Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 9, Number 6 (2013), pp. 533–538

http://www.ripublication.com/gjpam.htm

Quantum Algorithm for Modified Survival Time of Bacteria Problem by Central Limit Theorem

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Abstract

A quantum algorithm for a modified survival time of bacteria problem by the central limit theorem and its example are reported. When random variables Y_i and Z_i $[1 \leq i \leq n.\ i$ and n are integers.] become y_q as each probability $1/k_1$ and z_s as each probability $1/k_2$ $[1 \leq q \leq k_1, 1 \leq s \leq k_2, k_1k_2 = K.\ q, k_1, s, k_2$ and K are integers. $y_1 < y_2 < \cdots < y_{k_1}, z_1 < z_2 < \cdots < z_{k_2}$.], respectively, and a start survival time $[M_0]$ becomes a final survival time $[M_n = M_0Y_1Z_1Y_2Z_2 \cdots Y_nZ_n]$, one example in orders that reach at M_n is obtained. A computational complexity of a classical computation is K^n. The computational complexity becomes about $3(log_2K)n$ by the quantum algorithm that uses quantum phase inversion gates, quantum inversion about mean gates, the log normal distribution and the standard normal distribution. Therefore, a polynomial time process becomes possible.

AMS subject classification: Primary 81-08; Secondary 68R05, 68W40.
Keywords: Quantum algorithm, modified survival time of bacteria problem, central limit theorem, computational complexity, log normal distribution, standard normal distribution, polynomial time.

1. Introduction

Haroche and Wineland diveloped methods for measuring and manipulating individual quantum particles, which were the very first steps towards building a quantum computer [1]. On the other hand, Deutsch and Jozsa discovered the quantum algorithm of a high-speed process by a parallel computation that uses quantum entangled states [2–4]. After that, Shor found the method of solving the factoring in a polynomial time [3–5], and Grover showed the algorithm for the databace search in a square root time [3, 6, 7]. A quantum algorithm for the vertex coloring problem by the central limit theorem has recently been reported by Fujimura [8]. Its computational complexity becomes a polynomial time. In the survival time of bacteria problem [9], its modified problem that is set up is examined this time. Therefore, its result is reported.

2. Modified Survival Time of Bacteria Problem

It is assumed that this problem follows two factors. When random variables Y_i and Z_i [$1 \le i \le n$. i and n are integers.] become y_q as each probability $1/k_1$ and z_s as each probability $1/k_2$ [$1 \le q \le k_1$, $1 \le s \le k_2$, $k_1k_2 = K$. q, k_1, s, k_2 and K are integers. $y_1 < y_2 < \cdots < y_{k_1}, z_1 < z_2 < \cdots < z_{k_2}$.], respectively, and a start survival time [M_0] becomes a final survival time [$M_n = M_0Y_1Z_1Y_2Z_2 \cdots Y_nZ_n$], one example in orders that reach at M_n is searched.

3. Quantum Algorithm

It is assumed that Y_i and Z_i [$1 \le i \le n$. i and n are integers.] becomes y_q as each probability $1/k_1$ and z_s as each probability $1/k_2$ [$1 \le q \le k_1$, $1 \le s \le k_2$, $k_1k_2 = K$. q, k_1, s, k_2 and K are integers. $y_1 < y_2 < \cdots < y_{k_1}, z_1 < z_2 < \cdots < z_{k_2}$.], respectively, a start survival time [M_0] becomes a final survival time [$M_n = M_0Y_1Z_1Y_2Z_2 \cdots Y_nZ_n$], the minimum values of y_q and z_s are y_{min} and z_{min}, respectively, the maximum values of y_q and z_s are y_{max} and z_{max}, respectively, and $Y_1Z_1Y_2Z_2 \cdots Y_nZ_n$ follows the log normal distribution [9]. In $X_i = ln(Y_iZ_i)[X = ln(M_n/M_0) = ln(Y_1Z_1) + ln(Y_2Z_2) + \cdots + ln(Y_nZ_n)]$, a mean is $\mu_i = \left(\sum_{s=1}^{k_2}\sum_{q=1}^{k_1} ln(y_qz_s)\right)/K$, and a dispersion is $\sigma_i^2 = \left(\sum_{s=1}^{k_2}\sum_{q=1}^{k_1} (ln(y_qz_s) - \mu_i)^2\right)/K$. Therefore, when a total mean is $\mu = \sum_{i=1}^{n} \mu_i = \mu_i n$ and a total dispersion $\sigma^2 = \sum_{i=1}^{n} \sigma_i^2 = \sigma_i^2 n$, $\left(\sum_{i=1}^{n} X_i - \mu\right)/\sigma$ follows the normal distribution from the central limit theorem. When the standard normal distribution $f(m)$ is $\int_0^m (e^{-m^2/2}/(2\pi)^{1/2})dm$, and values of $\int_{u_p}^{v_p} (e^{-m^2/2}/(2\pi)^{1/2})dm$ are $1/2^2, 1/2^4, 1/2^6, 1/2^8, \cdots$ and $1/2^{2p}$ [p is a positive integer], each value of m is assumed u_p and v_p that these range are contained a value $((ln(M_n/M_0)) - \mu)/\sigma$ that is searched. u_p and v_p are obtained from the table of $f(m)$. Each total number of the data between $u_p\sigma + \mu$ and $v_p\sigma + \mu$ is $K^n/2^2, K^n/2^4, K^n/2^6, K^n/2^8, \cdots$, respectively. A height at x is $\int_{x-(w/2)}^{x+(w/2)} (K^n e^{-((x-\mu)/\sigma)^2/2}/((2\pi)^{1/2}\sigma))dx[= H(x)$. w is an effective unit width, for example, about $(ln(y_{max}z_{max}/(y_{min}z_{min})))/(K-1)$.].

Next, a quantum algorithm is shown as the following.

First of all, quantum registers $|a_1>, |a_2>, \cdots, |a_n>, |b>$ and $|c>$ are prepared. When α is the minimum integer that is log_2K or more, each of $|a_f>$ that f is an integer from 1 to n is consisted of α quantum bits [= qubits]. States of $|a_1>, |a_2>, \cdots, |a_n>$, $|b>$ and $|c>$ are $a_1, a_2, \cdots, a_n, b$ and c, respectively.

Step 1: Each qubit of $|a_1>, |a_2>, \cdots, |a_n>, |b>$ and $|c>$ is set $|0>$.

Step 2: The Hadamard gate $\boxed{H}$ [3, 4] acts on each qubit of $|a_1>, |a_2>, \cdots, |a_{n-1}>$ and $|a_n>$. It changes them for entangled states. The total states are $(2^{\alpha})^n$.

Step 3: It is assumed that a quantum gate (A) doesn't change $|b>$ in $a_f < K$, or it changes $|b>$ for $|b+1>$ in the others of a_f. As a target state for $|b>$ is 0, quantum phase inversion gates (PI) and quantum inversion about mean gates (IM) [3, 6, 7] act on $|b>$. When β is the minimum even integer that is $(2^{\alpha}/K)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is β, because they are a couple. Next, an observation gate (OB) observes $|b>$. These actions are repeated sequentially from $|a_1>$ to $|a_n>$. Therefore, each state of $|a_f>$ is $0, 1, \cdots, K-2$ and $K-1$, and the total states become $K^n[=W_0]$.

Step 4: It is assumed that a quantum gate (B) changes $|b>$ for $|b+ln(y_1z_1)>, |b+ln(y_1z_2)>, \cdots, |b+ln(y_{k_1}z_{k_2-1})>$ and $|b+ln(y_{k_1}z_{k_2})>$ in $a_f = 0, 1, \cdots, K-2, K-1$, respectively. This action repeats from 1 to n at f. Therefore, $|b>$ becomes from $|n(ln(y_{min}z_{min}))>$ to $|n(ln(y_{max}z_{max}))>$.

Step 5: It is assumed that a quantum gate (C_1) doesn't change $|c>$ in $u_1\sigma+\mu \leq b \leq v_1\sigma+\mu$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included in $u_1\sigma+\mu \leq b \leq v_1\sigma+\mu$ is $W_1 \approx K^n/2^2$. When γ_1 is the minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|c>$ is $\gamma_1 \approx 2$. Next, (OB) observes $|c>$, and the data of W_1 remain.

Similarly, (C_j) [$2 \leq j \leq g-1$. j is an integer. g that is an integer follows $W_0/H(ln(M_n/M_0)) = 1/(\int_{ln(M_n/M_0)-(w/2)}^{ln(M_n/M_0)+(w/2)} (e^{-((ln(M_n/M_0)-\mu)/\sigma)^2/2}/((2\pi)^{1/2}\sigma))dx) \approx 2^{2g}$.] doesn't change $|c>$ in $u_j\sigma+\mu \leq b \leq v_j\sigma+\mu$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included in $u_j\sigma+\mu \leq b \leq v_j\sigma+\mu$ is $W_j \approx K^n/2^{2j}$. When γ_j is the minimum even integer that is $(W_{j-1}/W_j)^{1/2}$ or more, the total number that (PI) and (IM) act on $|c>$ is $\gamma_j \approx 2$. Next, (OB) observes $|c>$, and the data of W_j remain. These actions are repeated sequentially from 2 to $g-1$ at j.

(C_g) doesn't change $|c>$ in $u_g\sigma+\mu \leq b \leq v_g\sigma+\mu$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included in $u_g\sigma+\mu \leq b \leq v_g\sigma+\mu$ is $W_g \approx H(ln(M_n/M_0)) \approx K^n/2^{2g}$. When γ_g is the minimum even integer that is $(W_{g-1}/W_g)^{1/2}$ or more, the total number that (PI) and (IM) act on $|c>$ is $\gamma_g \approx 2$. Next, (OB) observes $|a_1>, |a_2>, \cdots, |a_n>, |b>$ and $|c>$, and one of the data of W_g remains. Therefore, one example of orders that reach at M_n is obtained.

4. Numerical Computation

It is assumed that there are $n=3, y_1 = 1/2 = y_{min}, y_2 = 1, y_3 = 2 = y_{max}, z_1 = 1/3 = z_{min}, z_2 = 1 = z_{max}, ln(1/6) \approx -1.792, ln(1/2) \approx -0.6931$, $ln(1/3) \approx$

$-1.099, ln1 = 0, ln(2/3) \approx -0.4059, ln2 \approx 0.6931, k_1 = 3, k_2 = 2, K = 6, M_0 = 1, M_3 = 4, \mu_i = -0.5495, \mu = \sum_{i=1}^{3} \mu_i = 3\mu_i \approx -1.649, \sigma_i^2 \approx 0.6222, \sigma^2 = \sum_{i=1}^{3} \sigma_i^2 \approx 3\sigma_i^2 \approx 1.867, \sigma \approx 1.366, K^n = 6^3 = 216, w = (ln(y_{max}z_{max}/(y_{min}z_{min})))/(K - 1) = (ln(2/(1/6)))/5 = (ln12)/5 \approx 0.4970$, $H(ln(M_3/M_0)) = H(ln4) \approx H(1.386) \approx 3, g = 3, u_1 \approx 0.6491, u_2 \approx 1.471, u_3 \approx 1.982$ and $v_1 = v_2 = v_3 \approx 2.404$.

First of all, $|a_1>, |a_2>, |a_3>, |b>$ and $|c>$ are prepared. When α is the minimum integer that is $log_2 K = log_2 6 \approx 2.585 \leq 3 = \alpha$, each of $|a_f>$ that f is the integer from 1 to 3 is consisted of $\alpha = 3$ qubits. States of $|a_1>, |a_2>, |a_3>, |b>$ and $|c>$ are a_1, a_2, a_3, b and c, respectively.

Step 1: Each qubit of $|a_1>, |a_2>, |a_3>, |b>$ and $|c>$ is set $|0>$.

Step 2: $\boxed{\text{H}}$ acts on each qubit of $|a_1>, |a_2>$ and $|a_3>$. It changes them for entangled states. The total states are $(2^\alpha)^n = (2^3)^3 = 8^3 = 512$.

Step 3: (A) doesn't change $|b>$ in $a_f < K = 6$, or it changes $|b>$ for $|b+1>$ in the others of a_f. As the target state for $|b>$ is 0, (PI) and (IM) act on $|b>$. When β is the minimum even integer that is $(2^\alpha/K)^{1/2} = (2^3/6)^{1/2} = (8/6)^{1/2} \approx 1.155 \leq 2 = \beta$, the total number that (PI) and (IM) act on $|b>$ is $\beta \approx 2$. Next, (OB) observes $|b>$. These actions are repeated sequentially from $|a_1>$ to $|a_3>$. Therefore, each state of $|a_f>$ is $0, 1, 2, 3, 4$ and 5, and the total states become $K^n = 6^3 = 216[= W_0]$.

Step 4: (B) changes $|b>$ for $|b+ln(1/6)>, |b+ln(1/2)>, |b+ln(1/3)>, |b+ln1>, |b+ln(2/3)>$ and $|b+ln2>$ in $a_f = 0, 1, 2, 3, 4$ and 5, respectively. This action repeats from 1 to 3 at f. Therefore, $|b>$ becomes $|-5.376>$ to $|2.079>$.

Step 5: (C_1) doesn't change $|c>$ in $u_1\sigma + \mu \approx -0.7623 \leq b \leq v_1\sigma + \mu \approx 1.635$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included in $-0.7623 \leq b \leq 1.635$ is $W_1 \approx 6^3/2^2$. When γ_1 is the minimum even integer that is $(W_0/W_1)^{1/2} \approx (6^3/(6^3/2^2))^{1/2} = 2 \leq 2 = \gamma_1$, the total number that (PI) and (IM) act on $|c>$ is $\gamma_1 \approx 2$. Next, (OB) observes $|c>$, and the data of W_1 remain.

(C_2) doesn't change $|c>$ in $u_2\sigma+\mu \approx 0.3604 \leq b \leq v_2\sigma+\mu \approx 1.635$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included in $0.3604 \leq b \leq 1.635$ is $W_2 \approx 6^3/2^4$. When γ_2 is the minimum even integer that is $(W_1/W_2)^{1/2} \approx ((6^3/2^2)/(6^3/2^4))^{1/2} = 2 \leq 2 = \gamma_2$, the total number that (PI) and (IM) act on $|c>$ is $\gamma_2 \approx 2$. Next, (OB) observes $|c>$, and the data of W_2 remain.

(C_3) doesn't change $|c>$ at $u_3\sigma + \mu \approx 1.060 \leq b \leq v_3\sigma + \mu \approx 1.635$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and

(IM) act on $|c>$. The number of the data that is included in $1.060 \leq b \leq 1.635$ is $W_3 \approx H(ln(M_3/M_0)) = H(ln4) \approx 3 \approx 6^3/2^6$. When γ_3 is the minimum even integer that is $(W_2/W_3)^{1/2} \approx ((6^3/2^4)/(6^3/2^6))^{1/2} = 2 \leq 2 = \gamma_3$, the total number that (PI) and (IM) act on $|c>$ is $\gamma_3 \approx 2$. Next, (OB) observes $|a_1>, |a_2>, |a_3>, |b>$ and $|c>$, and one of the data of W_3 remains. For example, when a_1, a_2, a_3, b and c are 3, 5, 5, 1.386 and 0, respectively, it is obtained that one example of orders is 1, 2 and 2.

5. Discussion and Summary

The computational complexity of this quantum algorithm $[=S]$ becomes the following. In the order of the actions by the gates, the number of them is αn at $\boxed{\text{H}}$, n at (A), $\beta n \approx 2n$ at (PI) and (IM), n at (OB), n at (B), about g at (C_j) $[1 \leq j \leq g.$ j is the integer.], about $2g$ at (PI) and (IM), and about g at (OB). Therefore, S becomes about $(\alpha + 5)n + 4g$. In the example of the section 4, S is 36. The computational complexity of the classical computation $[=Z]$ is $K^n = 6^3 = 216$. After all, S/Z becomes $1/6$. When n is large enough, S becomes about $(\alpha + 5)n + 4g \approx 3(log_2 K)n$, where α is about $log_2 K$, and the maximum value of g is about $(n/2)log_2 K$, and S/Z is about $3(log_2 K)n/K^n$. For example, as for $K = 6$ and $n = 100$, S/Z is about $1/10^{75}$.

Therefore, a polynomial time process becomes possible.

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