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Uses of Real Options and Futures Contract to Evaluate a Palladium Mining Project Considering Pricerisk

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Abstract

TheDiscounted Cash Flow (DCF)method isgenerally used in mining industries for mining project evaluation, due to mainly the simplicity of the method. But unfortunately, depending on commodityprice risk and managerial operational flexibilities, the DCF method is oftenunsuitable for determining mining project values. Therefore, theobjective of this paper is to approximate a hypothetical palladium mining project values using real options and futures contract. Commodity price is an imperative factor for valuing any mining project.As a consequence, hedging and futures contract strategies have been used as optionsfor managing commodity prick risk and determining the project values. This will obviously help mining industries to approximate the project values and to make an exact investment decision through managing price risk.

Keywords: Real options, Futures contract. Partial differential equation, Finite difference method.

1. Introduction

In the real world to take the right decision for choosing the most money-making investment strategy for anindustry or a project is the utmost essentialmatter. Theimprovements in technology, unexpected price fluctuations, competition with other companies, political and environmental changes, consumer tastes etc., areall vitalrisk factors which will affect for the decision making [2]. One of the traditionalmethods in project evaluation is Discounted Cash Flow (DCF). This DCF method estimates the present value of the expected future cash flows. That means it uses future free cash flow projections and discounts them to arrive at a present value, which is used to evaluate the potential for investment.

In DCF method, a manager has to predict a cash flow outcome for some period in the future. One of the mainlimitations of the DCF method involves the use of a single risk adjusted discount rate to represent all sources of uncertainty in the project. This has the effect of masking the true sources of uncertainty as there are many factors which can influence the value of a mining project including technical, commodity price, exchange rate and market elements etc.[11]. Besides this, the riskiness of the project may change over time depending on how uncertainties unfold and management has to react to these uncertainties [15]. In a practical sense, it is also very difficult to determine the exact risk adjusted discount rate.

The DCF method also ignores the flexibility of management to adapt real options and to revise later decisions.In reality the mine manager has some flexibility to elect or to delay the project, undertake further exploration or development, accelerate or decelerate the mining rate, temporarily close or abandon the operation altogether etc. Such flexibility allows management to either limit downside losses or magnify upside returns, and consequently the expectations about project return, over the project's economic life [11].

As a consequence financial (hedging, future contracts) and real options strategies are trying to employed in today’s mining projects, and the researchers and evaluators are now attempting to explain the advantages of real options over the more traditional methods.

In 1977, Professor Myers firstlyintroduced the term "Real options" in a well-known paper, observing that corporate investment opportunities may be viewed as call options on real assets [14]. In 1985, Brennan and Schwartz formulated a mathematical model for finding the value and optimal production policy of a natural resource investment [5] .These authors considered that the arbitrage opportunities are unavailablefor trading in the real i.e. the natural resources as like as the financial assets in the futures market.After, Brennan and Schwartz, the real options was gradually introduced by several authors such as [4], [10], [16] in to the natural resource industries.There exists a scare literaturein mining project valuationsthroughreal options underuncertainty, and most of the studies werebased ontheoretical work, see [5],[6], [8] and [9] .Therefore, in the present study, real options and future contacts have been used to approximatethe mining project values numerically taking an example of a hypothetical palladium mine.

2. The Stochastic Model:

We consider one factor stochastic model to represent the palladiumprice fluctuations. We assume that the dynamics of the palladium price (i.e. output price), Z will follow the Geometric Brownian Motion(GBM) model (1).

$$dZ = (r - c)\, Zdt + \sigma\, Z\, dw \qquad (1)$$

Where, Z denote the spot unit price of the palladium r is the risk free rate of interest, c is the mean convenience yield on holding one unit of the palladium. σ

isthe volatility of the return of palladiumand dw is the Wiener incrementof GBM.

Now the value of the developed mine is defined as, $H = H(Z,Q)$,where Q is the total reserve. Itis possible to hedge the commodity price risk with the helpof futurescontract.

2.1 Futures contract:In finance, a futures contract is a standardized contract between two parties which allows to buy or sell a specified asset (commodity or some kind of financial assets) of standardized quantity and quality for price agreed upon today(the future price or strike price) with delivery and payment occurring at future date(i.e. the delivery date). The party to the contract who is agreeing to buy the underlying asset in the future i.e. the buyer of the contract, is said to be **long** in the position whereasthe party is agreeing to sell the asset in the future, i.e. the seller of the contract, is said to be **short** in the position .

Suppose the palladium spot price is Z , and its delivers for the future contract in time t , so the value of the future contact is $F(Z,t)$.

Applying Ito's lemma to the value of palladium futures we obtain:

$$dF(Z,t) = \frac{\partial F}{\partial Z} dZ - \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial Z^2} Z^2 \sigma^2 dt \tag{2}$$

If thepalladium futures market is arbitrage free, then an investor can make a portfolio of long one unit of palladium and short in $\left(\frac{\partial F}{\partial Z}\right)^{-1}$ units of the futures contract which help to hedge his pricerisk and earn a risk free interest rate[9]. Therefore, in time interval dt, this portfolio becomes

$$dZ + c\,Z\,dt - \frac{dF}{F_Z} = r\,Z\,dt \tag{3}$$

If there is no arbitrage opportunity, then a mining industry/company with long position for investment in a mining development (i.e. already developed just for extraction) project $H = H(Z, Q)$ and $\frac{H_Z}{F_Z}$ short position in futures contract of commodity hedges his price risk and should earn a return equal to at least the risk free interest rate [9]. Therefore;

$$dH + q(Z - C)dt - \frac{H_Z}{F_Z} dF = rH\,dt \tag{4}$$

Here C is denoted the total cost (Milling sales, CAPEX, OPEX etc.) of per unit Palladium and q is the operating rate.

Applying Ito's Lemma in, $H = H(Z, Q)$

$$dH = \frac{\partial H}{\partial Z} dZ + \frac{\partial H}{\partial Q} dQ + \frac{1}{2}\frac{\partial^2 H}{\partial Z^2}(dz)^2 \tag{5}$$

Using (3) and (5), we obtain from the equation (4)

$$\frac{\partial H}{\partial Z}dZ+\frac{\partial H}{\partial Q}dQ+\frac{1}{2}\frac{\partial^2 H}{\partial Z^2}(dz)^2+q(Z-C)dt+\frac{\partial H}{\partial Z}(r\,Z\,dt-dZ-cZ\,dt)=rH\,dt \tag{6}$$

The reserve of the mine is changed as $dQ=-q\,dt$, where q is the operating rate. Then from equation (6), we obtain

$$\frac{\partial H}{\partial Q}(-q\,dt)+\frac{1}{2}\frac{\partial^2 H}{\partial Z^2}(dz)^2+rZ\frac{\partial H}{\partial Z}dt-c\,Z\frac{\partial H}{\partial Z}dt+q(Z-C)dt=rH\,dt$$

$$\Rightarrow -q\frac{\partial H}{\partial Q}dt+\frac{1}{2}Z^{z}\sigma^2\frac{\partial^2 H}{\partial Z^2}dt+r\,Z\frac{\partial H}{\partial Z}dt-cZ\frac{\partial H}{\partial Z}+q(Z-C)dt=r\,H\,dt$$

$$\Rightarrow -q\frac{\partial H}{\partial Q}+\frac{1}{2}Z^{z}\sigma^2\frac{\partial^2 H}{\partial Z^2}+(r-c)Z\frac{\partial H}{\partial Z}+q(Z-C)-rH=0 \tag{7}$$

Subject to the boundary condition:
$H\ (Z,0)=0$

This means that when reserves are finished, the value of the mine is zero.To be complete the mathematical description and solve the PDE (7), we can consider that

$$\frac{\partial H}{\partial Z}(0,Q)=0$$
$$\frac{\partial H}{\partial Z}(\infty,Q)=0$$

There is no analytical solution for this type of PDE (7). Therefore, we will approximate the numerical solutions of the PDE (7) using the finite difference method (FDM) and MatLab software, which will apparently give the numerical values of the project.

2.2 PDE discretization in Finite difference Method (FDM)

To find the numericalsolution of the PDE (7) we will discretize itby FDM.

$$\frac{1}{2}Z^2\sigma^2\frac{\partial^2 H}{\partial Z^2}-q\frac{\partial H}{\partial Q}+(r-c)Z\frac{\partial H}{\partial Z}-r\,H\ \ +q(Z-C)=0$$

$$\Rightarrow\frac{H_{n,\,j+1}-H_{n,\,j}}{\Delta Q}-\frac{\sigma^2}{2q}(n\Delta Z)^2\left(\frac{H_{n+1,\,j}-2\,H_{n,\,j}+H_{n-1,\,j}}{(\Delta Z)^2}\right)-\frac{1}{q}(r-c)\,n\Delta Z\left(\frac{H_{n+1,\,j}-H_{n-1,\,j}}{2\Delta Z}\right)$$

$$+\frac{1}{q}r\ H_{n,\,j}-(n\Delta Z-C)=0$$

$$\Rightarrow H_{n,\,j+1}=\left(\frac{1}{2}\frac{\sigma^2 n^2}{q}\Delta Q-\frac{n}{2q}(r-c)\,\Delta Q\right)H_{n-1,\,j}\ \ +\left(1-\frac{\sigma^2 n^2}{q}\Delta Q-\frac{r}{q}\Delta Q\right)H_{n,\,j}+$$

$$\left(\frac{1}{2}\frac{\sigma^2 n^2}{q}\Delta Q+\frac{n}{2q}(r-c)\Delta Q\right)H_{n+1,\,j}+(n\Delta Z-C)\Delta Q$$

$$\Rightarrow V_{n,\, j+1} = A_n\, V_{n-1,\, j} + B_n\, V_{n,\, j} + C_n\ \ V_{n+1,\, j} + D_n \tag{8}$$

$$A_n = \frac{1}{2}\frac{\Delta Q}{q}\left(n^2\sigma^2 - n\left(r - c\right)\right)$$

$$B_n = 1 - \frac{\Delta Q}{q}\left(n^2\sigma^2 + r\ \right)$$

$$C_n = \frac{1}{2}\frac{\Delta Q}{q}\left(n^2\sigma^2 + n\left(r - c\right)\right)$$

$$D_n = \Delta Q\left(n\,\Delta P - C\ \right)$$

3. Numerical simulations and discussions

For showing the numerical results anddiscussions, weconsider the following example ofhypothetical palladium mining.

Illustration with example: Like financial options an investor can choose the real options for investment money in mining development project for extraction. The investor needs to pay the owner of the developed mine for buying the options for investment in developed mining. So the owner of the mine can sell the project and new investor buys the developed project paying the money to the owner for extraction. Before going to investment, the investors needs to find the approximate project values depending on the reserve and other parameter such as price of the palladium commodity, convenience yields etc. For the palladium price uncertainty, the investor can do futures contract for managing prick risk and making profits. Because, commodity price is the main important factor forproject values. Similarly, the owner of the mine can calculate the value of the mine before going to investment in extraction/developed mine just accumulating the costs associated before the development phase (such as geological study, exploration etc.) and the cost involved in extraction/production phase.

Due to the limitations of getting free data (real/authentic) from a mining industry, we consider an example of a hypothetical palladium mining to solve the PDE (7) numerically, and approximate the project values considering some available real options. Suppose that the hypothetical palladium mine has already developed and can be extracted 546,990 ounces with an average palladium grade of 4.34 g/t, during January 2003 to December 2005.The average ore production rate was at 1,191,000 tonnes/ year and the palladium production rate was yearly around182,300 ounces. Suppose for buying the options, the investor need to pay a fixed amount at US$42,802,000to the ownerof the mine.The total milling and sales costs includingCAPEX, OPEX etc. for extractionsare estimated atUS$43,997,996. During this project, the total corporates and the royalties taxes are assumed to beUS$27,896,490.The average palladium spot price during 2003 to 2005 is assumed to be $285.10 per ounce. The depreciation and amortisation charges are presumed to be zero, the risk free interest rate 7.00 %.The price volatility of palladium is 13% and convenience yield for holding palladium is 2.00%.

In these example thetotal costs of each ounce of palladium is US$201.65.

Numerically project values areshown belowthrough MatLabprogram.

Fig.1shows the maximum project value both in linear and in log scale. When the reserve is 546,990oz and operating rate 182,300 oz of palladium, the maximum project value is US$9,919,100.

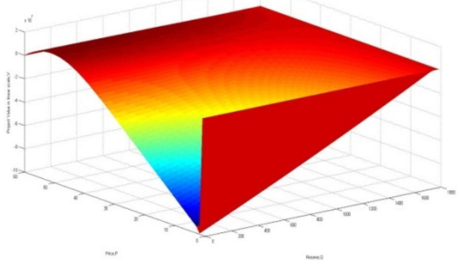

Fig.1.(a) in linear scale.

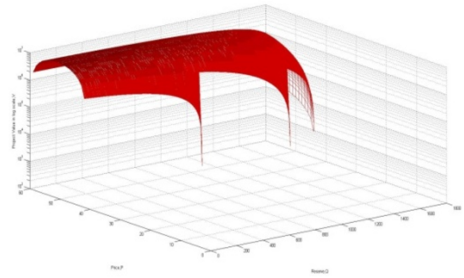

Fig.1.(b) in log scale.

Fig.1. Project value of the mine in linear scale (Fig.1.(a)) and in log scale (Fig.1.(b)) when reserve $Q = 546{,}990$ oz, $r = 7.00\%$, $\sigma = 13\%$, $c = 2.00\%$, $q = 182{,}300$ oz.

Accelerate or decelerate options: When the commodity price increases perceptibly then the managerial may accept the real option and accelerate the mine operating rate by increasing the parameter value q. For doing this, even though additional costs may be needed but itmay help to acquire higher project values as well as returns (Fig.2).

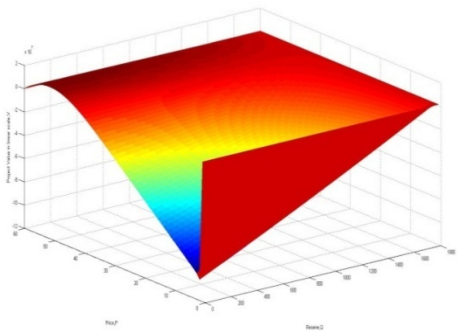

Fig. 2.(a) in linear scale.

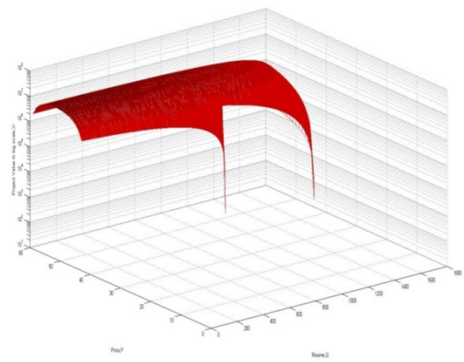

Fig. 2.(b) in log scale.

Fig.2. Project value of the mine in linear scale (Fig.2.(a)) and in log scale (Fig.2.(b)) when reserve $Q = 546{,}990$ oz., $r = 7.00\%$, $\sigma = 13\%$, $c = 2.00\%$, $q = 215{,}300$ oz.

Fig.2shows the maximum project value both in linear and in log scale. When the reserve is 546,990 oz and operating rate 215,300 ozof palladium, the maximum project value increases and becomes US$10,129,300, whereas if the operating rate is 182,300 oz of palladium, the maximum project value is US$ 9,919,100.

Conversely, if the commodity price decreases considerably, then managerial should reduce the production rate by controlling the parameter, q . That will provide a lower project value and returns at that time (Fig.3.), but may increase the life of mine. This option gives the chance to wait until favorable conditions return for obtaining higher commodity price as well as a higher project value.

Fig.3shows the maximum project value both in linear and in log scale. When the reserve is 546,990 ozand operating rate 149,300 ozof palladium, the maximum project value decreases and becomes US$8,552,400, because when the operating rate is 182,300 ozof palladium, the maximum project value is US$ 9,919,100.

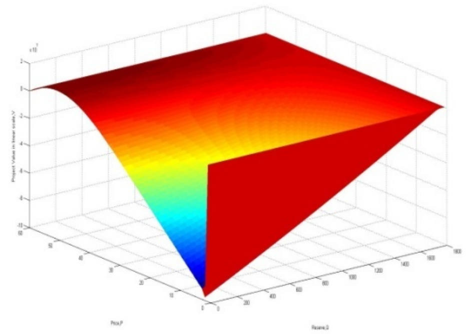

Fig. 3.(a) in linear scale.

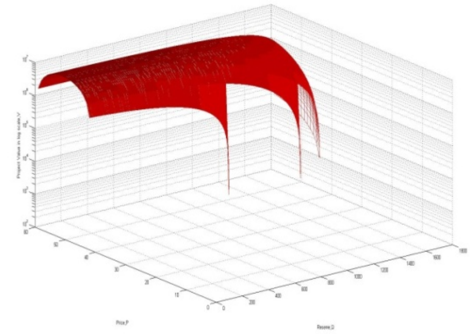

Fig.3.(b) in log scale.

Fig.3. Project value of the mine in linear scale (Fig.3.(a)) and in log scale (Fig.3.(b)) when reserve $Q = 546{,}990$ oz, $r = 7.00\%$, $\sigma = 13\%$, $c = 2.00\%$, $q = 149{,}300$ oz.

Delay and abandon option: If the maximum project value is 0, that is the total cash flows of the project is negative, then mine managerial may choose the delay option to start the operation or may adopt the abandon option and leave the project permanently.

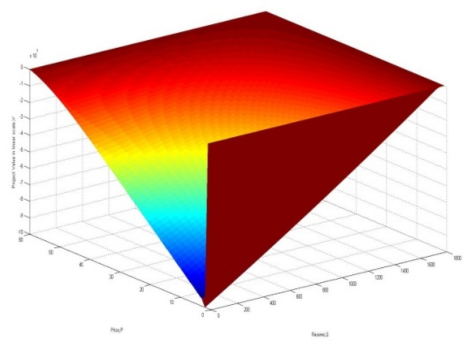

Fig. 4.(a) in linear scale.

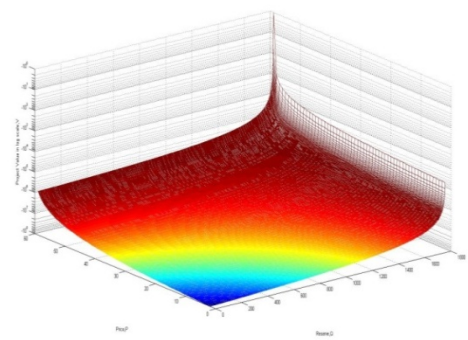

Fig. 4.(b) in log scale.

Fig.4. Project value of the mine in linear scale (Fig.4.(a)) and in log scale (Fig.4.(b)) when reserve $Q = 546{,}990$ oz, $r = 7.00\%$, $\sigma = 13\%$, $c = 2.00\%$, $q = 215{,}300$ oz

Fig.4 shows that the maximum project is 0 that means the total cash flow of this project is negative. Therefore, the mine managerial should take the delay option to wait for favourable commodity price or adopt the abandon option and leave the project permanently. For this case, the lowest palladium price is US$ 208.60 per ounce for which the project value becomes negative and there is no positive cash flow. This might be considered as the critical palladium price for this project. Therefore, mining project values are greatly influenced by the commodity price.

4. Conclusion

We consider real options, futures contracts as well as managerial flexibilities to approximate a hypothetical palladium mining project values. In the traditional DCF methods, it may not be possible to consider these types of real as well as financial options and managerial flexibilities. When the commodity price increases, the managerialcan adopt the option to accelerate the mine operation rate which gives the benefits to get a higher project values. Conversely, when the commodity price decreases then management may adopt the option to decrease the mine operationrate that gives a chance to wait until favorable conditions to obtain a higher project value.

Furthermore, due to the large mining costs as well as significant commodity price depreciation ,the mine managerial candelay the project, take the temporary closure option or adopt the abandon option. When the maximum project value is 0, thenmanagementcan choose the temporary closure option and wait until a favourable commodity price returns or he can adopt the abandon option and leave the project permanently. Due to the limitations of getting free data (real/authentic) from a mining industry, we consider an example of a hypothetical palladium mining to solve the PDE (7) numerically and approximate the project values. Therefore, to approximate real palladium mining project values, it is needed to collect the real data from the mining company and historical palladium prices from the stock market to determine the exact palladium price volatility. After collecting real data and calculating exact volatility using this PDE and Program, it is possible to approximate the real project values.

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