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Reverse Super Edge-Magic Labeling in Extended Duplicate Graph of Path

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Abstract

In this paper we prove that the extended Duplicate graph of a path is a reverse super edge-magic.

KEYWORDS: Graph labeling, reverse super edge-magic labeling, extended duplicate graphs.

1.Introduction:

The concept of graph labeling was introduced by Kotzig and Rosa(5) . Graph labeling is an assignment of integers to the edges or vertices or both subject to certain conditions. Labeled graphs serve as useful models in a broad range of applications such as circuit design, communication network addressing, X-ray crystallography, radar, astronomy, database management and coding theory. Over the past three decades various labeling of graphs such as cordial labeling, prime labeling , binary labeling , mean labeling , arithmetic labeling , graceful labeling , harmonious labeling etc have investigated in the literature(2).

In (6) S. Venkata Ramana etal introduced a reverse super edge-magic labeling. They proved $K_{m,n}$ has a reverse super edge – magic labeling for all m and n and C_n has a reverse super edge-magic labeling for all n≥3. They also proved C_n is reverse super edge – magic if and only if n is odd.

2.Preliminaries:

In this section , we give the basic notations relevant to this paper. Let G = G(V,E) be a finite , simple and undirected graph . By labeling we mean one- to- one mapping that carries a set of graph elements onto a set of numbers called labels (usually set of integers). A reverse edge-magic labeling of a graph G(V,E) is a bijection f:V∪E → $\{1,2,3, \ldots . |V \cup E|\}$ such that for all edges xy, f(xy) - {f(x)+f(y)} is *constant.* A reverse edge-magic labeling of G(V,E) is called a *reverse super edge-magic* if f(V) = {1,2,3,……. $|V|$} and f(E) = { $|V|+1, |V|+2,$……. $|V \cup E|$} . Let G(V,E) be a simple graph. A *duplicate graph* of G is DG = (V_1,E_1) where the vertex set V_1 = V∪V^1 and V∩V^1= ∅ and f: V→ V^1 is bijective (for $\vartheta \epsilon V^1$, we write f(V) = V^1 for convenience) and the edge set E_1 of DG is defined as : the edge ab is in E if and onlyif both ab^1 and a^1b are edges in E_1 . Clearly the duplicate graph of the path graph is disconnected. We give the following definition Thirusangu etal (2010) . Let DG= (V_1,E_1) be a *duplicate graph of the path graph* G(V,E) . We add an edge between any one vertex from V to any other vertex in V^1 , except the terminal vertices of V and V^1 . For convenience let us take V_2 ϵ V and V_2^1 ϵV^1 and thus the edge (V_2, V_2^1) is formed . This graph is called *extended duplicate graph* of the path graph P_m and is denoted by EDG(P_m).

3. Reverse Super Edge-magic Labeling for EDG(P_m)

In this section, we present an algorithm and prove the existence of reverse super edge-magic labeling for EDG(P_m) .

We assign the name to all edges in the following manner : In *even paths* , the edges $(v_1, v_2^{'})$,$(v_2^{'}, v_2)$,$(v_3, v_4^{'})$………$(v_m^{'}, v_{m+1})$ are named as e_1,e_2,e_3,……e_m; the edge $(v_2, v_2^{'})$is named as e_{m+1} ; and the edges and the edges $(v_1^{'}, v_2)$, $(v_{2,} v_3^{'})$, (v_3^1, v_4)…… $(v_m, v_{m+1}^{'})$ are named as $e_1^{'}, e_2^{'}, e_3^{'}, \ldots \ldots .., e_m^{'}$ respectively.

In case of *odd path*s , the edges $(v_1, v_2^{'})$,$(v_2^{'}, v_2)$,$(v_3, v_4^{'})$……… $v_m, v_{m+1}^{'})$) are named as e_1,e_2,e_3,……e_m ; the edge $(v_2, v_2^{'})$is named as e_{m+1} ; and the edges and the edges $(v_1^{'}, v_2)$, $(v_{2,} v_3^{'})$, (v_3^1, v_4)…… $(v_m^{'}, v_{m+1})$ are named as $e_1^{'}, e_2^{'}, e_3^{'}, \ldots \ldots .., e_m^{'}$ respectively.

Algorithm

Input: EDG(P_m)with 2m+2 edges .

Step1: Denote the vertices of EDG(P_m) as V=$\{v_1, v_2, v_3, \ldots, v_m, v_{m+1}, v_1^{'}, v_2^{'}, v_3^{'}, \ldots \ldots, v_m^{'}, v_{m+1}^{'}\}$ and the edges as E = $\{e_1, e_2, e_3, \ldots, e_m, e_{m+1}, e_1^{'}, e_2^{'}, e_3^{'}, \ldots \ldots, e_m^{'}, \}$

Step2 : If m is even , then m=2n , n∈N . The EDG(P_m) is of the form EDG(P_{2n}).
Define f:V→ $\{1,2,3, \ldots \ldots, |V|\}$ and f :E→ $\{|V|+1, |V|+2, |V|+3, \ldots \ldots \ldots, |V \cup E|\}$.
For 1≤ $i \leq (m+2)/2$.
f(v_{2i-1}) = (m+2)- i f($v_{2i-1}^{'}$) = (m+1) + i
For 1≤ $i \leq m/2$,
f(v_{2i}) = i f($v_{2i}^{'}$)= (2m+3)- i
f$(e_{2i-1}) = (4m+5) - 2i$ f$(e_{2i}) = (4m+4) - 2i$

f$(e'_{2i-1}) = (2m+1) + 2i$ f$(e'_{2i}) = (2m+2) + 2i$ and f$(e_{m+1}) = 3m+3$.

Step3: If m is odd , then m= 2n+1 ; n∈N . The EDG(P_m) is of the form EDG(P_{2n+1}).
For 1$\leq i \leq (m+1)/2$.
f(v_{2i-1}) = (m+2)- i f(v'_{2i-1}) = (m+1) + i
f(v_{2i}) = i f(v'_{2i})= (2m+3)- i
f$(e_{2i-1}) = (2m+2) + 2i$ f$(e'_{2i-1}) = (4m+4) - 2i$
For 1$\leq i \leq (m-1)/2$,
f$(e_{2i}) = (2m+1) + 2i$ f$(e'_{2i}) = (4m+5) - 2i$ and f$(e_{m+1}) = 3m+3$.

Step4:- Define f^*:E →N such that $f^*(v_i, v'_i)$= f $(v_i v'_i) - \{f(v_i) + f(v'_i)\}$

Output **:** Labeled EDG(P_m).

Theorem 3.1: The Extended Duplicate Graph of path P_m , m≥2 admits reverse super edge- magic labeling.

Proof: Let EDG(P_m) be an Extended Duplicate graph of Path P_m . Clearly EDG(P_m) has 2m+2 vertices and 2m+1 edges.

Denote the set of vertices as V = { $v_1, v_2, v_3, \dots, v_m, v_{m+1}, v'_1, v'_2, v'_3, \dots\dots, v'_m,$ }.

Denote the set of edges as E = { $e_1, e_2, e_3, \dots, e_m, e_{m+1}, e'_1, e'_2, e'_3, \dots\dots, e'_m,$ }.

Case 1 : In this case , We prove the Theorem for even paths. Let P_m be the path ,where m=2n; n∈N

To get a reverse super edge-magic labeling , define a map f:V→ $\{1,2,3, \dots\dots, |V|\}$ and f :E→ $\{|V|+1, |V|+2, |V|+3, \dots\dots\dots, |V \cup E|\}$ as given in step 2 of the above algorithm. Therefore the vertices $v_2, v_4, v_6, \dots., v_m, v_{m+1}, v_{m-1}, v_{m-3}, \dots\dots., v_3, v_1$ recieves consecutive numbers such as 1,2,3,…..,m+1 and $v'_1, v'_3, v'_5, \dots\dots, v'_{m+1}, v'_m, v'_{m-2}, v'_{m-4}, \dots\dots.., v'_4, v'_2$ receives the consecutive numbers such as m+2, m+3, m+4, ……. , 2m+2.

The edges $e_{m+1}, e_m, e_{m-1}, \dots\dots\dots\dots.., e_3, e_2, e_1$ recieves consecutive numbers 3m+3, 3m+4 , 3m+5 , ……..,4m+3 and the edges $e'_1, e'_2, e'_3, \dots\dots, e'_{m-2}, e'_{m-1}, e'_m$ recieves consecutive numbers 2m+3, 2m+4 , 2m+5 ,………………., 3m+2.

Define the induced function f^*:E →N such that $f^*(v_i, v'_i)$= f $(v_i v'_i) - \{f(v_i) + f(v'_i)\}$

From the definition of EDG(P_m) , the (2m+1) edges of EDG(P_m) are in following five forms , namely (v_{2i-1}, v'_{2i}) , (v'_{2i-1}, v_{2i}) where 1$\leq i \leq m/2$; (v_{2i}, v'_{2i-1}) , (v'_{2i}, v_{2i-1}) , where j = i+1 , 1$\leq i \leq m/2$, and (v_2, v'_2) .

Now $f^*(E)$ is computed as follows :

(i) For any i, where 1$\leq i \leq m/2$,

$$f^*(v_{2i-1}, v'_{2i}) = \text{f}\,(v_{2i-1}v'_{2i}) - \{f(v_{2i-1}) + f(v'_{2i})\}$$

$= f(e_{2i-1}) - \{f(v_{2i-1}) + f(v'_{2i})\}$
$=$ [(4m+5)-2i] - {[(m+2)-I] +[(2m+3)-i]} = m
$f^*(v'_{2i-1}, v_{2i}) = f(v'_{2i-1}v_{2i}) - \{f(v'_{2i-1}) + f(v_{2i})\}$
$= f(e'_{2i-1}) - \{f(v'_{2i-1}) + f(v_{2i})\}$
= [(2m+1)-2i] - {[(m+1)+i] + i} = m

(ii) For any j , where j = i+1 and $1 \leq i \leq m/2$,
$f^*(v_{2i}, v'_{2i-1}) =$ f $(v_{2i}v'_{2i-1}) - \{f(v_{2i}) + f(v'_{2i-1})\}$
$= f(e'_{2i}) - \{f(v_{2i}) + f(v'_{2i-1})\}$
= [(2m+2)+2i] - {i +[(m+1)+j]} = m
$f^*(v'_{2i}, v_{2i-1}) = f(v'_{2i}v_{2i-1}) - \{f(v'_{2i}) + f(v_{2i-1})\}$
$= f(e_{2i}) - \{f(v'_{2i}) + f(v_{2i-1})\}$
= [(4m+4)-2i] - {[(2m+3)-i] +[(m+2)-j]} = m

(iii) For the edge (v_2, v'_2)
$f^*(v_2, v'_2) =$ f $(v_2v'_2) - \{f(v_2) + f(v'_2)\} = f(e_{m+1}) - \{f(v_2) + f(v'_2)\}$
= (3m+3) - {1 + (2m+2)} = m

Thus $f^*(E)$ = m and hence EDG(P_m) admits reverse super edge-magic labeling with m as a reverse magic constant.

Case 2: In this case , We prove the Theorem for odd paths .

Let P_m be the path ,where m=2n+1; n∈N. Consider the paths of the type P_{2m+1} , n∈N.

To get a reverse super edge-magic labeling , define a map f:V→ $\{1,2,3, \dots\dots, |V|\}$ and f :E→ $\{|V|+1, |V|+2, |V|+3, \dots\dots\dots, |V \cup E|\}$ as given in step 3 of the above algorithm.

Therefore the vertices $v_2, v_4, v_6, \dots., v_m, v_{m+1}, v_{m-1}, v_{m-3}, \dots\dots., v_3, v_1$ recieves consecutive numbers such as 1,2,3,…..,m+1 and $v'_1, v'_3, v'_5, \dots\dots, v'_{m+1}, v'_m, v'_{m-2}, v'_{m-4}, \dots\dots.., v'_4, v'_2$ receives the consecutive numbers such as m+2, m+3, m+4, ……. , 2m+2.

The edges $e_{m+1}, e_m, e_{m-1}, \dots\dots\dots\dots.., e_3, e_2, e_1$ recieves consecutive numbers 3m+3, 3m+4 , 3m+5 , ……..,4m+3 and the edges $e'_1, e'_2, e'_3, \dots\dots, e'_{m-2}, e'_{m-1}, e'_m$ recieves consecutive numbers 2m+3, 2m+4 , 2m+5 ,………………., 3m+2.

Define the induced function f^*:E →N such that $f^*(v_i, v'_i)$= f $(v_iv'_i) - \{f(v_i) + f(v'_i)\}$

From the definition of EDG(P_m) , the (2m+1) edges of EDG(P_m) are in following five forms , namely (v_{2i-1}, v'_{2i}) , (v'_{2i-1}, v_{2i}) where $1 \leq i \leq (m+1)/2$; (v_{2i}, v'_{2i-1}) , (v'_{2i}, v_{2i-1}), where j = i+1 , $1 \leq i \leq (m-1)/2$, and (v_2, v'_2) .

Now $f^*(E)$ is computed as follows :

(i) For any i, where $1 \leq i \leq (m+1)/2$,
$f^*(v_{2i-1}, v'_{2i}) =$ f $(v_{2i-1}v'_{2i}) - \{f(v_{2i-1}) + f(v'_{2i})\}$
$= f(e_{2i-1}) - \{f(v_{2i-1}) + f(v'_{2i})\}$

$= [(4m+5)-2i] - \{[(m+2)-I] + [(2m+3)-i]\} = m$

$f^*(v'_{2i-1}, v_{2i}) = f(v'_{2i-1}v_{2i}) - \{f(v'_{2i-1}) + f(v_{2i})\}$

$= f(e'_{2i-1}) - \{f(v'_{2i-1}) + f(v_{2i})\}$

$= [(2m+1)-2i] - \{[(m+1)+i] + i\} = m$

(ii) For any j , where j = i+1 and $1 \leq i \leq (m-1)/2$,

$f^*(v_{2i}, v'_{2j-1}) = f(v_{2i}v'_{2j-1}) - \{f(v_{2i}) + f(v'_{2j-1})\}$

$= f(e'_{2i}) - \{f(v_{2i}) + f(v'_{2j-1})\}$

$= [(2m+2)+2i] - \{i + [(m+1)+j]\} = m$

$f^*(v'_{2i}, v_{2j-1}) = f(v'_{2i}v_{2j-1}) - \{f(v'_{2i}) + f(v_{2j-1})\}$

$= f(e_{2i}) - \{f(v'_{2i}) + f(v_{2j-1})\}$

$= [(4m+4)-2i] - \{[(2m+3)-i] + [(m+2)-j]\} = m$

(iii) For the edge (v_2, v'_2)

$f^*(v_2, v'_2) = f(v_2v'_2) - \{f(v_2) + f(v'_2)\}$

$= f(e_{m+1}) - \{f(v_2) + f(v'_2)\}$

$= (3m+3) - \{1 + (2m+2)\} = m$

Thus $f^*(E) = m$ and hence EDG(P_m) admits reverse super edge-magic labeling with m as a reverse magic constant.

ILLUSTRATION : Reverse super edge-magic labeling for the graphs EDG(P_6) and EDG(P_7) are shown in the Fig 1.

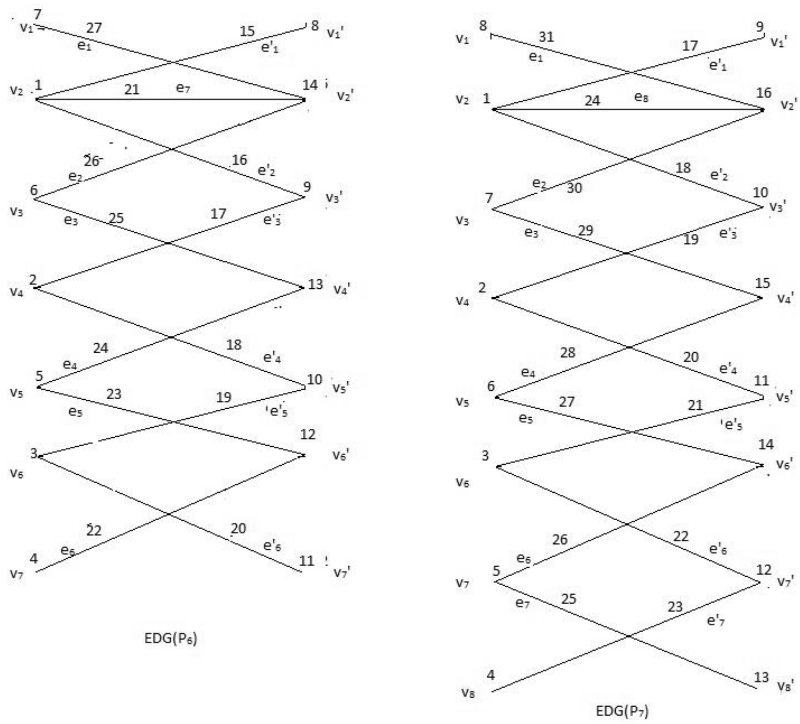

Fig 1:Reverse Super Edge-Magic Labelling

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