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Fuzzy Optimum Solution Through Pascal's Triangle Graded Mean

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Abstract

This paper deals with the identification of fuzzy optimum solution as the minimum transportation cost by applying Pascal's Triangle Graded Mean for trapezoidal fuzzy numbers through Fuzzy Induced North-West Corner Rule. This proposed method helps us to find the minimum transportation cost or fuzzy optimum solution of trapezoidal fuzzy numbers by comparing with the optimum solution of Fuzzy Graded Mean in a simple manner through suitable numerical example.

Keywords: Fuzzy Number, Optimum Solution, Pascal's Triangle, Graded Mean

Introduction

Operation Research Optimization Technique is divided into two programming problems. One is Linear Programming problem and another one is Integer Programming problem. Transportation problem is one of the sub classes of Linear Programming Problem in which the object is transported to various quantities of a single commodity that are initially stored at various origins to different destinations in such a way that the total transportation cost is minimum. The basic transportation problem was originally developed by Hitchcock [1]. Then Dantzig [3] developed the efficient methods of solution derived from the simplex algorithm in the year 1947. We get an initial basic feasible solution for the transportation problem by using North-West Corner method, Least Cost Method and Vogel's Approximation method. Charnes and Cooper [2] developed the stepping stone method which is the alternative method for finding the optimal solution. Lotfi.A.Zadeh [4] introduced the concept of fuzzy set theory in the year 1965.Chen and Hsieh [5] and [6] proposed fuzzy graded mean integration representation. In this proposed method the graded mean representation of Pascal's Triangle is used to find the fuzzy optimum solution as the minimum transportation cost with the help of fuzzy numbers using Fuzzy Induced Northwest Corner Rule. In this rule, the fuzzy numbers are taken as transportation cost in the source and destination nodes respectively. Here the sum of the demand is equal to the sum of the supply which was taken in reverse order as trapezoidal fuzzy numbers and it is suitable for fuzzy operations. In this paper the preliminaries of the transportation problem, Representation of fuzzy numbers, Graded Mean Representation, and then the description of proposed method, algorithm and the procedure for obtaining the optimum solution are illustrated by comparing the Fuzzy Graded Mean Approach with the Pascal's Triangle Graded Mean with a numerical example.

Preliminaries

Consider 'm' origins (or sources) O_i, i=1, 2, 3…m and 'n' destinations D_j, j =1, 2, 3…n. At each origin O_i, let a_i be the amount of a homogenous product that we want to transport to D_j, in order to satisfy the demand for b_j units of the product there. A penalty c_{ij} is associated with transportation of a unit of the product from source i to the destination j. The penalty could represent transportation cost, delivery time, quantity of goods delivered under-used capacity, etc. A variable x_{ij} represent the unknown quantity to be transported from origins O_i to destination D_j. However, in the real world, all transportation problems are not single objective linear programming problems. The mathematical form of the above said problem is as follows:

$$\text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \text{ subject to } \begin{array}{l} \sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,3..m \\ \sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,3..n \end{array}$$

$x_{ij} \geq 0, i = 1,2..m, j = 1,2,..n$, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ (balanced condition).The balanced condition is both a necessary and sufficient condition for the existence of a feasible solution to the transportation problems.

Representation of Generalized (Trapezoidal) Fuzzy Number

In general, a generalized fuzzy number A is described at any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions:

- μ_A is a continuous mapping from R to [0,1],
- μ_A(x)=0, $-\infty \prec x \leq c$,
- $\mu_A(x) = L(x)$ is strictly increasing on [c,a]
- $\mu_A(x) = w, a \leq x \leq b,$

- $\mu_A(x) = R(x)$ is strictly decreasing on [b,d],
- $\mu_A(x) = 0, d \le x \prec \infty$ Where $0 \prec w \le 1$ and a, b, c and d are real numbers.

We denote this type of generalized fuzzy number as $A = (c, a, b, d; w)_{LR}$. When w=1, we denote this type of generalized fuzzy number as $A = (c, a, b, d)_{LR}$. When L(x) and R(x) are straight line, then A is Trapezoidal fuzzy number, we denote it as (c, a, b, d).

Operations on Trapezoidal Fuzzy Number

The operations on Trapezoidal fuzzy numbers [9] are given as follows:

- *Addition:* $A \oplus B = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$
- *Subtraction:* $A-B = (a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1)$
- *Multiplication:*

$A \otimes B=\{\min(a_1b_1,a_1b_4,a_4b_1,a_4b_4),\min(a_2b_2,a_2b_3,a_3b_2,a_3b_3),\max(a_2b_2,a_2b_3,a_3b_2,a_3b_3), \max(a_1b_1,a_1b_4,a_4b_1,a_4b_4)\}$

Graded Mean Integration Representation

In 1998, Chen and Hsieh [5] and [6] proposed graded mean integration representation for representing generalized fuzzy number. Suppose L^{-1}, R^{-1} are inverse functions of L and R respectively, and the graded mean h-level value of generalized fuzzy number $A = (c, a, b, d; w)_{LR}$ is $h[L^{-1}(h) + R^{-1}(h)]/2$. Then the graded mean integration representation of generalized fuzzy umber based on the integral value of graded mean h-level is

$$P(A) = \frac{\int_0^w h\left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right)dh}{\int_0^w h\,dh},$$

where h is between 0 and w, 0<w ≤1;

$$P(A) = \frac{c + 2a + 2b + d}{6}.$$

Pascal's Triangle Graded Mean Approach

The Graded Mean Integration Representation for generalized fuzzy number was introduced by Chen and Hsieh [5] and [6].But the present approach is a very simple one for analyzing fuzzy variables to get the optimum solution to the transportation problem. This procedure is taken from the following Pascal's triangle. We take the coefficients of fuzzy variables as Pascal's triangle numbers.

Then we just add and divide by the total of Pascal's number and we call it as Pascal's Triangle Graded Mean Approach.

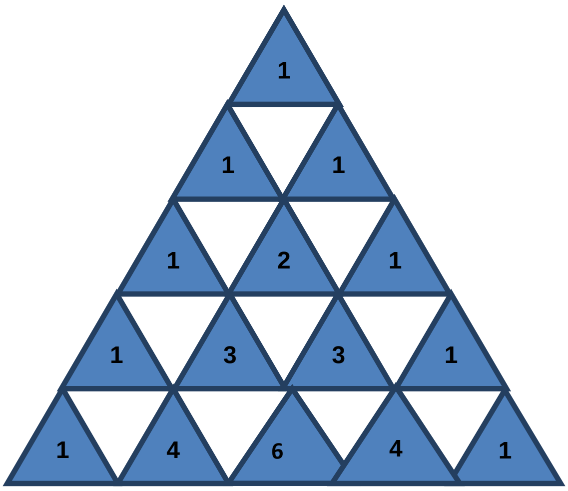


Figure 1: Pascal's Triangle

The following are the Pascal's triangular approach:

Let $A=(a_1,a_2,a_3,a_4)$ be a trapezoidal fuzzy number then we can take the coefficient of fuzzy numbers from Pascal's triangles and apply the graded mean approach we get the following formula:

$$P(A)=\frac{a_1+3a_2+3a_3+a_4}{8};$$

The coefficients of a_1,a_2,a_3,a_4 are 1, 3, 3, 1.This approach can be extended for n-dimensional Pascal's Triangular fuzzy order also.

Description of The Proposed Method

Induced Fuzzy North West Corner Rule

Consider the fuzzy transportation problem (FTP),

Minimize $Z=\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\,\tilde{x}_{ij}$ subject to

$$\sum_{j=1}^{n}\tilde{x}_{ij}\le a_i,\, i=1,2,3..m$$

$$\sum_{i=1}^{m}\tilde{x}_{ij}\ge b_j,\, j=1,2,3..n$$

$\tilde{x}_{ij}\ge 0, i=1,2..m, j=1,2,..n$, where $\tilde{a}_i=(a_1,a_2,a_3,a_4)$,

$\tilde{b}_i=(b_1,b_2,b_3,b_4)$, $\tilde{c}_{ij}=(c_{ij},c_{ij},c_{ij},c_{ij})$ represents the uncertain supply and demand for the transportation problems. Here the sum of the trapezoidal fuzzy numbers in the demand is equal to the sum of the trapezoidal fuzzy numbers in the supply which was taken in reverse order.

Algorithm

The algorithm for this Induced Fuzzy North West Corner Rule is as follows:

Step: 1 Allocate the cells with trapezoidal fuzzy number.

Step: 2 Start with a cells at the upper left (north-west) corner of the transportation matrix.

Step: 3Allocate the maximum possible value to $x_{11} = \min(a_1, b_1)$.

Step: 4 If $b_1 < a_1$, move right horizontally to the second column and make second allocation of the magnitude $x_{12} = \min(a_1 - x_{11}, b_2)$ in the cell (1, 2) using fuzzy operations.

Step: 5 If $b_1 > a_1$, move down vertically to the second row and make second allocation of the magnitude $x_{21} = \min(a_2, b_1 - x_{11})$ in the cell (2, 1) using fuzzy operations.

Step: 6 suppose if $b_1 = a_1$ then $x_{12} = \min(a_1 - a_1, b_2)$ (or) $x_{21} = \min(a_2, b_1 - b_1)$ using fuzzy operations.

Step: 7 repeat the process by moving down towards the lower right corner of transportation until the requirements are satisfied.

Step: 8 Then Total Transportation Cost = $\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\, \tilde{x}_{ij}$

Step: 8 finally apply the Pascal's Triangle Graded Mean $P(A) = \dfrac{a_1 + 3a_2 + 3a_3 + a_4}{8}$, which gives the minimum cost.

Numerical Example

The following illustration given in table: 1 and in table: 2 explain the working rule to obtain the optimum transportation cost.

Table 1:

	DESTINATION				***Supply***
SOURCE	(1,3,5,7)	(9,11,13,15)	(48,12,14)	(2,6,8,10)	**(90,80,70,35)**
	(5,7,9,11)	(2,3,4,5)	(7,8,9,10)	(2,4,10,12)	**(80,55,50,30)**
	(10,11,12,13)	(4,9,14,15)	(9,11,13,15)	(2,3,4,5)	**(80,65,45,25)**
Demand	(30,50,60,70)	(20,40,60,80)	(25,35,45,55)	(15,40,35,45)	

Here the trapezoidal fuzzy numbers are taken in the source and destination nodes respectively.

Table 2:

	DESTINATION				***Supply***
SOURCE	***(30,50,60,70)*** (1,3,5,7)	***(20,20,20,5)*** (9,11,13,15)	(48,12,14)	(2,6,8,10)	**~~(90,80,70,35)~~** **(20,20,20,5)**
	(5,7,9,11)	***(15,20,40,60)*** (2,3,4,5)	***(20,15,30,15)*** (7,8,9,10)	(2,4,10,12)	**~~(80,55,50,30)~~** **(20,15,30,15)**
	(10,11,12,13)	(4,9,14,15)	***(10,5,30,35)*** (9,11,13,15)	***(15,40,35,45)*** (2,3,4,5)	**~~(80,65,45,25)~~** **(45,35,40,15)**
Demand	**(30,50,60,70)**	**~~(20,40,60,80)~~** **(15,20,40,60)**	**~~(25,35,45,55)~~** **(10,5,30,35)**	**(15,40,35,45)**	

The Total Transportation Cost is given by

$\mathbf{T_{TC}} = (30, 50, 60, 70) \otimes (1, 3, 5, 7) + (20, 20, 20, 5) \otimes (9, 11, 13, 15) + (15,20,40,60) \otimes (2,3,4,5) + (20,15,30,15) \otimes (7,8,9,10) + (10,5,30,35) \otimes (9,11,13,15) + (15,40,35,45) \otimes (2,3,4,5)$

$= (30,150,300,490) + (45,220,260,300) + (30, 60,160,225) + (105,120,270,200) + (90, 55,390,525) + (30,105,160,225)$

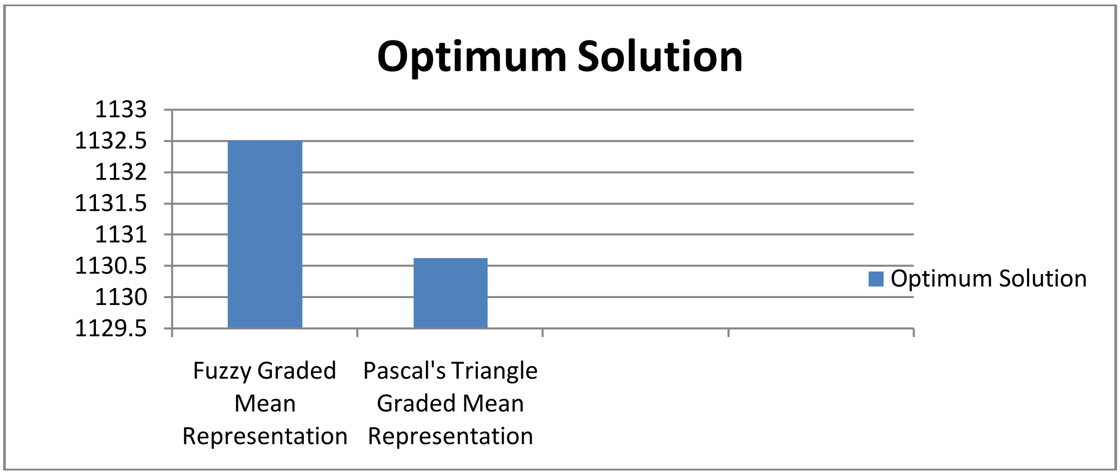
$= (75,370,560,790) + (135,180,430,425) + (120,160,550,750)$

$= (210, 550, 990, 1215) + (120,160,550,750)$

$= (330, 710, 1540,1965)$

Table 3:

Graded Mean Approach	***Calculation***	***Optimum Solution***
FGMA-Fuzzy Graded Mean Approach	$\mathbf{T_{TC}} = \frac{1}{6}\left(330 + 2(710) + 2(1540) + 1965\right)$	**1132. 5**
PTGMA-Pascal's Triangle Graded Mean Approach	$\mathbf{T_{TC}} = \frac{1}{8}\left(330 + 3(710) + 3(1540) + 1965\right)$	**1130.625**



Conclusion

The main aim of this paper is to introduce Pascal's Triangle Graded Mean in Induced Fuzzy North -West Corner Rule to obtain the optimum solution to the transportation problem. Through this simple method we obtain the minimum transportation cost by applying fuzzy operations on trapezoidal fuzzy numbers. Here we get the minimum optimum solution through Pascal's Triangle Graded Mean Approach when comparing to Fuzzy Graded Mean Approach. Finally we obtained the minimum transportation cost.

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