Filtergain at time k.

$Z_{(k)}$ = Measurement data at time k.

$H_{(k)}$ = Predicted measurement based on state at time k before update.

$P_{(k)}^-$ = State covariance matrix at time k before update.

$P_{(k)}^+$ = State covariance matrix at time k after update.

$Q_{(k)}$ = Process noise matrix at time k.

R = Measurement noise covariance matrix.

$g \left(Z_{(k)}, X_{(k)}^- \right)$ = Modified gain function.

I = Identity matrix.

The main difference between the Extended Kalman Filter (EKF) and Modified Gain Extended Kalman Filter (MGEKF) is the function g in the covariance update.

To determine the modified gain function g we write:

$$\begin{bmatrix} (\varphi - \hat{\varphi}) \\ (\theta - \hat{\theta}) \end{bmatrix} = g \begin{bmatrix} (x - \hat{x}) \\ (y - \hat{y}) \\ (z - \hat{z}) \end{bmatrix} \quad (20)$$

Where,

$\hat{\varphi}$ = Estimated bearing from the states \hat{x} , \hat{y} and \hat{z} .

$\hat{\theta}$ = Estimated elevation from the states \hat{x} , \hat{y} and \hat{z} .

Since ' g ' is not a function of target velocity we removed those states for the derivation of ' g '.

The measurement matrix H is given by

$$H = \begin{bmatrix} 0 & 0 & 0 & \frac{-\sin \hat{\varphi}}{\hat{r}_{xy}} & \frac{\cos \hat{\varphi}}{\hat{r}_{xy}} & 0 \\ 0 & 0 & 0 & \frac{\cos \hat{\varphi} \cos \hat{\theta}}{\hat{r}_{xyz}} & \frac{\sin \hat{\varphi} \cos \hat{\theta}}{\hat{r}_{xyz}} & \frac{-\sin \hat{\theta}}{\hat{r}_{xyz}} \end{bmatrix} \quad (21)$$

III. UPDATED MEASUREMENT OF BEARING DATA

If the range in horizontal plane is

$$r_{xy} = \sqrt{x^2 + y^2}$$

Then the estimated range be

$$\hat{r}_{xy} = \sqrt{\hat{x}^2 + \hat{y}^2}$$

By adding r_{xy} with \hat{r}_{xy} we can get,

$$r_{xy} + \hat{r}_{xy} = x \cos \varphi + y \sin \varphi + \hat{x} \cos \hat{\varphi} + \hat{y} \sin \hat{\varphi} \quad (22)$$

Adding both sides with $-x \cos \hat{\varphi} - y \sin \hat{\varphi} - \hat{x} \cos \varphi - \hat{y} \sin \varphi$

$$r_{xy} + \hat{r}_{xy} = \frac{(x - \hat{x})(\cos \varphi - \cos \hat{\varphi}) + (y - \hat{y})(\sin \varphi - \sin \hat{\varphi})}{1 - \cos(\varphi - \hat{\varphi})} \quad (23)$$

Similarly,

$$r_{xy} - \hat{r}_{xy} = \frac{(x - \hat{x})(\cos \varphi + \cos \hat{\varphi}) + (y - \hat{y})(\sin \varphi + \sin \hat{\varphi})}{1 + \cos(\varphi - \hat{\varphi})} \quad (24)$$

$$\begin{aligned} 2\hat{r}_{xy} = (x - \hat{x}) & \left[\frac{\cos \varphi - \cos \hat{\varphi}}{1 - \cos(\varphi - \hat{\varphi})} - \frac{\cos \varphi + \cos \hat{\varphi}}{1 + \cos(\varphi - \hat{\varphi})} \right] \\ & + (y - \hat{y}) \left[\frac{\sin \varphi - \sin \hat{\varphi}}{1 - \cos(\varphi - \hat{\varphi})} - \frac{\sin \varphi + \sin \hat{\varphi}}{1 + \cos(\varphi - \hat{\varphi})} \right] \end{aligned} \quad (25)$$

Taking Eq. (25) and simplifying we get

$$\begin{aligned} \left[\frac{\cos \varphi - \cos \hat{\varphi}}{1 - \cos(\varphi - \hat{\varphi})} - \frac{\cos \varphi + \cos \hat{\varphi}}{1 + \cos(\varphi - \hat{\varphi})} \right] &= -2 \frac{\cos \varphi \cos(\varphi - \hat{\varphi}) - \cos \hat{\varphi}}{1 - \cos^2(\varphi - \hat{\varphi})} \\ &= -2 \frac{\sin \varphi}{\sin(\varphi - \hat{\varphi})} \end{aligned} \quad (26)$$

$$\begin{aligned} \left[\frac{\sin \varphi - \sin \hat{\varphi}}{1 - \cos(\varphi - \hat{\varphi})} - \frac{\sin \varphi + \sin \hat{\varphi}}{1 + \cos(\varphi - \hat{\varphi})} \right] &= 2 \frac{\sin \varphi \cos(\varphi - \hat{\varphi}) - \sin \hat{\varphi}}{1 - \cos^2(\varphi - \hat{\varphi})} \\ &= 2 \frac{\cos \varphi}{\sin(\varphi - \hat{\varphi})} \end{aligned} \quad (27)$$

Now the coefficients of $(x - \hat{x})$ and $(y - \hat{y})$ are then,

$$\hat{r}_{xy} = \frac{-\sin \varphi}{\sin(\varphi - \hat{\varphi})} (x - \hat{x}) + \frac{\cos \varphi}{\sin(\varphi - \hat{\varphi})} (y - \hat{y}) \quad (28)$$

Equation (28) rewritten as

$$\sin(\varphi - \hat{\varphi}) = \frac{-\sin \varphi (x - \hat{x}) + \cos \varphi (y - \hat{y})}{\hat{r}_{xy}} \quad (29)$$

IV. UPDATED MEASUREMENT OF ELEVATION DATA

As can be seen from the above analysis

$$\tan^{-1} \frac{y}{x} = \varphi \text{ generates}$$

$$\sin(\varphi - \hat{\varphi}) = \frac{-\sin \varphi (x - \hat{x}) + \cos \varphi (y - \hat{y})}{\hat{r}_{xy}} \quad (30)$$

In the similar way

$$\tan^{-1} \frac{\sqrt{x^2 + y^2}}{z} = \tan^{-1} \frac{r_{xy}}{z} = \theta \text{ generates,}$$

$$\sin(\theta - \hat{\theta}) = \frac{-\sin \theta (z - \hat{z}) + \cos \theta (r_{xy} - \hat{r}_{xy})}{\hat{r}_{xyz}} \quad (31)$$

From the Eq. (24) we get

$$r_{xy} - \hat{r}_{xy} = \frac{(x - \hat{x}) \cos\left(\frac{\varphi + \hat{\varphi}}{2}\right) + (y - \hat{y}) \sin\left(\frac{\varphi + \hat{\varphi}}{2}\right)}{\cos\left(\frac{\varphi - \hat{\varphi}}{2}\right)} \quad (32)$$

Now replacing Eq. (32) in Eq. (31),

$$\sin(\theta - \hat{\theta}) = \frac{-\sin \theta (z - \hat{z})}{\hat{r}_{xyz}} + \frac{\cos \theta}{\hat{r}_{xyz}} \left[\frac{(x - \hat{x}) \cos\left(\frac{\varphi + \hat{\varphi}}{2}\right) + (y - \hat{y}) \sin\left(\frac{\varphi + \hat{\varphi}}{2}\right)}{\cos\left(\frac{\varphi - \hat{\varphi}}{2}\right)} \right] \quad (33)$$

By rearranging the Eq. (33) becomes,

$$\begin{aligned} \sin(\theta - \hat{\theta}) = & \frac{\cos \theta \cos\left(\frac{\varphi + \hat{\varphi}}{2}\right)}{\hat{r}_{xyz} \cos\left(\frac{\varphi - \hat{\varphi}}{2}\right)} (x - \hat{x}) + \frac{\cos \theta \sin\left(\frac{\varphi + \hat{\varphi}}{2}\right)}{\hat{r}_{xyz} \cos\left(\frac{\varphi - \hat{\varphi}}{2}\right)} (y - \hat{y}) \\ & + \frac{-\sin \theta (z - \hat{z})}{\hat{r}_{xyz}} \end{aligned} \quad (34)$$

$$\begin{bmatrix} (\varphi - \hat{\varphi}) \\ (\theta - \hat{\theta}) \end{bmatrix} = \begin{bmatrix} \frac{-\sin \varphi}{\hat{r}_{xy}} & \frac{\cos \varphi}{\hat{r}_{xy}} & 0 \\ \frac{\cos \theta \cos(\frac{\varphi + \hat{\varphi}}{2})}{\hat{r}_{xyz} \cos(\frac{\varphi - \hat{\varphi}}{2})} & \frac{\cos \theta \sin(\frac{\varphi + \hat{\varphi}}{2})}{\hat{r}_{xyz} \cos(\frac{\varphi - \hat{\varphi}}{2})} & \frac{-\sin \theta}{\hat{r}_{xyz}} \end{bmatrix} \begin{bmatrix} (x - \hat{x}) \\ (y - \hat{y}) \\ (z - \hat{z}) \end{bmatrix} \quad (35)$$

In practical the true bearing and elevation angles are not available, the measured bearing and elevations are used to calculate the modified gain. Hence the Eq.(35) can be rewritten as,

$$\begin{bmatrix} (\varphi - \hat{\varphi}) \\ (\theta - \hat{\theta}) \end{bmatrix} = \begin{bmatrix} \frac{-\sin \varphi_m}{\hat{r}_{xyz} \sin \theta_m} & \frac{\cos \varphi_m}{\hat{r}_{xyz} \sin \theta_m} & 0 \\ \frac{\cos \theta_m \cos(\frac{\varphi_m + \hat{\varphi}}{2})}{\hat{r}_{xyz} \cos(\frac{\varphi_m - \varphi}{2})} & \frac{\cos \theta_m \sin(\frac{\varphi_m + \hat{\varphi}}{2})}{\hat{r}_{xyz} \cos(\frac{\varphi_m - \varphi}{2})} & \frac{-\sin \theta_m}{\hat{r}_{xyz}} \end{bmatrix} \begin{bmatrix} (x - \hat{x}) \\ (y - \hat{y}) \\ (z - \hat{z}) \end{bmatrix} \quad (36)$$

By considering the velocity components v_x, v_y and v_z , g is given by

$$g = \begin{bmatrix} 0 & 0 & 0 & \frac{-\sin \varphi_m}{\hat{r}_{xyz} \sin \theta_m} & \frac{\cos \varphi_m}{\hat{r}_{xyz} \sin \theta_m} & 0 \\ 0 & 0 & 0 & \frac{\cos \theta_m \cos(\frac{\varphi_m + \hat{\varphi}}{2})}{\hat{r}_{xyz} \cos(\frac{\varphi_m - \varphi}{2})} & \frac{\cos \theta_m \sin(\frac{\varphi_m + \hat{\varphi}}{2})}{\hat{r}_{xyz} \cos(\frac{\varphi_m - \varphi}{2})} & \frac{-\sin \theta_m}{\hat{r}_{xyz}} \end{bmatrix} \quad (37)$$

V. RESULTS AND DISCUSSIONS

The simulated data with different measurement inaccuracies of simulation has been generated for validating the algorithm. At the instant of the first angle measurement, own-ship is considered at the origin. It has also been assumed that the target movement has a constant velocity and travels in a straight path. Fig. 1 shows the tracking angles of the observer and target. With the initial assumption of target at 10km range and an initial velocity of 10m/s (nearly equal to the real speed of the vehicles in underwater environment), the initial state has been defined as

$$X_0 = [10 \ 10 \ 10 \ r_m \cos \varphi_m(0) \sin \theta_m(0) \ r_m \sin \varphi_m(0) \sin \theta_m(0) \ r_m \cos \theta_m(0)] \quad (38)$$

Where $r_m, \theta_m(0)$ and $\varphi_m(0)$ are initial bearing and elevation measurements. Here simulation 1 and simulation 2 are for initial range measurements of 15km and 20km at target bearing of 0.5° and course of 45° with zero mean white gaussian noise and r.m.s level of 0.3 degree. The target is assumed to be moving at a constant course and speed of 135° and 10 m/s accordingly. Observer motion is assumed to be stationary. Bearing and elevation measurements are taken for every 1 second for 1000 Monte Carlo updates. Fig. 2 shows the trajectories of ownship, true target and

predicted target. Initial covariance matrix is assumed as per the standard procedure in [25].

The performance of the algorithm is evaluated in terms of :

i. The percentage fit error (PFE) in x and y

$$PFE_x = 100 * \frac{norm(x-\hat{x})}{norm(x)} \quad (39)$$

$$PFE_y = 100 * \frac{norm(y-\hat{y})}{norm(y)} \quad (40)$$

$$PFE_z = 100 * \frac{norm(z-\hat{z})}{norm(z)} \quad (41)$$

ii. The root mean square position error

$$RMSPE = \sqrt{\frac{1}{N} \sum_{i=1}^N \frac{(x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 + (z_i - \hat{z}_i)^2}{2}} \quad (42)$$

iii. The root mean square velocity error

$$RMSVE = \sqrt{\frac{1}{N} \sum_{i=1}^N \frac{(v_{x_i} - \hat{v}_{x_i})^2 + (v_{y_i} - \hat{v}_{y_i})^2 + (v_{z_i} - \hat{v}_{z_i})^2}{2}} \quad (43)$$

iv. The root sum square position error

$$RSSPE = \sqrt{(x - \hat{x})^2 + (y - \hat{y})^2 + (z - \hat{z})^2} \quad (44)$$

v. The root sum square velocity error

$$RSSVE = \sqrt{(v_x - \hat{v}_x)^2 + (v_y - \hat{v}_y)^2 + (v_z - \hat{v}_z)^2} \quad (45)$$

where x, y and z are the measurements, \hat{x}, \hat{y} and \hat{z} are the estimated target positions, v_x, v_y and v_z are the measurements, \hat{v}_x, \hat{v}_y and \hat{v}_z are the estimated target velocities in x, y and z coordinates, respectively.

VI. CONCLUSIONS

In this paper convergence issues pertaining to the new designed algorithm, wherein EKF has been extended to the BEOT method in respect of underwater environment has been analyzed. The estimated errors are plotted in Fig.3 to Fig.7. It has been observed that the convergence duration in respect of range is 200seconds, for bearing 50 seconds, for velocity is 200 seconds, for course is 600 seconds and elevation is 800 seconds,

which indicate the applicability/suitability of this method for aiding the AUV its safe navigation. The root mean square errors in position (RMSPE) and velocity (RMSVE) are shown in Fig.8 and Fig.9. The root sum square errors in position (RSSPE) and velocity (RSSVE) are shown in Fig.10 to Fig.14. It is observed that the errors are small and settled down after a filter learns the dynamics.

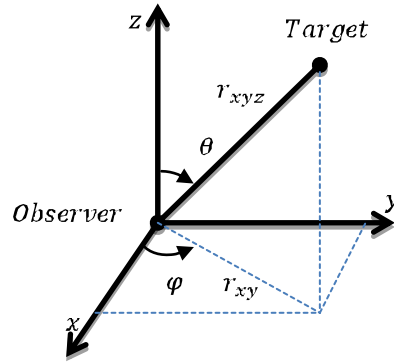


Fig. 1 Tracking Angles

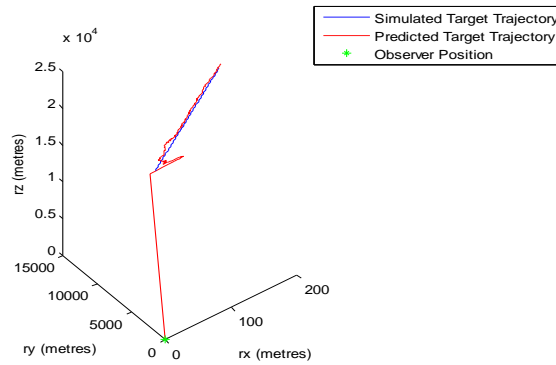


Fig. 2 Observer, True Target and Predicted Target Trajectories

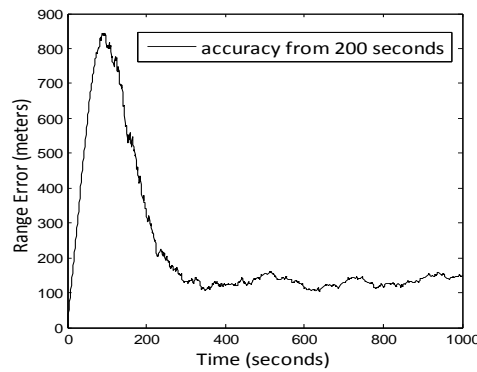


Fig. 3 Range Error Vs Time

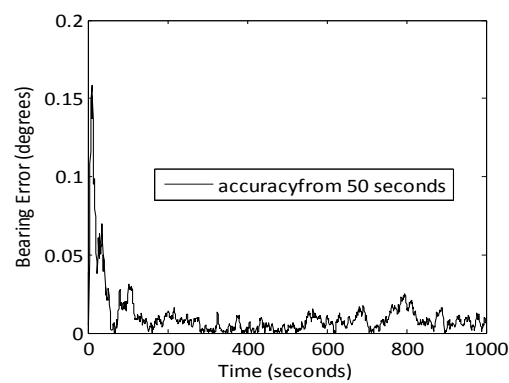


Fig. 4 Bearing Error Vs Time

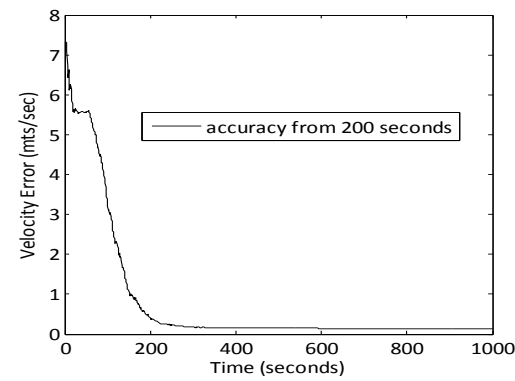


Fig. 5 Velocity error Vs Time

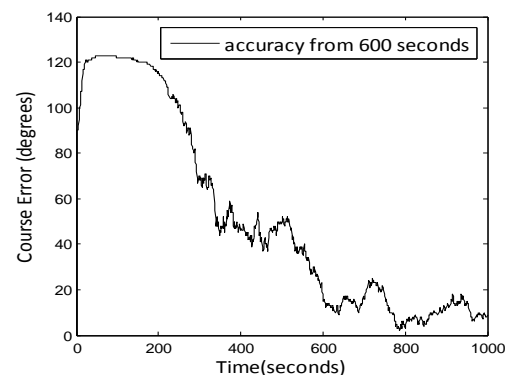


Fig. 6 Course Error Vs Time

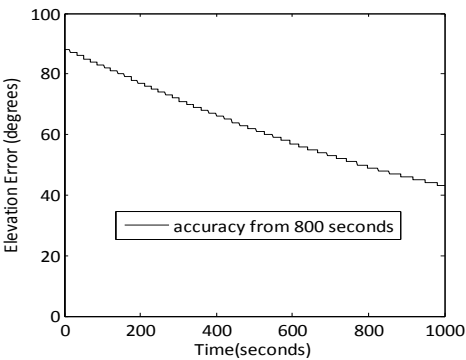


Fig. 7Elevation Error Vs Time

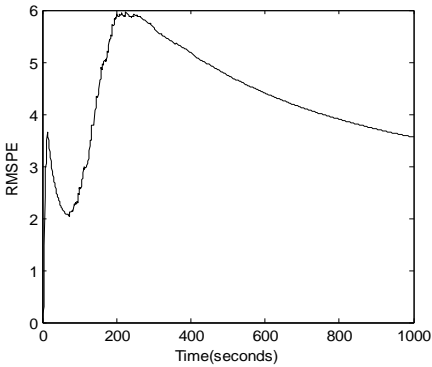


Fig. 8RMSPE Vs Time

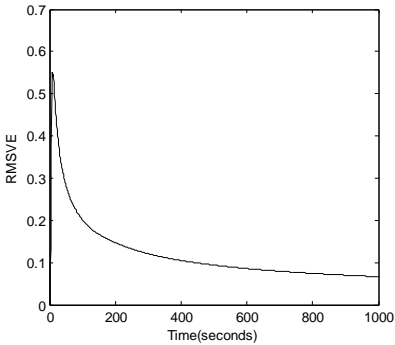


Fig. 9RMSVE Vs Time

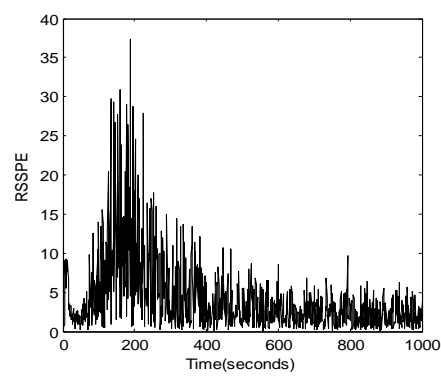


Fig. 10RSSPE Vs Time

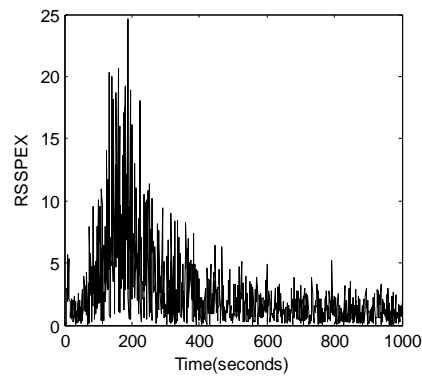


Fig. 11RSSPEX Vs Time

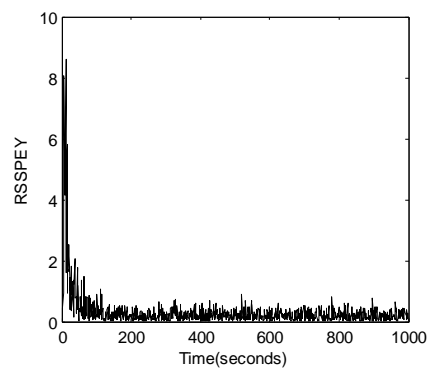


Fig. 12RSSPEY Vs Time

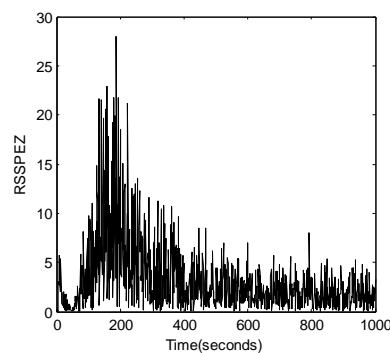


Fig. 13 RSSPEZ Vs Time

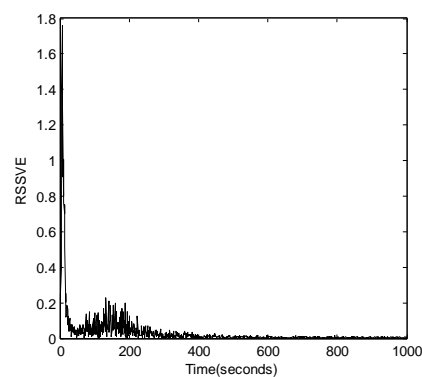


Fig. 14 RSSVE Vs Time

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