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Analysis of an M[x]/G/1 feedback Queue with Two stages of Repair times, General Delay Time

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ABSTRACT

In this paper an $M^{[x]}/G\ /1$ Queueing model with two stages of service is described. On completion of a service, the server will go for a vacation with probability p or remain staying back in the system for providing the service to the next customer with probability 1-p, if any. The system subject to breakdown at random. In this model, repair process does not start immediately. There is a delay in getting into the repair process. Repair takes place in two stages. Moreover service time, Vacation time, Delay time & Repair time follows general distribution. Steady state solution & Performance measures are derived.

Mathematics Subject Classification: 60K25; 60K30

Keywords: Bernoulli vacation, Probability generating function, Steady state, Two stages of service, Stages of repair, Delay time, mean queue length, mean waiting time.

Introduction

Queueing systems with server vacations and/or random system breakdowns have been studied by numerous researchers. Queueing modelling is being used enormously and effectively in congestion problems. In recent years many authors have studied batch arrival queueing systems. Choudhury and Madan [1], analysed a two stage batch arrival queueing system assuming that the server vacation is the modified Bernoulli schedule vacation under an N-policy. Chang and Ke [2], investigated an $M^{[x]}/G\ /1$ retrial queueing system with a modified vacation policy by applying the supplementary variable technique. A remarkable and excellent surveys on the earlier works of vacation models have been reported by Doshi [3], Takagi [4]. Many authors including Choudhury and Madan [5]. Anabosi and Madan [6], incorporated the concept of Bernoulli scheduled server vacation on non-Markovian queues. Maraghi et al. [7], have analysed steady state solution of batch arrival queueing system with random breakdowns and Bernoulli schedule server vacations having general vacation time. Takine [8], Choudhury [9], Madan, Abu-Dayyeh and Saleh [10], Anabosy and Madan [11], Madan and Al-Rawwash [12], Madan and Choudhury [13], Liu,Xu and Tian [14], Ke [15], and Wang and Li [16], have studied a batch arrival queue with breakdown followed by repair.

In this paper we study the batch arrival queueing system $M^{[x]}/G\ /1$ in which , after every service completion the server has the option to leave for a vacation with probability p or continue service with probability 1-p. Moreover, we assume that the server may breakdown randomly. The repair process does not necessarily start immediately after a breakdown, thus there may be a delay before starting repairs. We assume that the service times, vacation times, extended vacation times, repair times and delay times each have a general distribution while the breakdown times are exponentially distributed.

This paper is organized as follows. The mathematical description of the model is given in section 2. In section 3 all the equations governing the mathematical system in the steady state are formulated. The supplementary variable technique is used in this section to obtain the closed form of the probability generating function of the queue length. The average queue size and the average waiting time are given in section 4. Conclusions are given section 5.

2. Mathematical Description of the model

a) Customers arrive at the system in batches of variable size in a compound Poisson process and they are provided one by one service on a 'first come'-first served basis. Let $\lambda c_i(i = 1,2,3 \ldots.)$ be the first order probability that a batch of i customers arrives at the system during a short interval of time $(t, t + dt)$, where $0 \leq c_i \leq 1$ and $\sum_{i=1}^{\infty} c_i = 1$ and $\lambda > 0$ is the mean arrival rate of batches.

b) Each customer undergoes two stages of service provided by a single server on a first come first served basis. The service time follows general(arbitrary)distribution with distribution function $G_i(s)$ and density function $g_i(s)$. Let $\mu_i(x)dx$ be the conditional probability density of service completion during the interval $(x, x + dx)$,given that the elapsed time is x,so that

$$\mu_i(x) = \frac{g_i(x)}{1 - G_i(x)} \qquad (1)$$

and therefore

$$g_i(s) = \mu_i(s)e^{-\int_0^s \mu_i(x)dx} \quad i = 1,2 \qquad (2)$$

c) As soon as a service is completed, the server may go for a vacation with probability $p(0 \leq p \leq 1)$ or it may continue to serve the next customer $(1 - p)$.

d)The server's vacation time follows a general(arbitrary) distribution with distribution function $V(s)$ and density function $v(s)$. Let $B(x)dx$ be the conditional probability of a completion of a vacation during the interval $(x, x + dx)$,so that

$$\beta(x) = \frac{v(x)}{1 - V(x)} \tag{3}$$

And, therefore

$$v(s) = \beta(s)e^{-\int_0^s \beta(x)dx} \tag{4}$$

e) On returning from vacation the server instantly starts serving the customer at the head of the queue, if any.

f) The system may breakdown at random and breakdowns are assumed to occur according to a Poisson stream with mean breakdown rate $\alpha > 0$.

g) The server's delay time follows a general(arbitrary) distribution with distribution function $W(s)$ and density function $w(s)$. Let $\varphi(x)dx$ be the conditional probability of a completion of a vacation during the interval $(x, x + dx)$, so that

Delay time $\varphi(x) = \frac{w(x)}{1-W(x)}$ and $w(s) = \varphi(s)e^{-\int_0^s \varphi(x)dx}$

h) The server's delay time follows a general(arbitrary) distribution with distribution function $H_i(x)$ and density function $h_i(x)$. Let $\gamma(x)dx$ be the conditional probability of a completion of a vacation during the interval $(x, x + dx)$, so that

Repair time $\gamma(x) = \frac{h_i(x)}{1-H_i(x)}$ $\quad i = 1,2$ and $h_i(s) = \gamma(s)e^{-\int_0^s \gamma(x)dx}$

g) Various stochastic process involved in the system are assumed to be independent of each other.

3. Equations governing the System

$P_n^{(i)}(x,t)$: Probability that at time t,the server is active providing service and there are $n(n \geq 0)$ customers in the queue excluding the one being served in the first stage and the elapsed service time for this customer is x.Consequently, $P_n^{(1)}(t) = \int_0^\infty P_n^{(1)}(x,t)\,dx$ denotes the probability that at time t there are n customers in the queue excluding the one customer in the first stage of service irrespective of the value of x.

$V_n(x,t)$:Probability that at time t, the server is on vacation with elapsed vacation time x and there are $n(n > 0)$ customers waiting in the queue for service. Consequently, $V_n(t) = \int_0^\infty V_n(x,t)\,dx$ denotes the probability that at time t there are n customers in the queue and the server is on vacation irrespective of the value of x.

$D_n(x,t)$: Probability that at time t, the server is on vacation with elapsed vacation time x and there are $n(n > 0)$ customers waiting in the queue for service. Consequently, $D_n(t) = \int_0^\infty D_n(x,t)\,dx$ denotes the probability that at time t there are n customers in the queue and the server is on delay irrespective of the value of x.

$R_n(t)$:Probability that at time t,the server is inactive due to system breadown and the system is under repair, while there are $n(n \geq 0)$ customers in the queue.

$Q(t)$:Probability that at time$t$,there are no customers in the system and the server is idle but available in the system.

The queueing model is then, governed by the following set of differential-difference equations:

$$\frac{\partial}{\partial x}P_n^{(1)}(x) = -(\lambda + \mu_1(x) + \alpha_1)P_n^{(1)}(x) + \lambda\sum_{i=1}^{n-1} c_i\, P_{n-i}^{(1)}(x) \qquad (1)$$

$$\frac{\partial}{\partial x}P_0^{(1)}(x) = -(\lambda + \mu_1(x) + \alpha_1)P_0^{(1)}(x) \qquad (2)$$

$$\frac{\partial}{\partial x}P_n^{(2)}(x) = -(\lambda + \mu_1(x) + \alpha_2)P_n^{(1)}(x) + \lambda\sum_{i=1}^{n-1} c_i\, P_{n-i}^{(2)}(x) \qquad (3)$$

$$\frac{\partial}{\partial x}P_0^{(2)}(x) = -(\lambda + \mu_2(x) + \alpha_2)P_0^{(2)}(x) = 0 \qquad (4)$$

$$\frac{\partial}{\partial x}V_n(x) = -\big(\lambda + \beta(x)\big)V_n(x) = \lambda\sum_{i=1}^{n} c_i\, V_{n-i}(x) \qquad (5)$$

$$\frac{\partial}{\partial x}V_0(x) = -\big(\lambda + \beta(x)\big)V_0(x) \qquad (6)$$

$$\frac{\partial}{\partial x}D_n(x) + \big(\lambda + \varphi(x)\big)D_n(x) = \lambda\sum_{i=1}^{n} c_i\, D_{n-i}(x) \qquad (7)$$

$$\frac{\partial}{\partial x}D_n(x) = 0 \qquad (8)$$

$$\frac{\partial}{\partial x}R_n^{(1)}(x) + \big(\lambda + \gamma_1(x)\big)R_n^{(1)}(x) = \lambda\sum_{i=1}^{n} c_i\, R_{n-i}^{(1)}(x) \qquad (9)$$

$$\frac{\partial}{\partial x}R_0^{(1)}(x) = \big(\lambda + \gamma_1(x)\big)R_0^{(1)}(x) \qquad (10)$$

$$\frac{\partial}{\partial x}R_n^{(2)}(x) + \big(\lambda + \gamma_2(x)\big)R_n^{(2)}(x) = \lambda\sum_{i=1}^{n} c_i\, R_{n-i}^{(2)}(x) \qquad (11)$$

$$\frac{\partial}{\partial x}R_0^{(2)}(x) = \big(\lambda + \gamma_2(x)\big)R_0^{(2)}(x) \qquad (11a)$$

$$P_n^{(1)}(0) = (1-p)\int_0^\infty P_{n+1}^{(2)}(x)\mu_2(x)dx + \int_0^\infty V_{n+1}^{(2)}(x)\beta(x)dx + \int_0^\infty R_{n+1}^{(2)}(x)\gamma(x)dx + \lambda C_{n+1}Q \qquad (12)$$

$$P_n^{(2)}(0) = \int_0^\infty P_n^{(1)}(x)\mu_1(x)dx \qquad (13)$$

$$V_n(0) = p\int_0^\infty P_n^{(2)}(x)\mu_2(x)dx \qquad (14)$$

$$D_n(0) = \alpha\left[\int_0^\infty P_{n-1}^{(1)}(x)dx + \int_0^\infty P_{n-1}^{(2)}(x)dx\right] \qquad (15)$$

$$D_0(0) = 0 \qquad (16)$$

$$R_n^{(1)}(0) = \int_0^\infty D_n(x)\varphi(x)dx \qquad (17)$$

$$R_n^{(2)}(0) = \int_0^\infty R_n^{(1)}(x)\gamma_1(x)dx \tag{18}$$

We assume that initially there are no customers in the system and the server is idle. So the initial conditions are

$$P_n(0) = 0; \quad n = 0,1,2 \ldots.; \; V_0(0) = V_n(0) = 0; \qquad Q(0) = 1$$

We define the following probability generating functions

$$\left.\begin{array}{c} P_q^{(i)}(x,z,t) = \sum_{n=0}^{\infty} z^n P_n(x,t), \qquad P_q^{(i)}(z,t) = \sum_{n=0}^{\infty} z^n P_n \qquad i = 1,2 \\ V_q(x,z,t) = \sum_{n=0}^{\infty} z^n V_n(x,t), \qquad V_q(z,t) = \sum_{n=0}^{\infty} z^n V_n(t) \\ , \\ C(z) = \sum_{n=1}^{\infty} C_n Z^n, \end{array}\right\}$$

Multiply equation (10) by z^n & adding to (11) & using Probability generating function

$$\frac{\partial}{\partial x} P_q^{(1)}(x,z) + (\lambda + \lambda c(z) + \mu_1(x) + \alpha_1) P_q^{(1)}(x,z) = 0 \tag{19}$$

$$\frac{\partial}{\partial x} P_q^{(2)}(x,z) + (\lambda + \lambda c(z) + \mu_2(x) + \alpha_2) P_q^{(2)}(x,z) = 0 \tag{20}$$

$$\frac{\partial}{\partial x} V_q(x,z) + (\lambda - \lambda c(z) + \mu_2(x) + \beta(x)) V_q(x,z) = 0 \tag{21}$$

$$\frac{\partial}{\partial x} D_q(x,z) + \big(\lambda - \lambda c(z) + \varphi(x)\big) D_q(x,z) = \alpha_1 z \int_0^\infty P_q^{(1)}(x,z)dx + \alpha_2 z \int_0^\infty P_q^{(2)}(x,z)dx \tag{22}$$

$$\frac{\partial}{\partial x} R_q^{(1)}(x,z) + \big(\lambda + \gamma_1(x) - \lambda c(z)\big) R_q^{(1)}(x,z) = 0 \tag{23}$$

$$\frac{\partial}{\partial x} R_q^{(2)}(x,z) + \big(\lambda + \gamma_2(x) - \lambda c(z)\big) R_q^{(2)}(x,z) = 0 \tag{24}$$

$$\lambda Q = \int_0^\infty R_0(x)\gamma(x)dx + \int_0^\infty V_0(x)\beta(x)dx + (1-p)\int_0^\infty P_0^{(2)}(x,z)\mu_2(x)dx \tag{25}$$

$$zP_q^{(1)}(0,z) = (1-p)\int_0^\infty P_q^{(2)}(x,z)\mu_2(x)dx + \int_0^\infty V_q(x,z)\beta(x)dx + \int_0^\infty R_q^{(2)}(x,z)\gamma(x)dx + \lambda c(z)Q - \lambda Q \tag{26}$$

$$P_q^{(2)}(0,z) = \int_0^\infty P_q^{(1)}(x,z)\mu_1(x)dx \tag{27}$$

$$V_q(0,z) = p\int_0^\infty P_q^{(2)}(x,z)\mu_2(x)dx \tag{28}$$

$$R_q^{(1)}(0,z) = \int_0^\infty D_q(x,z)\varphi(x)dx \tag{29}$$

$$R_q^{(2)}(0,z) = \int_0^\infty R_q^{(1)}(x,z)\gamma_1(x)dx \tag{30}$$

$$D_q(0,z) = z\left[\alpha_1 P_q^{(1)}(z) + \alpha_2 P_q^{(2)}(z)\right] \tag{31}$$

Integrating Equation(19),

$$P_q^{(i)}(z) = P_q^{(i)}(0,z)e^{-(\lambda+\lambda c(x)+\alpha)x-\int_0^x \mu_i(x)dx} \tag{32}$$

$$P_q^{(i)}(z) = P_q^{(i)}(0,z)\left[\frac{1-\bar{G}_\iota(\lambda-\lambda c(z)+\alpha)}{\lambda+\lambda c(z)+\alpha}\right] \tag{33}$$

Multiplying equation (33) by $\mu_i(x)$ and integrating

$$\int_0^\infty P_q^{(i)}(x,z)\,\mu_i(x)dx = P_q^{(i)}(0,z)\bar{G}_1(\lambda-\lambda c(z)+\alpha) \tag{34}$$

$$V_q(0,z) = pP_q^{(2)}(0,z)\overline{G_2}(\lambda+\lambda c(z)+\alpha_2) \tag{35}$$

$$V_q(0,z) = V_q(0,z)e^{-(\lambda+\lambda c(z))x-\int_0^x \beta(t)dt}\overline{G_2}(\lambda-\lambda c(z)+\alpha) \tag{36}$$

Integrating,

$$V_q(z) = \frac{pP_q^{(2)}(0,z)\overline{G_2}(\lambda-\lambda c(z)+\alpha_2)\left(1-\bar{V}\left(\lambda-\lambda c(z)\right)\right)}{\lambda-\lambda c(z)} \tag{37}$$

$$\int_0^\infty V_q(x,z)\,\beta(x)dx = pP_q^{(2)}(0,z)\overline{G_2}(\lambda-\lambda c(z)+\alpha_2)\bar{V}\left(\lambda-\lambda c(z)\right) \tag{38}$$

$$D_q(x,z) = D_q(0,z)e^{-(\lambda-\lambda c(z))x-\int_0^x \varphi(t)dt}$$

$$= z\left[\alpha_1 P_q^{(1)}(z)\alpha_2 P_q^{(2)}(z)\right]\left[\frac{1-\bar{W}\left(\lambda-\lambda c(z)\right)}{\lambda-\lambda c(z)}\right] \tag{39}$$

$$D_q(z) = \frac{z\alpha_1 P_q^{(1)}(0,z)[1-\bar{G}_1(\alpha)\bar{G}_2(\alpha)]}{\lambda-\lambda c(z)+\alpha_1} \tag{40}$$

$$\int_0^\infty D_q(x,z)\,\varphi(x)dx$$

$$= \frac{\alpha z\left[P_q^{(1)}(0,z)\right]\bar{W}\left(\lambda-\lambda c(z)\right)\left(1-\bar{G}_1(a)\bar{G}_2(a)\right)}{(\lambda-\lambda c(z)+\alpha)} \tag{41}$$

$$R_q^{(1)}(x,z) = R_q^{(1)}(0,z)e^{-(\lambda-\lambda c(z))x-\int_0^x \gamma_1(t)dt}$$

$$= \frac{\alpha z P_q^{(1)}(0,z)\bar{W}\left(\lambda-\lambda c(z)\right)\left(1-\bar{G}_1(a)\bar{G}_2(a)\right)}{\lambda-\lambda c(z)+\alpha}e^{-(\lambda-\lambda c(z))x-\int_0^x \gamma_1(t)dt} \tag{42}$$

$$R_q^{(1)}(z) = \frac{\alpha z P_q^{(1)}(0,z)\bar{W}\left(\lambda-\lambda c(z)\right)\left(1-\bar{G}_1(a)\bar{G}_2(a)\right)\left(1-\bar{H}\left(\lambda-\lambda c(z)\right)\right)}{(\lambda-\lambda c(z)+\alpha)\left(\lambda-\lambda c(z)\right)} \tag{43}$$

$$\int_0^\infty R_q^{(1)}(x,z)\gamma_1(x)dx = \frac{\alpha z P_q^{(1)}(0,z)\bar{W}\big(1-\bar{G}_1(\alpha)\bar{G}_2(\alpha)\big)\overline{H_1}\big(\lambda-\lambda c(z)\big)}{\lambda-\lambda c(z)+\alpha} \quad (44)$$

Therefore

$$zP_q^{(1)}(0,z) = (1-p)P_q^{(1)}(0,z)\bar{G}_2(\lambda-\lambda c(z)+\alpha)\bar{G}_1(\lambda-\lambda c(z)+\alpha)$$
$$+pP_q^{(1)}(0,z)\bar{G}_1(a)\bar{G}_2(a)\bar{V}\ \big(\lambda-\lambda c(z)\big)$$
$$+\alpha z P_q'\ (0,z)\bar{W}\ \big((\lambda-\lambda c(z)+\alpha)(1-\bar{G}_1(a)\bar{G}(a)_2\big)$$
$$+\frac{\bar{H}_1\big(\lambda-\lambda c(z)\big)\bar{H}_2\big(\lambda-\lambda c(z)\big)}{\lambda-\lambda c(z)+\alpha}$$
$$+\lambda Q(c(z)-1) \quad (45)$$

$$P_q^{(1)}(0,z) = \frac{\lambda Q(C(z)-1)}{z-(1-p)\bar{G}_1(a)\bar{G}_2(a)-p\bar{G}_1(a)\bar{G}_2(a)\bar{V}(b)} - \frac{\alpha z\bar{W}(b)\,\overline{H}_1(b)\,\big(1-\bar{G}_1(a)\bar{G}_2(a)\big)\bar{H}_2(b)}{\lambda-\lambda c(z)+\alpha} \quad (46)$$

$$P_q^{(1)}(0,z) = \frac{-aQb}{a\big(z-(1-p)\bar{G}_1(a)\bar{G}_2(a)-p\bar{G}_1(a)\bar{G}_2(a)\bar{V}(b)\big)} \quad (47)$$
$$-\alpha z\bar{W}(b)\bar{H}_2(b)\bar{H}_1(b)[1-\bar{G}_1(a)\bar{G}_2(a)]$$

Therefore,

$$P_q^{(1)}(z) = \frac{-Qb[1-\bar{G}_1(a)]}{Dr} \quad (48)$$

$$P_q^{(2)}(z) = \frac{\bar{G}_1(a)[1-\bar{G}_2(a)]-Qb}{Dr} \quad (49)$$

$$D_q(z) = \frac{-z\alpha_1 Qb[1-\bar{G}_1(a)\bar{G}_2(a)]}{Dr}\ \bar{H}_1(b) \quad (50)$$

$$V_q(z) = \frac{-Q\bar{G}_1(a)\bar{G}_2(a)[1-\bar{V}(b)]}{Dr} \quad (51)$$

$$R_q^{(1)}(0,z) = \frac{-\alpha zQ\bar{W}(b)\big(1-\bar{G}_1(a)\bar{G}_2(a)\big)(1-\bar{H}_1(b))}{Dr} \quad (52)$$

$$R_q^{(2)}(0,z) = \frac{-\alpha zQ\bar{W}(b)\big(1-\bar{G}_1(a)\bar{G}_2(a)\big)(1-\bar{H}_2(b))\bar{H}_1(b)}{Dr} \quad (53)$$

Let $S_q(z)$ be the probability generating function

$$S_q(z) = P_q^{(1)}(z)+P_q^{(2)}(z)+D_q(z)+V_q(z)+R_q^{(1)}(0,z)+R_q^{(2)}(0,z)$$

$$S_q(z) = \frac{-Q}{Dr}[b-\bar{G}_1(a)\bar{G}_2(a)b+z\alpha b+z\alpha\bar{G}_1(a)\bar{G}_2(a)b+\bar{G}_1(a)\bar{G}_2(a)$$
$$-\bar{G}_1(a)\bar{G}_2(a)\bar{V}\ (b)+\alpha z\bar{W}(b)-\alpha z\bar{W}(b)\bar{H}_1(b)$$
$$-\alpha z\bar{W}(b)\bar{G}_1(a)\bar{G}_2(a)+\alpha z\bar{W}(b)\bar{G}_1(a)\bar{G}_2(a)\bar{H}_1(b)+z\bar{W}(b)$$
$$-z\bar{W}(b)\bar{G}_1(a)\bar{G}_2(a)$$
$$-z\bar{W}(b)\bar{H}_1(b)\bar{H}_2(b)] \quad (54)$$

Using Normalizing Condition $S_q(1) + Q = 1$, we can find the unknown factor Q, Then the utilization factor ρ can be calculated.

4 The Average Queue Size and the Average Waiting Time

Let L_q denote the mean number of customers in the queue under the steady state. Then

$$L_q = \frac{d}{dz} W_q(z)\Big|_{z=1}$$

$$\begin{aligned} Nr = {} & b - \bar{G}_1(a)\bar{G}_2(a)b + z\alpha b + z\alpha\bar{G}_1(a)\bar{G}_2(a)b + \bar{G}_1(a)\bar{G}_2(a) \\ & - \bar{G}_1(a)\bar{G}_2(a)\bar{V}(b) + \alpha z\bar{W}(b) - \alpha z\bar{W}(b)\bar{H}_1(b) \\ & - \alpha z\bar{W}(b)\bar{G}_1(a)\bar{G}_2(a) + \alpha z\bar{W}(b)\bar{G}_1(a)\bar{G}_2(a)\bar{H}_1(b) - z\bar{W}(b) \\ & - z\bar{W}(b)\bar{G}_1(a)\bar{G}_2(a) - z\bar{W}(b)\bar{H}_1(b)\bar{H}_2(b) \\ & + \bar{G}_1(a)\bar{G}_2(a)\bar{H}_1(b)\bar{H}_2(b) \qquad (55) \end{aligned}$$

$$\begin{aligned} Dr = {} & a\big[z - \bar{G}_1(a)\bar{G}_2(a) + p\bar{G}_1(a)\bar{G}_2(a) - p\bar{G}_1(a)\bar{G}_2(a)\bar{V}(b) \\ & - \alpha z\bar{W}(b)\bar{H}_1(b)\bar{H}_2(b)[1 - \bar{G}_1(a)\bar{G}_2(a)]\big] \qquad (56) \end{aligned}$$

$$\boldsymbol{L_q} = \frac{\boldsymbol{D'(1)N''(1) - N'(1)D''(1)}}{\boldsymbol{2(D'(1))^2}} \qquad \mathbf{(57)}$$

$$N'(1) = Q\begin{bmatrix} \lambda E(I)[1-\bar{G}_1(\alpha)] + \lambda E(I)[1-\bar{G}_2(\alpha)]\bar{G}_1(\alpha) \\ +\lambda E(I)\alpha[1-\bar{G}_1(\alpha)\bar{G}_2(\alpha)] - \lambda E(I)E(v)\bar{G}_1(\alpha)\bar{G}_2(\alpha) \\ +\lambda E(I)E(H_1)\alpha[1-\bar{G}_1(\alpha)\bar{G}_2(\alpha)] \\ +\lambda E(I)E(H_1)\alpha[1-\bar{G}_1(\alpha)\bar{G}_2(\alpha)] \end{bmatrix}$$

$$\begin{aligned} D'(1) = {} & -\lambda E(I)[1-\bar{G}_1(\alpha)\bar{G}_2(\alpha)] - \alpha \\ & +\alpha\big[1 - [\bar{G}'_1(\alpha)(-\lambda E(I))\bar{G}_2(\alpha) + \bar{G}_1(\alpha)(-\lambda E(I))\bar{G}'_2(\alpha)]\big] \\ & +p[\bar{G}'_1(\alpha)(-\lambda E(I))\bar{G}_2(\alpha) + \bar{G}_1(\alpha)(-\lambda E(I))\bar{G}'_2(\alpha)] \\ & -p[\bar{G}'_1(\alpha)(-\lambda E(I))\bar{G}_2(\alpha) + \bar{G}_1(\alpha)(-\lambda E(I))\bar{G}'_2(\alpha) + \lambda E(I)E(v)] \\ & -\alpha\begin{bmatrix} [1-\bar{G}_1(\alpha)\bar{G}_2(\alpha)] + \lambda E(I)E(v)[1-\bar{G}_1(\alpha)\bar{G}_2(\alpha)] \\ +\lambda E(I)E(H_1)[1-\bar{G}_1(\alpha)\bar{G}_2(\alpha)] + \lambda E(I)E(H_2)[1-\bar{G}_1(\alpha)\bar{G}_2(\alpha)] \\ +[\lambda E(I)\bar{G}'_1(\alpha)\bar{G}_2(\alpha) + \lambda E(I)\bar{G}'_2(\alpha)\bar{G}_1(\alpha)] \end{bmatrix} \end{aligned}$$

$N'(1), N''(1), D'(1)\ and D''(1)$ Into **(57)** we obtain L_q in closed form. Further, the mean waiting time of a customer could be found using $W_q = L_q/\lambda$.

5 Conclusions

In this paper we have studied a $M^{[X]}/G/1$ feedback queue with two stages of repair times, general delay times. The probability generating function of the number of customers in the queue is found using the Supplementary variable method. This paper clearly analyses the steady state results and some performance measures of the queueing system. The result of this paper is useful for computer communication network, and large scale industrial production lines.

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