A Mathematical Implementation of Time Series Method For Exchange Rate Forecasting

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Abstract

Exchange rate forecasting has become a new research topic in present time for best market strategy, planning of investment for investors in foreign project and also for business profit. The methods or processes which give the accurate forecasting result are not easy because there are several types of forecasting methods. Exchange Rate Forecasting is most important financial topic for individual investor, stock fund manager, financial analyst for their investment and stock market. Forecasting is to predict the future value of a particular field with the help of previous value or history. Various types of forecasting can be possible like exchange rate forecasting, stock market forecasting, gold price forecasting which affects the economy of a country. Now, there is a drastic change in Indian Rupee(INR) in terms of United State Dollar(USD) from 2009 to 2014. In exchange rate forecasting, there are various type of performance parameter on the basic of error by which we can consider which method gives the best forecasting result. These are mean square error, mean absolute error, root mean square error etc. Error can be calculate as the difference between the actual value and forecasted value. Many factor affects the Indian Rupee which are Economic Activities, Interest Rate, Stock, Money Supply. Time Series Forecasting is the only mathematical method to forecast Exchange Rate specially Dollar Rate Forecasting. Mainly three types of forecasting are available.

Keywords: Forecasting, Exchange Rate Forecasting, Time Series Method, Moving Average, Weighted Moving Average, Exponential Smoothing, Mean Square Error, Mean Absolute Error, Root Mean Square Error.

Introduction

Money is the desired and popular asset among the world. Money is the main thing by which all type of transaction can be happened whole around the world. Money is only

asset for buying some product. Investors also invest money in their firms and also invest in stock market. Various countries follow different currencies like India maintains Indian Rupee, United State maintains US Dollar, United Kingdom maintains Pound, Japan maintains Yen, Indian Rupee is only valid for Indian market. Indian Rupee is not valid in USA. Similarly in USA, US Dollar is valid and in India, US Dollar is not valid. So if anyone want to buy American product then he have to pay in dollar.

Forecasting is the process where predict the future value on a specific field with the help of past data records and concluded in the specific result. Forecasting start with the specific assumption based on the management's experience, knowledge and judgement. These estimates are predicted into the coming day, month, or year. There are several fields where forecasting is used like exchange rate forecasting more specifically dollar rate forecasting with respect to Indian Rupee, or Pound forecasting in Indian Rupees, gold price forecasting, weather forecasting, tide forecasting. Exchange Rate Forecasting is very important in modern time for world economy because of market strategy, investor to invest in foreign projects. Foreign exchange rate are most important and largest financial markets in the world with trading taking place twenty-four hours a day around the world and a large amount of dollars of currencies transacted each day through the world each day. Transactions in foreign exchanges market determine the rate at which currencies are exchanged, which determine the cost of purchasing foreign goods and foreign assets. So An exchange rate between two countries is the rate at which one currency will be exchanged to another. It is regarded as the one country's currency in terms of another.

In finance, Exchange Rate Forecasting is the method of prediction of future currency rate with the help of previous existing data. We consider interbank exchange rate 118 Japanese Yen(JPY ¥) to United State Dollar(USD $). It means that ¥188 will be exchanged for each 1 USD$ or 1 USD$ will be exchanged for ¥91. Exchange Rate are retained by the foreign exchange market, which gives services to the wide range of different types of buyers and sellers where currency trading is continuous. A currency pair is called the quotation of the relative value of a currency unit against the unit of another in the foreign exchange market. The quotation EUR/USD 1.3533 means that 1 Euro is able to buy 1.3533 US dollar. In other words, this is the price of a unit of Euro in US dollar. Here, EUR is called the "Fixed Currency", while USD is called the "Variable Currency". Same as the quotation USD/INR 58.58 means that 1 USD is able to buy 58.58 INDIAN Rupees .In other words, this is the price of a unit of USD in Indian Rupee. Here USD is the "Fixed Currency" ,while INR is the "Variable Currency".

Performance Parameter
Various types of performance parameter-
1. MAE- Mean Absolute Error
2. SSE- Sum of Square Error
3. MSE- Mean Square Error
4. RMSE- Root Mean Square Error

First we discuss how to calculate the Error? The difference between the actual value and forecasted value is Error.

Error= Actual Value - Forecasted Value

SSE(Sum of Square Error)= $\sum_1^N Error = \sum_1^N (Actual\ Value - Forecasted\ Value)$

MAE(Mean Absolute Error)=$\frac{\sum(Actual\ Value - Forecasted\ Value)}{T}$

MSE(Mean Square Error)=$\frac{\sum(Actual\ Value - Forecasted\ Value)^2}{T}$

RMSE(Root of Mean Square Error)=$\sqrt{\frac{\sum(Actual\ Value - Forecasted\ Value)^2}{T}}$

Methods

1. Time Series Forecasting Methods

The time series forecasting is the method where series of observations on variable calculated at successive points of times or consecutive same interval of times. The time span can be taken as every minutes, hours, days, weeks, months, years, or any other regular time intervals. The pattern of data is an essential factor or parameter in understanding how the time series has behaved in the past. If such behaviour can be expected to continue in the future, we can use the past data-pattern for guiding us to find out the best forecasting methods.

Various Methods of Time Series Forecasting-
1. Moving Average
2. Weighted Moving Average
3. Exponential Smoothing

1.1. Moving Average - The Moving Average method used the average of the most recent k data values in the time series as the forecast for the next period. k can be integer value like k = 1,2,3,4,5,......... . If k=3(we take the value of k is 3),then we take the 3 recent year average dollar value to forecast the next year dollar value. More specifically if we want to forecast the dollar value of year 2015,then we take the average dollar value of 2014,2013 and 2012 where k = 3.If we want to forecast the dollar value of year 2015,then we take the average dollar value of 2014,2013,2012 and 2011 where k=4. Mathematically, a moving average forecast of order k can be expressed as follows:

Moving Average Forecast of Order k

$$F_{t+1} = \frac{\sum(most\ recent\ k\ data\ values)}{k} = \frac{Y_t + Y_{t-1} + \cdots\cdots\cdots + Y_{t-k+1}}{k}$$

Where F_{t+1} = Forecast of the time series for period t+1

Y_t = Actual value of time series in period t

1.2. Weighted Moving Average - The Weight Moving Average method involves selecting the different weight for each data value and then compute weighted average of the most recent k values as the forecast. In most case, the most recent data holds the most weight and the weight decreases for the older data value.

$F_{t+1} = \frac{1}{6}(data1) + \frac{2}{6}(data2) + \frac{3}{6}(data3)$ (We have taken 3 data for forecast)

$F_{t+1} = \frac{1}{10}(data1) + \frac{2}{10}(data2) + \frac{3}{10}(data3) + \frac{4}{10}(data4)$ (We have taken 4 data for forecast)

Where F_{t+1} = Forecast of the time series for period t+1

1.3. Exponential Smoothing - The Exponential Smoothing also uses a weighted average of past time series values as a forecast; it is a special case of weighted moving average method in which we select only one weight- the weight for the recent observation. The weight for the other data values calculated automatically and become smaller as the observations move further into the past. The Exponential Equation is as follow :

Exponential Smoothing Forecast

$F_{t+1} = \alpha Y_t + (1 - \alpha)F_t$

Where

F_{t+1} = forecast of the time series for period t+1

Y_t = actual value of the time series in period t

F_t = forecast of the time series for the period t

α = Smoothing Constant($0 \leq \alpha \leq 1$)

Mathematical Implementation of Time Series Forecasting

Dataset for Time Series Forecasting

Year	Average $ Value	YEAR	Average $ Value	YEAR	Average $ Value	YEAR	Average $ Value
2003	46.35658	2006	45.16155	2009	48.30869	2012	53.38985
2004	45.22749	2007	41.21104	2010	45.71480	2013	58.51320
2005	44.02466	2008	43.48021	2011	46.63802	2015	60.92102

We have taken the value or rate of Dollar in terms of Indian Rupee on the day basis from 1st January,2003 to 31st December,2014. That means day-wise 365*12=4380 Dollar value in term of Indian Rupee have taken as a dataset. Then we take the 30 days of dollar values and average it. Thus we get month-wise average dollar rate in terms of Indian Rupee and we get 12*12=144 data of average monthly Dollar in terms of Indian Rupee. Again we take 12 month of average dollar values and again average them then we get 12 data of yearly average Dollar value in terms of Indian Rupee.

(A). Moving Average -
(1). If k=2(If 2 average yearly dollar value have taken)

(a) Forecasted Value:-

$F_{2003} = \dfrac{48.58632 + 47.6979}{2} = 48.14211$	$F_{2004} = \dfrac{46.35658 + 48.58632}{2} = 47.471$
$F_{2005} = \dfrac{45.22749 + 46.35658}{2} = 45.7920$	$F_{2006} = \dfrac{44.02466 + 45.22749}{2} = 44.626$
$F_{2007} = \dfrac{45.16155 + 44.02466}{2} = 44.5931$	$F_{2008} = \dfrac{41.21104 + 45.16155}{2} = 43.1862$
$F_{2009} = \dfrac{43.48021 + 41.21104}{2} = 42.3456$	$F_{2010} = \dfrac{48.30869 + 43.48021}{2} = 45.8944$
$F_{2011} = \dfrac{45.71480 + 48.30869}{2} = 45.8944$	$F_{2012} = \dfrac{46.63802 + 45.71480}{2} = 46.1764$
$F_{2013} = \dfrac{53.38985 + 46.63802}{2} = 50.0139$	$F_{2014} = \dfrac{58.51320 + 53.38985}{2} = 55.9515$

(b). Error :-

$Error_{2003} = 46.35658 - 48.14211 = -1.78553$	$Error_{2004} = 45.22749 - 47.471 = -2.24351$
$Error_{2005} = 44.02466 - 45.7920 = -1.76734$	$Error_{2006} = 45.16155 - 44.626 = 0.53555$
$Error_{2007} = 41.21104 - 44.5931 = -3.38206$	$Error_{2008} = 43.48021 - 43.1862 = 0.29401$
$Error_{2009} = 48.30869 - 42.3456 = 5.96309$	$Error_{2010} = 45.71480 - 45.8944 = -0.1796$
$Error_{2011} = 46.63802 - 47.0117 = -0.37368$	$Error_{2012} = 53.38985 - 46.1764 = 7.21345$
$Error_{2013} = 58.51320 - 50.0139 = 8.4993$	$Error_{2014} = 60.92102 - 55.9515 = 4.96952$

(c).Absolute Error :-

$E_{2003} = 1.78553$	$E_{2004} = 2.24351$	$E_{2005} = 1.76734$
$E_{2006} = 0.53555$	$E_{2007} = 3.38206$	$E_{2008} = 0.29401$
$E_{2009} = 5.96302$	$E_{2010} = 0.1796$	$E_{2011} = 0.37368$
$E_{2012} = 7.21345$	$E_{2013} = 8.4993$	$E_{2014} = 4.96952$

(d). Square Forecast Error :-

$E^2_{2003} = 3.188117$	$E^2_{2004} = 5.033337$	$E^2_{2005} = 3.1234$
$E^2_{2006} = 0.2868$	$E^2_{2007} = 11.4383$	$E^2_{2008} = 0.0864$
$E^2_{2009} = 35.5576$	$E^2_{2010} = 0.03225$	$E^2_{2011} = 0.1396$
$E^2_{2012} = 52.0338$	$E^2_{2013} = 72.2381$	$E^2_{2014} = 24.69612$

(e). Mean Square Error(MSE):-

MSE=

$$\frac{3.188117+5.0333+3.1234+0.2868+11.4348+0.0864+35.5576+0.03225+0.1396+52.0338+72.2381+24.69612}{12}$$

=17.3208

(2). If k=3(If 3 average yearly dollar value have taken)

   (a) Forecasted Value:-

$F_{2003} = \dfrac{48.58632 + 47.6919 + 45.6844}{3}$ $= 47.3208$	$F_{2004} = \dfrac{46.35658 + 48.58632 + 47.6919}{3}$ $= 47.5449$
$F_{2005} = \dfrac{45.22749 + 46.35658 + 48.58632}{3}$ $= 46.723$	$F_{2006} = \dfrac{44.02466 + 45.22749 + 46.35658}{3}$ $= 45.20291$
$F_{2007} = \dfrac{45.16155 + 44.02466 + 45.22749}{3}$ $= 44.80456$	$F_{2008} = \dfrac{41.21104 + 45.16155 + 44.02466}{3}$ $= 43.46575$
$F_{2009} = \dfrac{43.48021 + 41.21104 + 45.16155}{3}$ $= 43.28426$	$F_{2010} = \dfrac{48.30869 + 43.48021 + 41.21104}{3}$ $= 44.3333$
$F_{2011} = \dfrac{45.71480 + 48.30869 + 43.48021}{3}$ $= 45.8356$	$F_{2012} = \dfrac{46.63802 + 45.71480 + 48.30869}{3}$ $= 46.88717$
$F_{2013} = \dfrac{53.38985 + 46.63802 + 45.71480}{3}$ $= 48.5808$	$F_{2014} = \dfrac{58.5132 + 53.38985 + 46.63802}{3}$ $= 52.84702$

(b).Error:-

$Error_{2003} = 46.3565 - 47.3208 = -0.96422$	$Error_{2004} =45.22749 - 47.5449 = -2.31741$
$Error_{2005} =44.02466 - 46.723= -2.69834$	$Error_{2006} =45.16155 - 45.20291 = -0.04136$
$Error_{2007} =41.21104 - 44.80456 = -3.59352$	$Error_{2008} =43.48021 - 43.46575= 0.01446$
$Error_{2009} =48.30869 - 43.28426 =5.02443$	$Error_{2010} =45.71480 - 44.3333= 1.3815$
$Error_{2011} =46.63802 - 45.8356=0.80242$	$Error_{2012} =53.38985 - 46.88717 = 6.50268$
$Error_{2013} =58.51320 - 48.5808 = 9.9324$	$Error_{2014} =60.92102 - 52.84702 = 8.074$

(c).Absolute Error:-

$Error_{2003} = 0.96422$	$Error_{2004} = 2.31741$	$Error_{2005} = 2.69834$
$Error_{2006} = 0.04136$	$Error_{2007} = 3.59352$	$Error_{2008} = 0.01446$
$Error_{2009} = 5.02443$	$Error_{2010} = 1.3815$	$Error_{2011} = 0.80242$
$Error_{2012} = 6.50268$	$Error_{2013} = 9.9324$	$Error_{2014} = 8.074$

(d). Square Forecast Error :-

$Error^2_{2003} = 0.92972$	$Error^2_{2004} = 5.3703$	$Error^2_{2005} = 7.28103$
$Error^2_{2006} = 0.0017106$	$Error^2_{2007} = 12.91338$	$Error^2_{2008} = 0.00020909$
$Error^2_{2009} = 25.24489$	$Error^2_{2010} = 1.90854$	$Error^2_{2011} = 0.64387$
$Error^2_{2012} = 42.2848$	$Error^2_{2013} = 98.65256$	$Error^2_{2014} = 65.1894$

(e).Mean Square Error(MSE) :-
MSE=

$$\frac{0.92972+5.3703+7.28103+0.0017106+12.91338+0.00020909+25.24489+1.90854+0.64387+42.2848+98.65256+65.1894}{12}=21.7017$$

(3).If k=4(If 4 average yearly dollar value have taken)

(a).Forecasted Value:-

F_{2003} $$=\frac{48.58632+47.6919+45.6844+43.3327}{4}$$ $= 46.3238$	F_{2004} $$=\frac{46.35658+48.58632+47.6919+45.6844}{4}$$ $= 47.0798$
F_{2005} $$=\frac{45.22749+46.35658+48.58632+47.6919}{4}$$ $= 46.96557$	F_{2006} $$=\frac{44.02466+45.22749+46.35658+48.5863}{4}$$ $= 46.04876$
F_{2007} $$=\frac{45.15155+44.02466+45.22749+46.35658}{4}$$ $= 45.19257$	F_{2008} $$=\frac{41.21104+45.16155+44.02466+45.2274}{4}$$ $= 43.9061$
F_{2009} $$=\frac{43.48021+41.21104+45.16155+44.02466}{4}$$ $= 43.4693$	F_{2010} $$=\frac{48.30869+43.48021+41.21104+45.1615}{4}$$ $= 44.54037$
F_{2011} $$=\frac{45.71480+48.30869+43.48021+41.21104}{4}$$ $= 44.6786$	F_{2012} $$=\frac{46.63802+45.71480+48.30869+43.4802}{4}$$ $= 46.03543$
F_{2013} $$=\frac{53.38985+46.63802+45.71480+48.30869}{4}$$ $= 48.5128$	F_{2014} $$=\frac{58.51320+53.38985+46.63802+45.7148}{4}$$ $= 51.0639$

(b).Error:-

$Error_{2003} = 46.35658 - 46.3238 = 0.03278$	$Error_{2004} = 45.22749 - 47.0798 = -1.8523$
$Error_{2005} = 44.02466 - 46.96557 = -2.94091$	$Error_{2006} = 45.16155 - 46.04876 = -0.88721$
$Error_{2007} = 41.21104 - 45.19257 = -3.98153$	$Error_{2008} = 43.48021 - 43.9061 = -0.42589$
$Error_{2009} = 48.30869 - 43.4693 = 4.83939$	$Error_{2010} = 45.71480 - 44.54037 = 1.17443$
$Error_{2011} = 46.63802 - 44.6786 = 1.95942$	$Error_{2012} = 53.38985 - 46.03543 = 7.35442$
$Error_{2013} = 58.51320 - 48.5128 = 10.0004$	$Error_{2014} = 60.92102 - 51.0639 = 9.85712$

(c).Absolute Error:-

$Error_{2003} = 0.03278$	$Error_{2004} = 1.8523$	$Error_{2005} = 2.94091$
$Error_{2006} = 0.88721$	$Error_{2007} = 3.98153$	$Error_{2008} = 0.42589$
$Error_{2009} = 4.83939$	$Error_{2010} = 1.17443$	$Error_{2011} = 1.95942$
$Error_{2012} = 7.35442$	$Error_{2013} = 10.0004$	$Error_{2014} = 9.85712$

(d). Square Forecast Error:-

$Error_{2003}^2 = 0.001074$	$Error_{2004}^2 = 3.431015$	$Error_{2005}^2 = 8.6489$
$Error_{2006}^2 = 0.78714$	$Error_{2007}^2 = 15.85258$	$Error_{2008}^2 = 0.18138$
$Error_{2009}^2 = 23.41969$	$Error_{2010}^2 = 1.37928$	$Error_{2011}^2 = 3.83932$
$Error_{2012}^2 = 54.08749$	$Error_{2013}^2 = 100.008$	$Error_{2014}^2 = 97.1628$

(e).Mean Square Error(MSE):-

MSE=

$$\frac{0.001074+3.431015+8.6489+0.78714+15.85258+0.18138+23.41969+1.37928+3.83932+54.08749+100.008+97.1628}{12}$$

=25.7332

(B).Weighted Moving Average

(1).For 2 Data(3 average yearly dollar values have taken)

(a).Forecasted Value:-

$F_{2003} = \frac{1}{3}(47.6919) + \frac{2}{3}(48.58632) = 15.8973 + 32.3908 = 48.2881$
$F_{2004} = \frac{1}{3}(48.58632) + \frac{2}{3}(46.35658) = 16.19544 + 30.90438 = 47.0998$
$F_{2005} = \frac{1}{3}(46.35658) + \frac{2}{3}(45.22749) = 15.45219 + 30.15166 = 45.60385$
$F_{2006} = \frac{1}{3}(45.22749) + \frac{2}{3}(44.02466) = 15.0758 + 29.34977 = 44.42557$
$F_{2007} = \frac{1}{3}(44.02466) + \frac{2}{3}(45.16155) = 14.67488 + 30.1077 = 44.78258$
$F_{2008} = \frac{1}{3}(45.16155) + \frac{2}{3}(41.21104) = 15.05385 + 27.474026 = 42.52787$
$F_{2009} = \frac{1}{3}(41.21104) + \frac{2}{3}(43.48021) = 13.73701 + 28.9868 = 42.72381$
$F_{2010} = \frac{1}{3}(43.48021) + \frac{2}{3}(48.30869) = 14.4934 + 32.20579 = 46.69919$
$F_{2011} = \frac{1}{3}(48.30869) + \frac{2}{3}(45.71480) = 1610289 + 30.47653 = 46.57942$
$F_{2012} = \frac{1}{3}(45.71480) + \frac{2}{3}(46.63802) = 15.23826 + 31.09201 = 46.33027$
$F_{2013} = \frac{1}{3}(46.63802) + \frac{2}{3}(53.38985) = 15.546 + 35.59323 = 51.1392$
$F_{2014} = \frac{1}{3}(53.38985) + \frac{2}{3}(58.51320) = 17.7966 + 39.0088 = 56.8054$

(b).Error:-

E_{2003}=46.35658-48.2281= -1.87152	E_{2004}=45.22749-47.0998= -1.87231
E_{2005}=44.02466-45.60385= -1.57919	E_{2006}=45.16155-44.42557=0.73598
E_{2007}=41.21104-44.78258= -3.57154	E_{2008}=43.48021-42.5278=0.95234
E_{2009}=48.30869-42.72381=5.58488	E_{2010}=45.71480-46.69919=-0.98439
E_{2011}=46.63802-46.57942=0.0586	E_{2012}=53.38985-46.33027=7.05958
E_{2013}=58.51320-51.1392=7.374	E_{2014}=60.92102-56.8054=4.11562

(c).Absolute Error:-

E_{2003}=1.87152	E_{2004}=1.87231	E_{2005}=1.57919
E_{2006}=0.73598	E_{2007}=3.57154	E_{2008}=0.95234
E_{2009}=5.58488	E_{2010}=0.98439	E_{2011}=0.0586
E_{2012}=7.05958	E_{2013}=7.374	E_{2014}=4.11562

(d).Square Forecast Error:-

E^2_{2003}=3.50258	E^2_{2004}=3.50554	E^2_{2005}=2.49384
E^2_{2006}=0.54166	E^2_{2007}=12.7558	E^2_{2008}=0.90695
E^2_{2009}=31.1908	E^2_{2010}=0.96902	E^2_{2011}=0.003434
E^2_{2012}=49.83766	E^2_{2013}=54.3758	E^2_{2014}=16.9383

(e).Mean Square Error:-
MSE=
$$\frac{3.50258+3.50554+2.49384+0.54166+12.7558+0.90695+31.1908+0.96902+0.003434+49.83766+54.3758+16.9383}{12}=14.7517$$

(2). For 3 Data(3 average yearly dollar values have taken)

(a). Forecasted Value:-

$F_{2003} = \frac{1}{6}(\text{data1}) + \frac{2}{6}(\text{data2}) + \frac{3}{6}(\text{data3}) = \frac{1}{6}(45.6844) + \frac{1}{3}(47.6919) + \frac{1}{2}(48.58632)$ $= 7.61406 + 15.8973 + 24.29316 = 47.80452$
$F_{2004} = \frac{1}{6}(47.6919) + \frac{1}{3}(48.58632) + \frac{1}{2}(46.35658) = 7.94865 + 16.19544 + 23.17829$ $= 47.32238$
$F_{2005} = \frac{1}{6}(48.58632) + \frac{1}{3}(46.35658) + \frac{1}{2}(45.22749) = 8.09772 + 15.45219 + 22.61374$ $= 46.16365$
$F_{2006} = \frac{1}{6}(46.35658) + \frac{1}{3}(45.22749) + \frac{1}{2}(44.02466) = 7.72609 + 15.07583 + 222.01233$ $= 44.81425$
$F_{2007} = \frac{1}{6}(45.22749) + \frac{1}{3}(44.02466) + \frac{1}{2}(45.16155) = 7.5379 + 14.67488 + 22.580775$ $= 42.79355$
$F_{2008} = \frac{1}{6}(44.02466) + \frac{1}{3}(45.16155) + \frac{1}{2}(41.21104) = 7.33744 + 15.05385 + 20.60552$ $= 42.99681$

$F_{2009} = \frac{1}{6}(45.16155) + \frac{1}{3}(41.21104) + \frac{1}{2}(43.48021) = 7.526925 + 13.73701 + 21.740104$ $= 43.00404$
$F_{2010} = \frac{1}{6}(41.21104) + \frac{1}{3}(43.48021) + \frac{1}{2}(48.30869) = 6.868506 + 14.493403 + 24.154345$ $= 45.516254$
$F_{2011} = \frac{1}{6}(43.48021) + \frac{1}{3}(48.30869) + \frac{1}{2}(45.71480) = 7.2467 + 16.102896 + 22.8574$ $= 46.206996$
$F_{2012} = \frac{1}{6}(48.30869) + \frac{1}{3}(45.71480) + \frac{1}{2}(46.63802) = 8.051448 + 15.23826 + 23.31901$ $= 46.608718$
$F_{2013} = \frac{1}{6}(45.71480) + \frac{1}{3}(46.63802) + \frac{1}{2}(53.38985) = 7.619133 + 15.546006 + 26.6949$ $= 49.860039$
$F_{2014} = \frac{1}{6}(46.63802) + \frac{1}{3}(53.38985) + \frac{1}{2}(58.51320) = 7.773003 + 17.796616 + 29.2566$ $= 54.826219$

(b).Error:-

$Error_{2003} = 46.35658 - 47.80452 = -1.44794$	$Error_{2004} = 45.227449 - 47.32238 = -2.09489$
$Error_{2005} = 44.02466 - 46.16365 = -2.13899$	$Error_{2006} = 45.16155 - 44.81425 = 0.3473$
$Error_{2007} = 41.21104 - 42.79355 = -1.58251$	$Error_{2008} = 43.48021 - 42.99681 = 0.4834$
$Error_{2009} = 48.30869 - 43.00404 = 5.30465$	$Error_{2010} = 45.71480 - 45.516254 = 0.198546$
$Error_{2011} = 46.63802 - 46.206996 = 0.431024$	$Error_{2012} = 53.51320 - 49.860039 = 6.781132$
$Error_{2013} = 58.51320 - 49.860039 = 8.653161$	$Error_{2014} = 60.92102 - 54.826219 = 6.094801$

(c). Absolute Error:-

$Error_{2003} = 1.44794$	$Error_{2004} = 2.09489$	$Error_{2005} = 2.13899$
$Error_{2006} = 0.3473$	$Error_{2007} = 1.58251$	$Error_{2008} = 0.4834$
$Error_{2009} = 5.30465$	$Error_{2010} = 0.198546$	$Error_{2011} = 0.431024$
$Error_{2012} = 6.781132$	$Error_{2013} = 8.653161$	$Error_{2014} = 6.094801$

(d). Square Forecast Error:-

$Error_{2003}^2 = 2.09653$	$Error_{2004}^2 = 4.38856$	$Error_{2005}^2 = 4.57527$
$Error_{2006}^2 = 0.120617$	$Error_{2007}^2 = 2.504337$	$Error_{2008}^2 = 0.23367$
$Error_{2009}^2 = 28.139311$	$Error_{2010}^2 = 0.0394205$	$Error_{2011}^2 = 0.18578$
$Error_{2012}^2 = 45.98375$	$Error_{2013}^2 = 74.87719$	$Error_{2014}^2 = 37.146599$

(e). Mean Square Error(MSE):-

MSE=

$$\frac{2.09653 + 4.38856 + 4.57527 + 0.120617 + 2.504337 + 0.23367 + 28.139311 + 0.0394205 + 0.18578 + 45.98375 + 74.87719 + 37.146599}{12}$$

$= 16.6909$

(3).For 4 Data(4 average yearly dollar values have taken) :-

(a). Forecasted Value:-

$F_{2003} = \dfrac{1}{10}(43.3327) + \dfrac{1}{5}(45.6844) + \dfrac{3}{10}(47.6919) + \dfrac{2}{5}(48.58632)$ $= 4.33327 + 9.13688 + 14.30757 + 19.434528 = 47.212248$
$F_{2004} = \dfrac{1}{10}(45.6844) + \dfrac{1}{5}(47.6919) + \dfrac{3}{10}(48.58632) + \dfrac{2}{5}(46.35658)$ $= 4.56844 + 9.53838 + 14.575896 + 18.542632 = 47.225348$
$F_{2005} = \dfrac{1}{10}(47.6919) + \dfrac{1}{5}(48.58632) + \dfrac{3}{10}(46.35658) + \dfrac{2}{5}(45.22749)$ $= 4.76919 + 9.717264 + 13.906974 + 18.09097 = 46.484398$
$F_{2006} = \dfrac{1}{10}(48.58632) + \dfrac{1}{5}(46.35658) + \dfrac{3}{10}(45.22749) + \dfrac{2}{5}(44.02466)$ $= 4.858632 + 9.271316 + 13.568247 + 17.609864 = 45.308059$
$F_{2007} = \dfrac{1}{10}(46.35658) + \dfrac{1}{5}(45.22749) + \dfrac{3}{10}(44.02466) + \dfrac{2}{5}(45.16155)$ $= 4.635658 + 9.045498 + 13.20739 + 18.06462 = 44.953166$
$F_{2008} = \dfrac{1}{10}(45.22749) + \dfrac{1}{5}(44.02466) + \dfrac{3}{10}(45.16155) + \dfrac{2}{5}(41.21104)$ $= 4.522749 + 8.804932 + 13.548465 + 16.484416 = 43.360562$
$F_{2009} = \dfrac{1}{10}(44.02466) + \dfrac{1}{5}(45.16155) + \dfrac{3}{10}(41.21104) + \dfrac{2}{5}(43.48021)$ $= 4.402466 + 9.03231 + 12.363312 + 17.392084 = 43.190172$
$F_{2010} = \dfrac{1}{10}(45.16155) + \dfrac{1}{5}(41.21104) + \dfrac{3}{10}(43.48021) + \dfrac{2}{5}(48.30869)$ $= 4.516155 + 8.242208 + 13.044063 + 19.323476 = 45.125902$
$F_{2011} = \dfrac{1}{10}(41.21104) + \dfrac{1}{5}(43.48021) + \dfrac{3}{10}(48.30869) + \dfrac{2}{5}(45.71480)$ $= 4.121104 + 8.696042 + 14.492607 + 18.28592 = 45.595673$
$F_{2012} = \dfrac{1}{10}(43.48021) + \dfrac{1}{5}(48.30869) + \dfrac{3}{10}(45.71480) + \dfrac{2}{5}(46.63802)$ $= 4.348021 + 9.661738 + 13.71444 + 18.655208 = 46.379407$
$F_{2013} = \dfrac{1}{10}(48.30869) + \dfrac{1}{5}(45.71480) + \dfrac{3}{10}(46.63802) + \dfrac{2}{5}(53.38985)$ $= 4.830869 + 9.14296 + 13.991406 + 21.35594 = 49.321175$
$F_{2014} = \dfrac{1}{10}(45.71480) + \dfrac{1}{5}(46.63802) + \dfrac{3}{10}(53.38985) + \dfrac{2}{5}(58.51320)$ $= 4.571480 + 9.327604 + 16.016955 + 23.40528 = 53.321319$

(b).Error:-

$\text{Error}_{2003} = 46.35658 - 47.212248 = -0.855668$	$E_{2004} = 45.22749 - 47.225348 = -1.997858$
$E_{2005} = 44.02466 - 46.484398 = -2.459738$	$E_{2006} = 45.16155 - 45.308059 = -0.146509$
$E_{2007} = 41.21104 - 44.953166 = -3.742126$	$E_{2008} = 43.48021 - 43.366562 = 0.113648$
$E_{2009} = 48.30869 - 43.190172 = 5.118518$	$E_{2010} = 45.71480 - 45.125902 = 0.588898$
$E_{2011} = 46.63802 - 45.595673 = 1.042347$	$E_{2012} = 53.38985 - 46.379407 = 7.010443$
$E_{2013} = 58.51320 - 49.321175 = 9.192025$	$E_{2014} = 60.92102 - 53.321319 = 7.599701$

(c). Absolute Error:-

$E_{2003} = 0.855668$	$E_{2004} = 1.997858$	$E_{2005} = 2.459738$
$E_{2006} = 0.146509$	$E_{2007} = 3.742126$	$E_{2008} = 0.113648$
$E_{2009} = 5.118518$	$E_{2010} = 0.588898$	$E_{2011} = 1.042347$
$E_{2012} = 7.010443$	$E_{2013} = 9.192025$	$E_{2014} = 9.192025$

(d). Square Forecast Error:-

$E_{2003}^2 = 0.732167$	$E_{2004}^2 = 3.9914365$	$E_{2005}^2 = 6.050311$
$E_{2006}^2 = 0.0214648$	$E_{2007}^2 = 14.003507$	$E_{2008}^2 = 0.012915$
$E_{2009}^2 = 26.199226$	$E_{2010}^2 = 0.3468008$	$E_{2011}^2 = 1.086487$
$E_{2012}^2 = 49.14631106$	$E_{2013}^2 = 84.4933236$	$E_{2014}^2 = 57.755455$

(e).Mean Square Error:-

$$MSE = \frac{0.73+3.99+6.05+0.021+14.003+0.012+26.199+0.346+1.086+49.146+84.49+57.75}{12} = 20.3199$$

(C).Exponential Smoothing :-

(1).Where the value of α=o.2:-

(a).Forecasted value:-

$F_{2003} = \alpha Y_{2002} + (1-\alpha)F_{2002} = \alpha Y_{2002} + (1-\alpha)Y_{2002} = \alpha Y_{2002} + Y_{2002} - \alpha Y_{2002} = Y_{2002}$ $= 48.58632$
$F_{2004} = \alpha Y_{2003} + (1-\alpha)F_{2003} = (0.2 * 46.35658) + (0.8 * 48.58632) = 48.140372$
$F_{2005} = \alpha Y_{2004} + (1-\alpha)F_{2005} = (0.2 * 45.22749) + (0.8 * 48.140372) = 47.5577956$
$F_{2006} = \alpha Y_{2005} + (1-\alpha)F_{2005} = (0.2 * 44.02466) + (0.8 * 47.5577956) = 46.85116848$
$F_{2007} = \alpha Y_{2006} + (1-\alpha)F_{2006} = (0.2 * 45.16155) + (0.8 * 46.85116848) = 46.51324478$
$F_{2008} = \alpha Y_{2007} + (1-\alpha)F_{2007} = (0.2 * 41.21104) + (0.8 * 46.85116848) = 45.45280382$
$F_{2009} = \alpha Y_{2008} + (1-\alpha)F_{2008} = (0.2 * 43.48021) + (0.8 * 45.45280382) = 45.05828506$
$F_{2010} = \alpha Y_{2009} + (1-\alpha)F_{2009} = (0.2 * 48.30869) + (0.8 * 45.05828506) = 45.70836605$
$F_{2011} = \alpha Y_{2010} + (1-\alpha)F_{2010} = (0.2 * 45.71480) + (0.8 * 45.70836605) = 45.70965284$
$F_{2012} = \alpha Y_{2011} + (1-\alpha)F_{2011} = (0.2 * 46.63802) + (0.8 * 45.70965284) = 45.89532627$
$F_{2013} = \alpha Y_{2012} + (1-\alpha)F_{2012} = (0.2 * 53.38985) + (0.8 * 4589532627) = 47.39423102$
$F_{2014} = \alpha Y_{2013} + (1-\alpha)F_{20013} = (0.2 * 58.51320) + (0.8 * 47.39423102) = 49.61802482$

(b).Error:-

$Error_{2003} = 46.35658 - 48.58632$ $= -2.22974$	$Error_{2004} = 45.22749 - 48.140372$ $= -2.912882$
$Error_{2005} = 44.02466 - 47.5577956$ $= -3.5331356$	$Error_{2006} = 45.16155 - 46.85116848$ $= -1.68961848$
$Error_{2007} = 41.21104 - 46.51324478$ $= -5.30220478$	$Error_{2008} = 43.48021 - 45.45280382$ $= -1.97260382$
$Error_{2009} = 48.30869 - 45.05828506$ $= 3.25040495$	$Error_{2010} = 45.71480 - 45.70836605$ $= 0.00643395$

$Error_{2011} = 46.63802 - 45.70965284$ $= 0.92836716$	$Error_{2012} = 53.38985 - 45.89532627$ $= 7.49452373$
$Error_{2013} = 58.51320 - 47.39423102$ $= 11.11896898$	$Error_{2014} = 60.92102 - 49.61802482$ $= 11.30299518$

(c). Absolute Error:-

$Error_{2003} = 2.22974$	$Error_{2004} = 2.912882$	$Error_{2005} = 3.5331356$
$Error_{2006} = 1.68961848$	$Error_{2007} = 5.30220478$	$Error_{2008} = 1.97260382$
$Error_{2009} = 3.25040495$	$Error_{2010} = 0.00643395$	$Error_{2011} = 0.92836716$
$Error_{2012} = 7.49452373$	$Error_{2013} = 11.11896898$	$Error_{2014} = 11.30299518$

(d).Square Forecast Error:-

$Error_{2003}^2 = 4.971740468$	$Error_{2004}^2 = 8.484881546$	$Error_{2005}^2 = 12.48304717$
$Error_{2006}^2 = 2.854810608$	$Error_{2007}^2 = 28.11337553$	$Error_{2008}^2 = 3.891165831$
$Error_{2009}^2 = 10.56513234$	$Error_{2010}^2 = 0.0000413957$	$Error_{2011}^2 = 0.86186558$
$Error_{2012}^2 = 56.16788594$	$Error_{2013}^2 = 123.6314712$	$Error_{2014}^2 = 127.7577$

(e).Mean Square Error(MSE):-

MSE=
$$\frac{4.971+8.484+12.483+2.854+28.113+3.891+10.565+0.0000417+0.862+56.167+123.631+127.7577}{12} = 31.64859$$

(2).Where the value of α=0.3:-

(a).Forecasted Value:-

$F_{2003} = \alpha Y_{2002} + (1 - \alpha)F_{2002} = \alpha Y_{2002} + (1 - \alpha)Y_{2002} = \alpha Y_{2002} + Y_{2002} - \alpha Y_{2002} = Y_{2002}$ $= 48.58632$
$F_{2004} = \alpha Y_{2003} + (1 - \alpha)F_{2003} = (0.3 * 46.35658) + (0.7 * 48.58632) = 47.917398$
$F_{2005} = \alpha Y_{2004} + (1 - \alpha)F_{2005} = (0.3 * 45.22749) + (0.7 * 47.917398) = 47.1104256$
$F_{2006} = \alpha Y_{2005} + (1 - \alpha)F_{2005} = (0.3 * 44.02466) + (0.7 * 47.11042256) = 46.18469592$
$F_{2007} = \alpha Y_{2006} + (1 - \alpha)F_{2006} = (0.3 * 45.16155) + (0.7 * 46.18469592) = 44.87775214$
$F_{2008} = \alpha Y_{2007} + (1 - \alpha)F_{2007} = (0.3 * 41.21104) + (0.7 * 44.87775214) = 44.4777385$
$F_{2009} = \alpha Y_{2008} + (1 - \alpha)F_{2008} = (0.3 * 43.48021) + (0.7 * 44.4777385) = 44.17847995$
$F_{2010} = \alpha Y_{2009} + (1 - \alpha)F_{2009} = (0.3 * 48.30869) + (0.7 * 44.17847995) = 45.41754297$
$F_{2011} = \alpha Y_{2010} + (1 - \alpha)F_{2010} = (0.3 * 45.71480) + (0.7 * 45.41754297) = 45.50672008$
$F_{2012} = \alpha Y_{2011} + (1 - \alpha)F_{2011} = (0.3 * 46.63802) + (0.7 * 45.50672008) = 45.84611006$
$F_{2013} = \alpha Y_{2012} + (1 - \alpha)F_{2012} = (0.3 * 53.38985) + (0.7 * 45.84611006) = 47.9446592$
$F_{2014} = \alpha Y_{2013} + (1 - \alpha)F_{20013} = (0.3 * 58.51320) + (0.7 * 47.9446592) = 51.11522144$

(b).Error:-

$\text{Error}_{2003} = 46.35658 - 48.58632 = -2.22974$	$\text{Error}_{2004} = 44.22749 - 47.917398$ $= -3.689908$
$\text{Error}_{2005} = 44.02466 - 47.1104256$ $= -3.0857656$	$\text{Error}_{2006} = 45.16155 - 46.18469592$ $= -1.02314592$
$\text{Error}_{2007} = 41.2110 - 45.877752$ $= -4.66671214$	$\text{Error}_{2008} = 43.48021 - 44.4777385$ $= -0.9975285$
$\text{Error}_{2009} = 48.30869 - 44.17847995$ $= 4.13021005$	$\text{Error}_{2010} = 45.71480 - 45.41754297$ $= 0.297257$
$\text{Error}_{2011} = 46.63802 - 45.50672008$ $= 1.1312999$	$\text{Error}_{2012} = 53.38985 - 45.84611006$ $= 7.5437399$
$\text{Error}_{2013} = 58.51320 - 47.9446592$ $= 10.5685408$	$\text{Error}_{2014} = 60.92102 - 51.11522144$ $= 9.80579856$

(c).Absolute Error:-

$\text{Error}_{2003} = 2.22974$	$\text{Error}_{2004} = 3.689908$	$\text{Error}_{2005} = 3.0857656$
$\text{Error}_{2006} = 1.02314592$	$\text{Error}_{2007} = 4.66671214$	$\text{Error}_{2008} = 0.9975285$
$\text{Error}_{2009} = 4.13021005$	$\text{Error}_{2010} = 0.297257$	$\text{Error}_{2011} = 1.1312999$
$\text{Error}_{2012} = 7.5437399$	$Error_{2013} = 10.5685408$	$\text{Error}_{2014} = 9.80579856$

(d).Absolute Square Error:-

$Error^2_{2003} = 4.971740468$	$\text{Error}^2_{2004} = 13.61542105$	$Error^2_{2005} = 9.521949338$
$\text{Error}^2_{2006} = 1.046827574$	$Error^2_{2007} = 21.7782022$	$\text{Error}^2_{2008} = 0.9950631083$
$Error^2_{2009} = 17.05863506$	$\text{Error}^2_{2010} = 0.08836172405$	$Error^2_{2011} = 1.279839464$
$\text{Error}^2_{2012} = 56.90801168$	$Error^2_{2013} = 111.6940546$	$\text{Error}^2_{2014} = 96.1536854$

(e).Mean Square Error:-

MSE=
$$\frac{4.97174+13.61542+9.52194+1.04682+21.77820+0.99506+17.05863+0.08836+1.27983+56.90801+111.69405+96.15368}{12} =$$
27.92597

(3).Where the value of α=o.4:-

(a).Forecasted Value:-

$F_{2003} = \alpha Y_{2002} + (1-\alpha)F_{2002} = \alpha Y_{2002} + (1-\alpha)Y_{2002} = \alpha Y_{2002} + Y_{2002} - \alpha Y_{2002} = Y_{2002}$ $= 48.58632$
$F_{2004} = \alpha Y_{2003} + (1-\alpha)F_{2003} = (0.4*46.35658) + (0.6*48.58632) = 47.694422$
$F_{2005} = \alpha Y_{2004} + (1-\alpha)F_{2005} = (0.4*45.22749) + (0.6*47.694422) = 46.7076492$
$F_{2006} = \alpha Y_{2005} + (1-\alpha)F_{2005} = (0.4*44.02466) + (0.6*46.7076492) = 45.63445352$
$F_{2007} = \alpha Y_{2006} + (1-\alpha)F_{2006} = (0.4*45.16155) + (0.6*45.63445352) = 45.44529211$
$F_{2008} = \alpha Y_{2007} + (1-\alpha)F_{2007} = (0.4*41.21104) + (0.6*45.44529211) = 43.75159127$
$F_{2009} = \alpha Y_{2008} + (1-\alpha)F_{2008} = (0.4*43.48021) + (0.6*43.75159127) = 43.64303876$
$F_{2010} = \alpha Y_{2009} + (1-\alpha)F_{2009} = (0.4*48.30869) + (0.6*45.50929926) = 45.50929926$

$F_{2011} = \alpha Y_{2010} + (1-\alpha)F_{2010} = (0.4 * 45.71480) + (0.6 * 45.50929926) = 45.59149956$
$F_{2012} = \alpha Y_{2011} + (1-\alpha)F_{2011} = (0.4 * 46.63802) + (0.6 * 45.59149956) = 46.01010774$
$F_{2013} = \alpha Y_{2012} + (1-\alpha)F_{2012} = (0.4 * 53.38985) + (0.6 * 46.01010774) = 48.962004$
$F_{2014} = \alpha Y_{2013} + (1-\alpha)F_{2013} = (0.4 * 58.51320) + (0.6 * 48.962004) = 52.7824824$

(b).Error:-

$Error_{2003} = 46.35658 - 48.58632$ $= -2.22974$	$Error_{2004} = 44.22749 - 47.694422$ $= -3.466932$
$Error_{2005} = 44.02466 - 46.7076492$ $= -2.6829892$	$Error_{2006} = 45.16155 - 45.63445352$ $= -0.47290352$
$Error_{2007} = 41.21104 - 45.44529211$ $= -4.23425211$	$Error_{2008} = 43.48021 - 43.75159127$ $= -0.27138127$
$Error_{2009} = 48.30869 - 43.64303876$ $= 4.66565124$	$Error_{2010} = 45.71480 - 45.41754297$ $= 0.297257$
$Error_{2011} = 46.63802 - 45.50672008$ $= 1.131299$	$Error_{2012} = 53.38985 - 45.84611006$ $= 7.5437399$
$Error_{2013} = 58.51320 - 47.9446592$ $= 10.5685408$	$Error_{2014} = 60.92102 - 51.11522144$ $= 9.80579856$

(c).Absolute Error:-

$Error_{2003} = 2.22974$	$Error_{2004} = 3.689908$	$Error_{2005} = 3.0857656$
$Error_{2006} = 1.02314592$	$Error_{2007} = 4.66671214$	$Error_{2008} = 0.9975285$
$Error_{2009} = 4.13021005$	$Error_{2010} = 0.297257$	$Error_{2011} = 1.131299$
$Error_{2012} = 7.5437399$	$Error_{2013} = 10.5685408$	$Error_{2014} = 9.80579856$

(d).Square Forecast Error:-

$Error_{2003}^2 = 4.971740468$	$Error_{2004}^2 = 12.01961749$	$Error_{2005}^2 = 7.198431047$
$Error_{2006}^2 = 0.2236377392$	$Error_{2007}^2 = 17.92889093$	$Error_{2008}^2 = 0.073647793$
$Error_{2009}^2 = 21.76830149$	$Error_{2010}^2 = 0.042230554$	$Error_{2011}^2 = 1.095205031$
$Error_{2012}^2 = 54.46059582$	$Error_{2013}^2 = 91.22534503$	$Error_{2014}^2 = 66.23579427$

(e).Mean Square Error(MSE):-

$$MSE = \frac{4.971+12.019+7.198+0.223+17.928+0.073+21.768+0.0422+1.095+54.460+91.225+66.235}{12}$$

$$= 23.1036$$

Result For Mathematical Implementation of Time Series Forecasting

1. Moving Average (Time Series Forecasting) :-
(1.1). k=2(Value of k=2)

Year	Actual $ Value (INR)	Forecasted $ Value (INR)	Error	Absolute Error	Square Forecast Error	Mean Square Error(MSE)	Root Mean Square Error(RMSE)
2003	46.35	48.14	-1.78	1.78	3.18		
2004	45.22	47.47	-2.24	2.24	5.03		
2005	44.02	45.79	-1.76	1.76	3.12		
2006	45.16	44.62	+0.53	0.53	0.28	17.32	4.16
2007	41.21	44.59	-3.38	3.38	11.43		
2008	43.48	43.18	+0.29	0.29	0.08		
2009	48.30	42.34	+5.96	5.96	35.55		
2010	45.71	45.89	-0.17	0.17	0.03		
2011	46.63	47.01	-0.37	0.37	0.13		
2012	53.38	46.17	+7.21	7.21	52.03		
2013	58.51	50.01	+8.49	8.49	72.23		
2014	60.92	55.95	+4.96	4.96	24.69		

(1.2). k=3(Value of k=3)

Year	Actual $ Value (INR)	Forecasted $ Value (INR)	Error	Absolute Error	Square Forecast Error	Mean Square Error(MSE)	Root Mean Square Error (RMSE)
2003	46.35	47.32	-0.96	0.96	0.92		
2004	45.22	47.54	-2.31	2.31	5.37		
2005	44.02	46.72	-2.69	2.69	7.28		
2006	45.16	45.20	-0.04	0.04	0.001		
2007	41.21	44.80	-3.59	3.59	12.91	21.70	4.65
2008	43.48	43.46	+0.01	0.01	0.0002		
2009	48.30	43.28	+5.02	5.02	25.24		
2010	45.71	44.33	+1.38	1.38	1.90		
2011	46.63	45.83	+0.80	0.80	0.64		
2012	53.38	46.88	+6.50	6.50	42.28		
2013	58.51	48.58	+9.93	9.93	98.65		
2014	60.92	52.84	+8.07	8.07	65.18		

(1.3). k=4(Value of k=4)

Year	Actual Value (INR)	Forecasted Value(INR)	Error	Absolute Error	Square Forecast Error	Mean Square Error(MSE)	Root Mean Square Error (RMSE)
2003	46.35	46.32	+0.03	0.03	0.001		
2004	45.22	47.07	-1.85	1.85	3.43		
2005	44.02	46.96	-2.94	2.94	8.64	25.73	5.07
2006	45.16	46.04	-0.88	0.88	0.78		
2007	41.21	45.19	-3.98	3.98	15.85		
2008	43.48	43.90	-0.42	0.42	0.18		

Year							
2009	48.30	43.46	+4.83	4.83	23.41		
2010	45.71	44.54	+1.17	1.17	1.37		
2011	46.63	44.67	+1.95	1.95	3.83		
2012	53.38	46.03	+7.35	7.35	54.08		
2013	58.51	48.51	+10.00	10.00	100.00		
2014	60.92	51.06	+9.85	9.85	97.16		

2. Weighted Moving Average(Time Series Forecasting) –

(2.1). If 2 years of average Dollar Value

Year	Actual Value (INR)	Forecasted Value(INR)	Error	Absolute Error	Square Forecast Error	Mean Square Error (MSE)	Root Mean Square Error (RMSE)
2003	46.35	48.28	-1.87	1.87	3.50		
2004	45.22	47.09	-1.87	1.87	3.50		
2005	44.02	45.60	-1.57	1.57	2.49		
2006	45.16	44.42	+0.73	0.73	0.54	14.75	3.84
2007	41.21	44.78	-3.57	3.57	12.75		
2008	43.48	42.52	+0.95	0.95	0.90		
2009	48.30	42.72	+5.58	5.58	31.19		
2010	45.71	46.69	-0.98	0.98	0.96		
2011	46.63	46.57	+0.05	0.05	0.0034		
2012	53.38	46.33	+7.05	7.05	49.83		
2013	58.51	51.13	+7.37	7.37	54.37		
2014	60.92	56.80	+4.11	4.11	16.93		

(2.2). If 3 years of average Dollar Value

Year	Actual Value (INR)	Forecasted Value(INR)	Error	Absolute Error	Square Forecast Error	Mean Square Error(MSE)	Root Mean Square Error (RMSE)
2003	46.35	47.80	-1.44	1.44	2.09		
2004	45.22	47.32	-2.09	2.09	4.38		
2005	44.02	46.16	-2.13	2.13	4.57		
2006	45.16	44.81	+0.34	0.34	0.12	16.69	4.08
2007	41.21	44.79	-1.58	1.58	2.50		
2008	43.48	42.99	+0.48	0.48	0.23		
2009	48.30	43.004	+5.30	5.30	28.13		
2010	45.71	45.51	+0.19	0.19	0.039		
2011	46.63	46.20	+0.43	0.43	0.18		
2012	53.38	46.60	+6.78	6.78	45.98		
2013	58.51	49.86	+8.65	8.65	74.87		
2014	60.92	54.82	+6.09	6.09	37.14		

(2.3). If 4 years of average Dollar Value

Year	Actual Value (INR)	Forecasted Value(INR)	Error	Absolute Error	Square Forecast Error	Mean Square Error(MSE)	Root Mean Square Error(RMSE)
2003	46.35	47.21	-0.85	0.85	0.73	20.31	4.50
2004	45.22	47.22	-1.99	1.99	3.99		
2005	44.02	46.48	-2.45	2.45	6.05		
2006	45.16	45.30	-0.14	0.14	0.02		
2007	41.21	44.95	+3.74	3.74	14.003		
2008	43.48	43.36	+0.11	0.11	0.012		
2009	48.30	43.19	+5.11	5.11	26.19		
2010	45.71	45.12	+0.58	0.58	0.34		
2011	46.63	45.59	+1.04	1.04	1.08		
2012	53.38	46.37	+7.01	7.01	49.14		
2013	58.51	49.32	+9.19	9.19	84.49		
2014	60.92	53.32	+7.59	7.59	57.75		

3. Exponential Smoothing(Time Series Forecasting) -

(3.1). Where value of $\alpha=0.2$ ($\alpha=0.2$)

Year	Actual Value (INR)	Forecasted Value (INR)	Error	Absolute Error	Square Forecast Error	Mean Square Error (MSE)	Root Mean Square Error (RMSE)
2003	46.35	48.48	-2.22	2.22	4.97	31.64	5.62
2004	45.22	48.14	-2.91	2.91	8.48		
2005	44.02	47.55	-3.53	3.53	12.48		
2006	45.16	46.85	-1.68	1.68	2.85		
2007	41.21	46.51	-5.30	5.30	28.11		
2008	43.48	45.45	-1.97	1.97	3.89		
2009	48.30	45.05	+3.25	3.25	10.56		
2010	45.71	45.70	+0.0064	0.0064	0.000041		
2011	46.63	45.70	+0.92	0.92	0.86		
2012	53.38	45.89	+7.49	7.49	56.16		
2013	58.51	47.39	+11.11	11.11	123.63		
2014	60.92	49.61	+11.30	11.30	127.75		

(3.2). Where value of $\alpha=0.3$ ($\alpha=0.3$)

Year	Actual Value (INR)	Forecasted Value(INR)	Error	Absolute Error	Square Forecast Error	Mean Square Error(MSE)	Root Mean Square Error
2003	46.35	48.58	-2.22	2.22	4.97	27.92	5.28
2004	45.22	47.91	-3.68	3.68	13.61		
2005	44.02	47.11	-3.08	3.08	9.52		
2006	45.16	46.18	-1.02	1.02	1.04		
2007	41.21	45.87	-4.66	4.66	21.77		

2008	43.48	44.47	-0.99	0.99	0.99		
2009	48.30	44.17	+4.13	4.13	17.05		
2010	45.71	45.41	+0.29	0.29	0.08		
2011	46.63	45.50	+1.13	1.13	1.27		
2012	53.38	45.84	+7.54	7.54	56.90		
2013	58.51	47.94	+10.56	10.57	111.69		
2014	60.92	51.11	+9.80	9.80	96.15		

(3.3). Where value of $\alpha=0.4$($\alpha=0.4$)

Year	Actual Value (INR)	Forecasted Value (INR)	Error	Absolute Error	Square Forecast Error	Mean Square Error (MSE)	Root Mean Square Error
2003	46.35	48.58	-2.22	2.22	4.97		
2004	45.22	47.69	-3.46	3.46	12.01		
2005	44.02	46.70	-2.68	2.68	7.19		
2006	45.16	45.63	-0.47	0.47	0.22	23.103	4.806
2007	41.21	45.44	-4.23	4.23	17.92		
2008	43.48	43.75	-0.27	0.27	0.073		
2009	48.30	43.64	+4.66	4.66	21.76		
2010	45.71	45.50	+0.20	0.20	0.042		
2011	46.63	45.59	+1.04	1.04	1.09		
2012	53.38	46.01	+7.37	7.37	54.46		
2013	58.51	48.96	+9.55	9.55	91.22		
2014	60.92	52.78	+8.13	8.13	66.23		

Result of Time Series Forecasting

Methods	MAE	MSE	RMSE
Moving Average(k=2)	3.100	17.321	4.16
Moving Average(k=3)	3.445	21.701	4.65
Moving Average(k=4)	3.775	25.733	5.07
Weighted Moving Average(for 2 data)	2.979	17.751	3.84
Weighted Moving Average(for 3 data)	2.963	16.690	4.08
Weighted Moving Average(for 4 data)	3.322	20.319	4.50
Exponential Smoothing($\alpha=0.2$)	4.311	31.648	5.62
Exponential Smoothing($\alpha=0.3$)	4.097	27.925	5.28
Exponential Smoothing($\alpha=0.4$)	3.695	23.103	4.06

If the value of MSE is less, then that method gives the proper forecasted result then other. From the table, it is clear that MSE value of Moving average increasing constantly with the value of k is increased. So if we take two year average dollar values for moving average, then it gives the most accurate dollar value than k=3. Because MSE value where k= 2 gives less than MSE value where k=3. In case of Exponential Smoothing, If we increase the value of α ,the MSE value decrease. So in case of α= 0.4, the forecasted value will be most accurate.
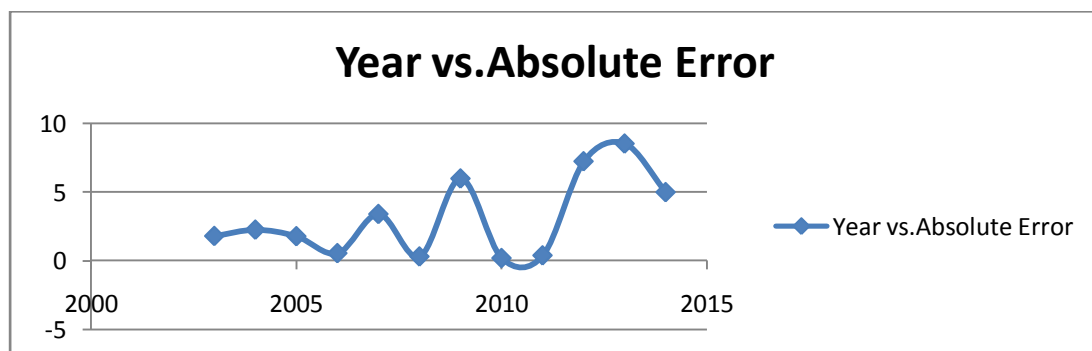
1. Moving Average(k=2)

a. Year vs. Actual Value & Year vs. Forecasted Value
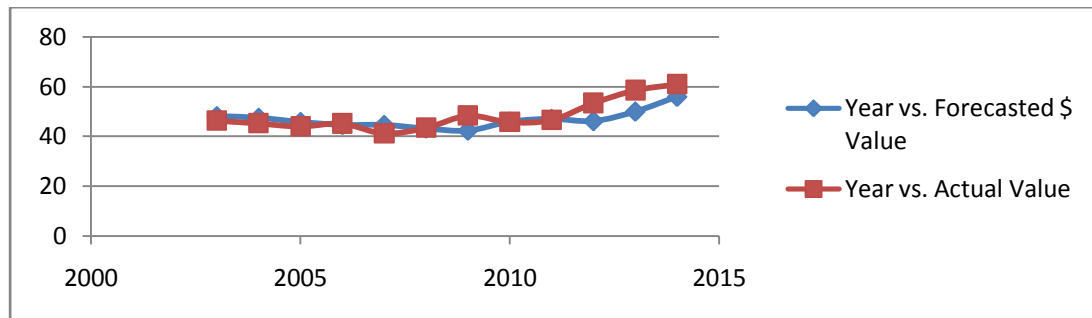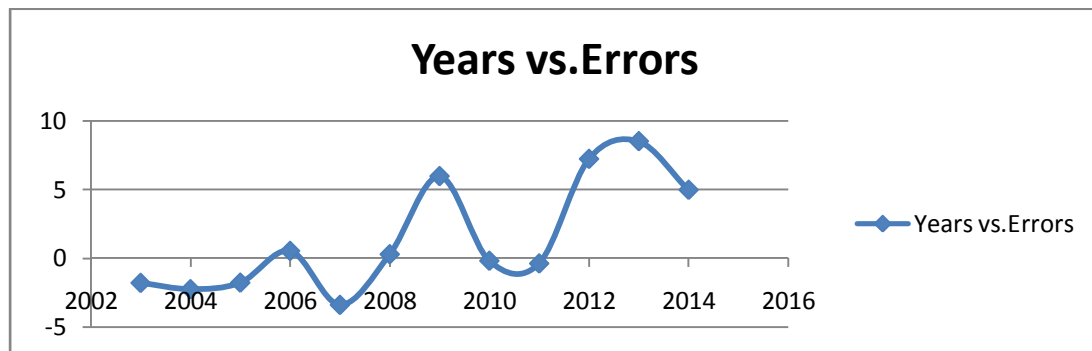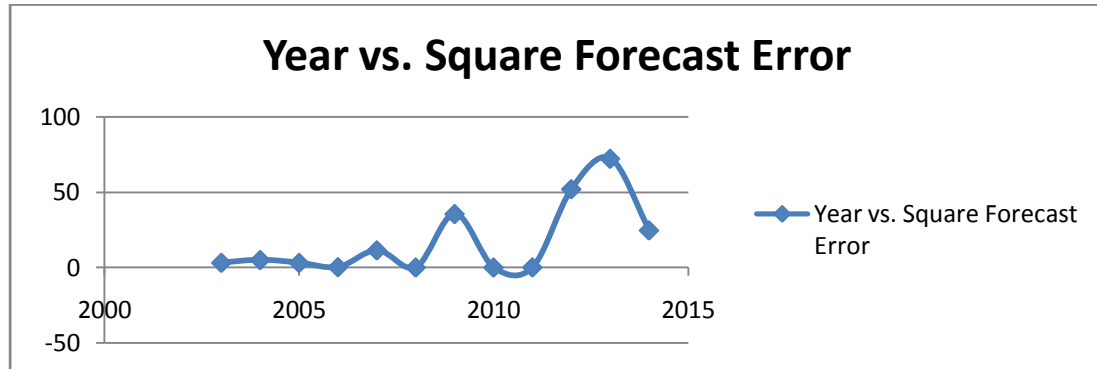

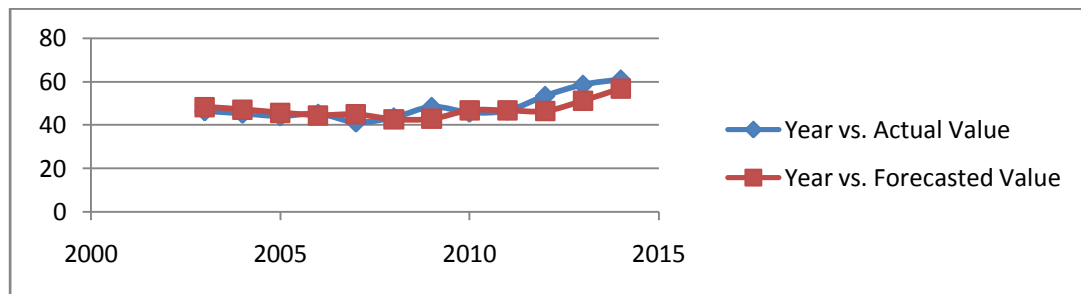b. Years vs. Errors

c. Years vs. Absolute Errors

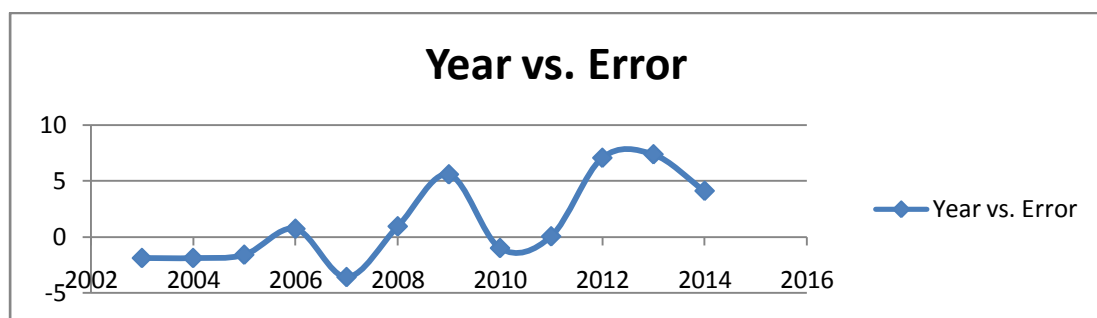d. Years vs. Square Forecast Errors

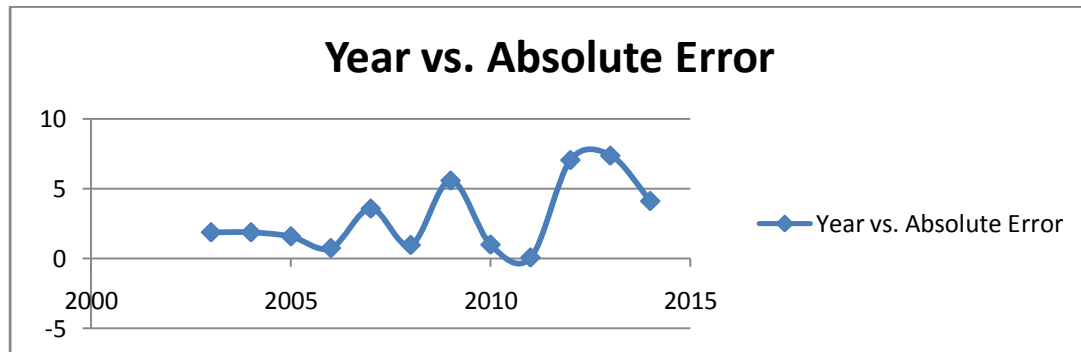4. Weighted Moving Average(For 2 Dollar Value)
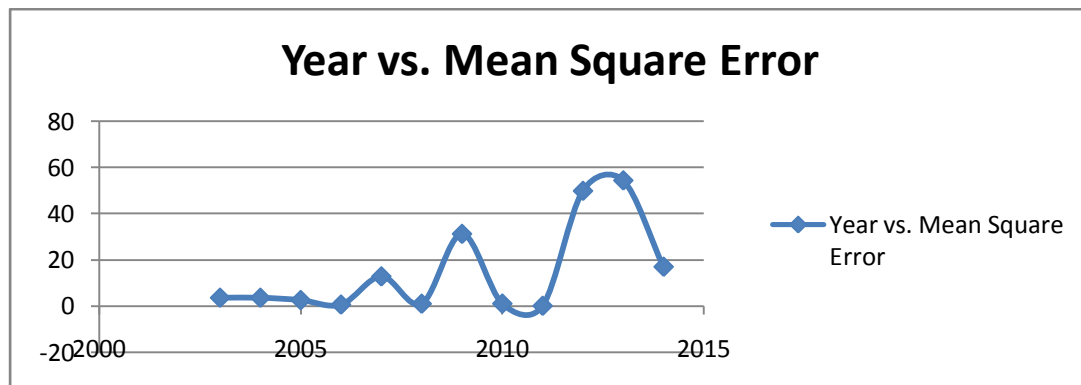
a. Year vs. Actual Value & Year vs. Forecasted Value
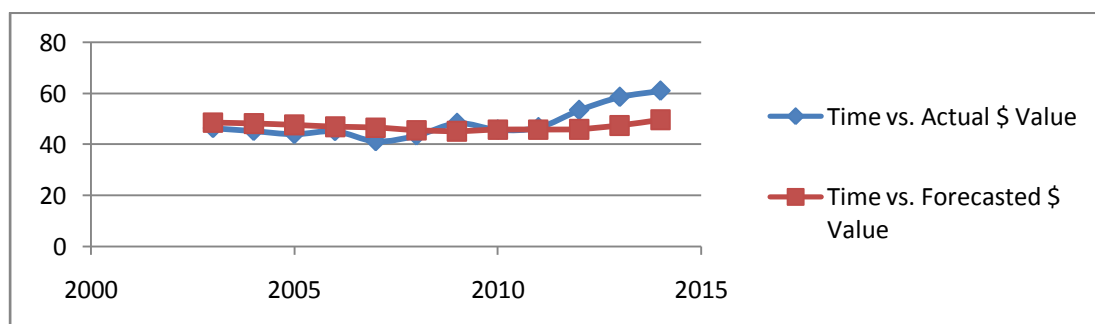

b. Years vs. Errors

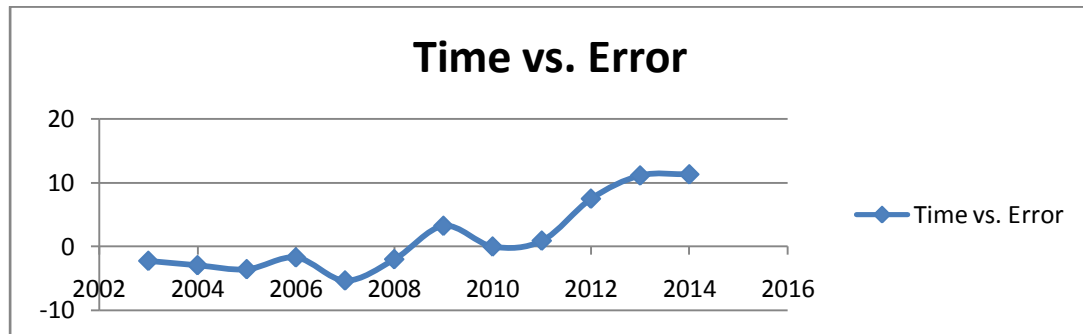c. Years vs. Absolute Errors

d. Years vs. Square Error
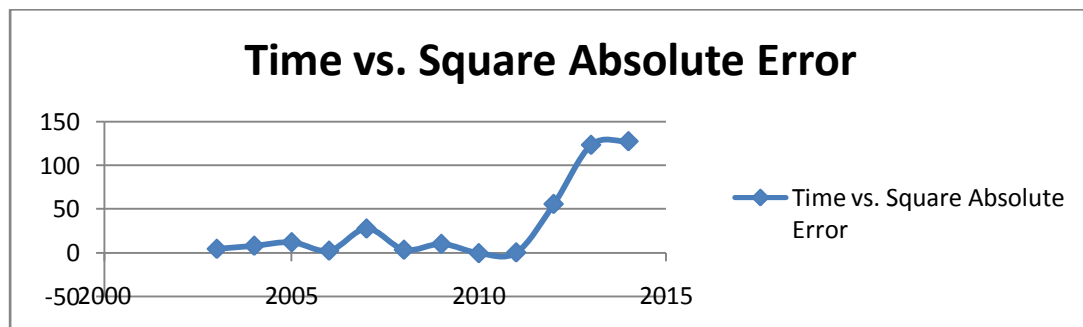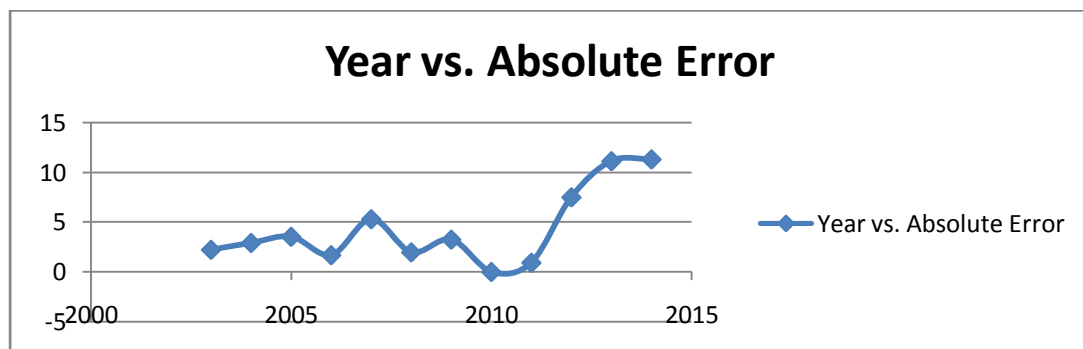

7. Exponential Smoothing -
   α=0.2
   Year vs. Actual Value & Year vs. Forecasted Value

Years vs. Error(Actual USD value - Forecasted USD value):-

Time vs. Error

Years vs. Absolute Error:-

Year vs. Absolute Error

Time vs. Square Absolute Error

Conclusion

This paper contents the mathematical implementation of time series forecasting for Exchange Rate Forecasting specifically Dollar Rate Forecasting. There are 3 types of time series methods and they have their own formulas. We take 12 years of average dollar value from 2003 to 2014 and this average dollar values are taken as actual value. Using three methods, first we find out the forecasted value and then find out the error. We use mainly two performance parameters which are Mean Square Error(MSE) and Root Mean Square Error(RMSE) for calculate the error between

forecasted value and actual value. We try to investigate the method which gives the best result with the help of these two performance parameters.

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