Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 9, Number 5 (2013), pp. 489-498

http://www.ripublication.com/gjpam.htm

Effect of a Poro-Piezo Ceramic Plate on the Reflection and Transmission of High Frequency Waves

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Abstract

The effects of poro-piezoelectricity on the interaction of high frequency waves in fluid immersed plate are studied in the present paper. The governing equations are formulated and their solution is obtained for a porous piezoelectric plate. The reflected and transmitted amplitude ratios are obtained. The behaviour of amplitude ratios relative to frequency, incidence angle and layer thickness is studied numerically for a particular model PZT. The studied problem is useful in various engineering fields.

1. Introduction

Piezoelectric materials are the materials which produce electric field when mechanical stress is applied and vice-versa. These materials have been widely used in many branches of science and technology. Nayfeh and Chien (1992) studied the reflection and transmission phenomena in fluid immersed piezoelectric plate. The effects of angle of incidence are studied on the velocity of surface waves. Clezio and Shuvalov (2004) use the octet formalism method to study the transmission of plane acoustic waves through an anisotropic, inhomogeneous and absorbing piezoelectric plate.

Porous piezoelectric materials have low acoustic impedance and higher efficiency compared to dense piezoelectric materials. Porous piezoelectric materials are widely used in ultrasonic transducers, hydrophones, pressure sensors and other piezoelectric devices (Arai et al. (1991), Hayashi et al. (1991), Mizumura et al. (1991)).The study of reflection and transmission coefficients from fluid saturated porous piezoelectric plate is important in various fields of engineering acoustics. Different studies (Gomez and Montero (1997), Gomez et al. (2000), Roncari et al. (2001), Bowen et al. (2004), Kumar et al. (2005)) have been made related to synthesis, characterizations, fabrication and microstructure of these materials. Vashishth and Gupta (2009) derived the constitutive equations for porous piezoelectric materials using Biot's theory and electric enthalpy density function definition. Wave propagation in transversely isotropic porous piezoelectric materials was studied analytically by Vashishth and Gupta (2009). Vashishth and Gupta (2011) proved the Uniqueness Theorem, Theorem of Reciprocity and general theorems in the linear theory of porous piezoelectricity. The reflection and transmission of waves from fluid loaded porous piezoelectric half space is studied by Vashishth and Gupta (2011).

In the present paper, the reflection-transmission phenomenon in porous piezoelectric plate is studied analytically. The constitutive equations and governing equations of motion are formulated. The behaviour of reflected and transmitted amplitude ratios relative to angle of incidence, frequency, porosity and thickness of the layer is also studied.

2. Governing equation and solution.

The constitutive equations for transversely isotropic porous piezoelectric material are

$$
\begin{aligned}
&\sigma_{11} = c_{11}u_{1,1} + c_{12}u_{2,2} + c_{13}u_{3,3} + m_{11}(U_{1,1} + U_{2,2} + U_{3,3}) + e_{31}\psi_{,3} + \zeta_{31}\psi^{*}_{,3},\\
&\sigma_{22} = c_{12}u_{1,1} + c_{11}u_{2,2} + c_{13}u_{3,3} + m_{11}(U_{1,1} + U_{2,2} + U_{3,3}) + e_{31}\psi_{,3} + \zeta_{31}\psi^{*}_{,3},\\
&\sigma_{33} = c_{13}u_{1,1} + c_{13}u_{2,2} + c_{33}u_{3,3} + m_{33}(U_{1,1} + U_{2,2} + U_{3,3}) + e_{33}\psi_{,3} + \zeta_{33}\psi^{*}_{,3},\\
&\sigma_{23} = c_{44}(u_{2,3} + u_{3,2}) + e_{15}\psi_{,2} + \zeta_{15}\psi^{*}_{,2},\\
&\sigma_{13} = c_{44}(u_{1,3} + u_{3,1}) + e_{15}\psi_{,1} + \zeta_{15}\psi^{*}_{,1},\\
&\sigma_{12} = c_{66}(u_{1,2} + u_{2,1}),\\
&\sigma^{*} = m_{11}u_{1,1} + m_{11}u_{2,2} + m_{33}u_{3,3} + R(U_{1,1} + U_{2,2} + U_{3,3}) + \tilde{\zeta}_{3}\psi_{,3} + e^{*}_{3}\psi^{*}_{,3},\\
&D_{1} = e_{15}(u_{1,3} + u_{3,1}) - \xi_{11}\psi_{,1} - A_{11}\psi^{*}_{,1},\\
&D_{2} = e_{15}(u_{2,3} + u_{3,2}) - \xi_{11}\psi_{,2} - A_{11}\psi^{*}_{,2},\\
&D_{3} = e_{31}u_{1,1} + e_{31}u_{2,2} + e_{33}u_{3,3} + \tilde{\zeta}_{3}(U_{1,1} + U_{2,2} + U_{3,3}) - \xi_{33}\psi_{,3} - A_{33}\psi^{*}_{,3},\\
&D^{*}_{1} = \zeta_{15}(u_{1,3} + u_{3,1}) - A_{11}\psi_{,1} - \xi^{*}_{11}\psi^{*}_{,1},\\
&D^{*}_{2} = \zeta_{15}(u_{2,3} + u_{3,2}) - A_{11}\psi_{,2} - \xi^{*}_{11}\psi^{*}_{,2},\\
&D^{*}_{3} = \zeta_{31}u_{1,1} + \zeta_{31}u_{2,2} + \zeta_{33}u_{3,3} + e^{*}_{3}(U_{1,1} + U_{2,2} + U_{3,3}) - A_{33}\psi_{,3} - \xi^{*}_{33}\psi^{*}_{,3} \qquad (1)
\end{aligned}
$$

Here, σ_{ij} and D_i (σ^{*} and D^{*}_{i}) are the stress tensor components and electric displacements for the solid (fluid) phases of the porous aggregate, respectively. ψ and ψ^{*} are the electric potentials corresponding to solid and fluid phase. c_{ij} are the elastic stiffness tensor components of porous bulk material. R is elastic constant corresponding to fluid phase. $e_{31}, e_{33}, e_{15}, \zeta_{31}, \zeta_{33}, \zeta_{15}, e^{*}_{3}, \tilde{\zeta}_{3}$ are piezoelectric coefficients. $\xi_{11}, \xi_{33}, A_{11}, A_{33}, \xi^{*}_{11}, \xi^{*}_{33}$ are the dielectric coefficients. m_{11}, m_{33} are elastic coupling coefficients.

The equations of motion are

$$\sigma_{ij,j} = \rho_{11}\ddot{u}_j + \rho_{12}\ddot{U}_j ,$$
$$\sigma^*_{,i} = \rho_{12}\ddot{u}_j + \rho_{22}\ddot{U}_j ,$$
$$D_{i,i} = 0 ,$$
$$D^*_{i,i} = 0 , \quad (2)$$

ρ_{11}, ρ_{12} and ρ_{22} are the dynamical coefficients.

Let us assume that

$$(u_i, U_i, \psi, \psi^*) = (A_i, B_i, G, H)\ \exp\left(\iota\omega\left(\frac{1}{c}x_1 + px_3 - t\right)\right), \quad (i = 1,3), \quad (3)$$

where p is unknown parameter and $(A_1, A_3, B_1, B_3, G, H)$ are the amplitudes associated with the harmonic wave. c is the apparent phase velocity.

The equations (1)-(3) imply

$$c_1 p^{10} + c_2 p^8 + c_3 p^6 + c_4 p^4 + c_5 p^2 + c_6 = 0 , \quad (4)$$

where coefficients c_i $(i = 1,2,..,6)$ are given in the Appendix. p_1, p_3 and p_5 correspond to slowness of quasi- P_1 mode, quasi S_1 mode and quasi- P_2 mode and p_7 and p_9 correspond to electric potential modes. These roots of the equation (4) are arranged such that

$$p_{i+1} = -p_i , \quad (i = 1,3,5,7,9) .$$

The mechanical displacements and electric potentials in the porous piezoelectric plate are represented as

$$(u_1, u_3, U_1, U_3, \psi, \psi^*) = \sum_{i=1}^{10} (1, u_{1i}, u_{2i}, u_{3i}, u_{4i}, u_{5i})\, A_{1i}\ \exp\left(\iota\omega\left(\frac{1}{c}x_1 + p_i x_3 - t\right)\right). \quad (5)$$

The formal solutions for the mechanical stresses and electrical displacements are obtained as

$$(\sigma_{31}, \sigma_{33}, \sigma^*, D_3, D_3^*) = \sum_{i=1}^{10} (d_{1i}, d_{2i}, d_{3i}, d_{4i}, d_{5i})\, A_{1i}\ \exp\left(\iota\omega\left(\frac{1}{c}x_1 + p_i x_3 - t\right)\right), \quad (6)$$

where $d_{ki}, (k = 1,2,...,5)$ are given in the Appendix.

3. Model of the problem

Let us consider a porous piezoelectric plate of thickness h which is immersed in a fluid. An elastic wave from the upper fluid half space making an angle θ with x_3 axis, is assumed to strike the interface $x_3 = 0$ which results into one reflected wave in upper fluid half space and one transmitted wave in lower fluid half space.

The normal displacement and normal stress in the upper/lower fluid half space can be written as

$$u_3^f = \rho^f c\left[U_1^f \exp\left(\iota\omega\left(\frac{1}{c}x_1 + p_f\, x_3 - t\right)\right) - U_2^f \exp\left(\iota\omega\left(\frac{1}{c}x_1 - p_f\, x_3 - t\right)\right)\right],$$

$$\sigma_{33}^f = \iota\omega\rho^f c\left[U_1^f \exp\left(\iota\omega\left(\frac{1}{c}x_1 + p_f\, x_3 - t\right)\right) - U_2^f \exp\left(\iota\omega\left(\frac{1}{c}x_1 - p_f\, x_3 - t\right)\right)\right], \qquad (7)$$

where $p^f = \frac{1}{c}\sqrt{\frac{c^2}{(c^f)^2} - 1}$.

Here, c_f and ρ^f is the longitudinal incident wave velocity and density in the upper/lower fluid medium.

The boundary conditions at the interface $x_3 = 0$ ($x_3 = h$) are

$$\sigma_{33} = (1-f)\sigma_{33}^f,\ \sigma^* = f\sigma_{33}^f,\ \sigma_{13} = 0,\ (1-f)\dot{u}_3 + f\,\dot{U}_3 = \dot{u}_3^f,\ D_3 = 0,\ D_3^* = 0\ , \qquad (8)$$

Making use of the equations (5)-(7) into the equation (8), we obtain following non-homogeneous system

$$\begin{bmatrix}
d_{21}+d_{31} & - - - - - - - - & d_{2,10}+d_{3,10} & \iota\omega\rho^f c & 0 \\
d_{31} & - - - - - - - - & d_{3,10} & \iota f\omega\rho^f c & 0 \\
d_{11} & - - - - - - - - & d_{1,10} & 0 & 0 \\
\begin{matrix}(1-f)u_{11}\\ +fu_{31}\end{matrix} & - - - - - - - - & \begin{matrix}(1-f)u_{1,10}\\ +fu_{3,10}\end{matrix} & \rho^f c & 0 \\
d_{41} & - - - - - - - - & d_{4,10} & 0 & 0 \\
d_{51} & - - - - - - - - & d_{5,10} & 0 & 0 \\
d_{21}e_1 & - - - - - - - - & d_{21}e_{10} & 0 & \iota(1-f)\omega\rho^f ce_2^f \\
d_{31}e_1 & - - - - - - - - & d_{3,10}e_{10} & 0 & \iota\omega\rho^f fce_2^f \\
d_{11}e_1 & - - - - - - - - & d_{1,10}e_{10} & 0 & 0 \\
\begin{matrix}((1-f)u_{11}\\ +fu_{31})e_1\end{matrix} & - - - - - - - - & \begin{matrix}((1-f)u_{1,10}\\ +fu_{3,10})e_{10}\end{matrix} & 0 & \rho^f ce_2^f \\
d_{41}e_1 & - - - - - - - - & d_{4,10}e_{10} & 0 & 0 \\
d_{51}e_1 & - - - - - - - - & d_{5,10}e_{10} & 0 & 0
\end{bmatrix}
\begin{bmatrix} A_{11} \\ A_{12} \\ A_{13} \\ A_{14} \\ A_{15} \\ A_{16} \\ A_{17} \\ A_{18} \\ A_{19} \\ A_{1,10} \\ R \\ T \end{bmatrix}
=
\begin{bmatrix} \iota\omega\rho^f c \\ \iota\omega f\rho^f c \\ 0 \\ \rho^f c \\ 0 \\ 0 \\ \iota(1-f)\omega\rho^f ce_1^f \\ \iota\omega f\rho^f ce_1^f \\ 0 \\ \rho^f ce_1^f \\ 0 \\ 0 \end{bmatrix}$$

where $e_i = \exp(\iota\omega p_i h), (i = 1,2,...,10), e_1^f = \exp(\iota\omega p^f h), e_2^f = \exp(-\iota\omega p^f h)$. (9)

On solving non-homogeneous system (9), the reflected and transmitted amplitude ratios can be obtained.

4. Numerical study

The reflected and transmitted amplitude ratios are calculated numerically for a particular model PZT. Following Gupta and Venketesh (2006), the elastic, piezoelectric, dielectric coefficients and other parameter used in the study for these ceramics are given in the tables 1. The fixed frequency is assumed 1MHz and fixed angle of incidence is taken $\theta = 5^\circ$ unless specified otherwise.

Table 1. Elastic constants, Piezoelectric constants and Dielectric constants of PZT-7H crystal.

$$c_{11} = 148.0e9, c_{12} = 76.2e9, c_{13} = 74.2e9, c_{33} = 131.0e9, c_{44} = 25.3e9, m_{11} = 8.8e9,$$
$$m_{33} = 5.2e9, R = 20e9, e_{15} = 9.3, e_{31} = -2.324, e_{33} = 10.99, \zeta_{15} = 2.72, \zeta_{31} = -0.48,$$
$$\zeta_{33} = 5.32, e_3^* = -3.6, \tilde{\zeta}_3 = -7.5, \xi_{11} = 3.984e-9, \xi_{33} = 2.081e-9, \xi_{11}^* = 11.8e-9,$$
$$\xi_{33}^* = 13.9e-9, A_{11} = 12.8e-9, A_{33} = 15.1e-9, f = 0.2, c_f\,(m/s) = 1500,$$
$$\rho(kg/m^3) = 7700, \rho_{11} = 0.66\times 5700, \rho_{12} - 0.15\times 5700, \rho_{22} = 0.66\times 5700,$$

Figure1 shows the behaviour of absolute value of reflection and transmission coefficients relative to angle of incidence for a PZT plate at fixed frequency. The behaviour is found to be very sensitive in respect to the incidence angle. The reflection and transmission coefficients show maxima and minima for different angles at fixed frequency. These maxima/minima correspond to the excitation of surface modes.

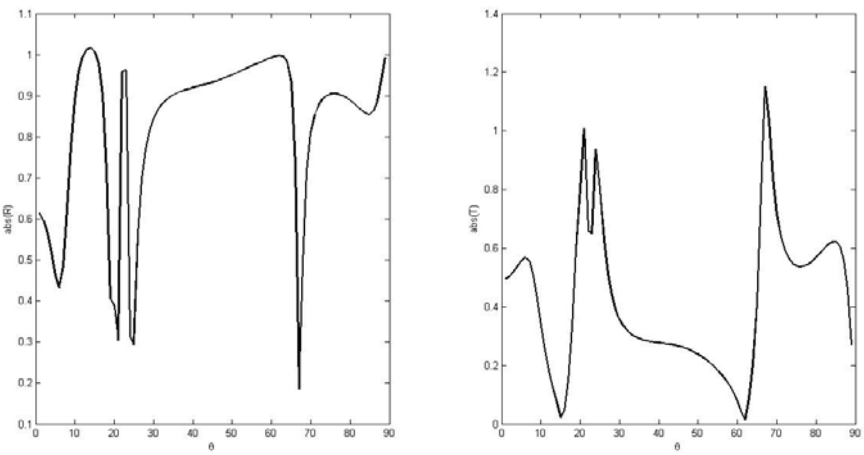

Figure 1. Behaviour of absolute value of reflection and transmission coefficients with the angle of incidence at fixed frequency.

The effect of thickness of the plate on the variation of reflection and transmission coefficients with the angle of incidence at fixed frequency is shown in the figure 2. The thickness of the layer is taken 1mm, 2mm, 3mm and 4mm in respective figures. The behaviour of amplitude ratio relative to incidence angle is not affected much due to change in the thickness of the layer.

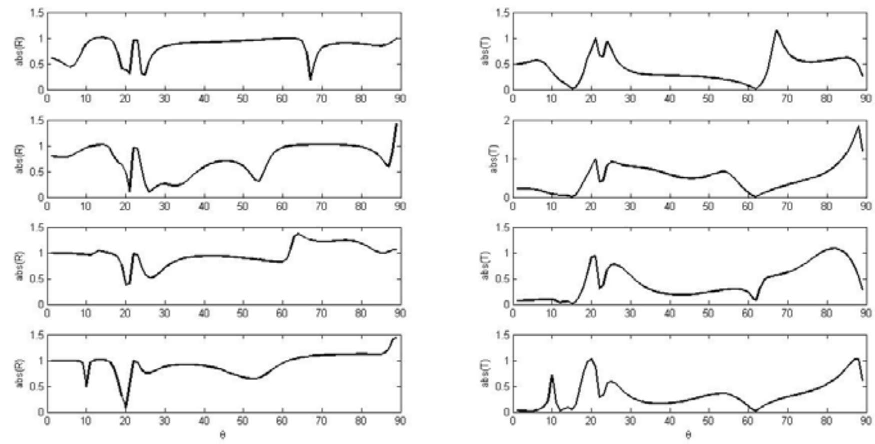

Figure 2. Effects of thickness of plate on the behaviour of reflection and transmission coefficients with the angle of incidence at fixed frequency.

Figure 3 shows the effects of frequency on the reflection and transmission coefficients at fixed angle of incidence. The behaviour of reflection and transmission coefficients with frequency is oscillatory and involves a system of resonances.

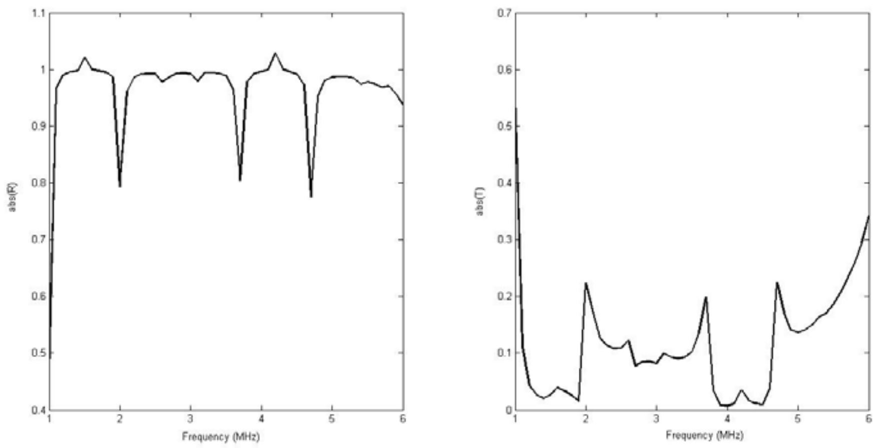

Figure 3 Behaviour of absolute value of reflection and transmission coeffcients with frequency at fixed angle of incidence.

The effects of thickness of the plate on the behaviour of reflection and transmission coefficients with frequency are also observed in the figure 4. The thickness of the layer is taken 1mm, 2mm, 3mm and 4mm in respective figures. The number of maxima or minima in transmission or reflection coefficients increases in number as the thickness of the layer increases. The frequencies at which transmission is very high are important. The small change in the thickness of the layer can result in a relatively large change in the gradient of the velocity profile.

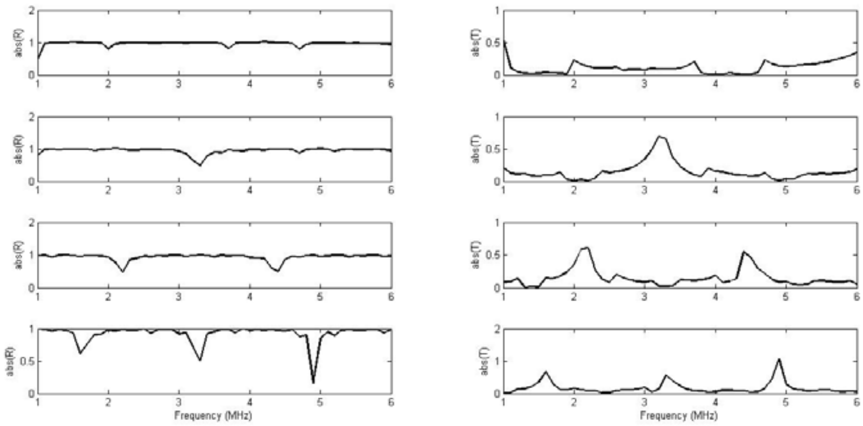

Figure 4 Effects of thickness of plate on the behaviour of reflection and transmission coeffcients with frequency.

The behaviour of absolute value of reflection and transmission coefficients with porosity is shown in the figure 5. It is observed that the absolute value of reflection coefficient decrease with increase in porosity while absolute value of transmission coefficients increases with porosity. Thus the amount of energy reflected back decreases with increase in the porosity of the plate.

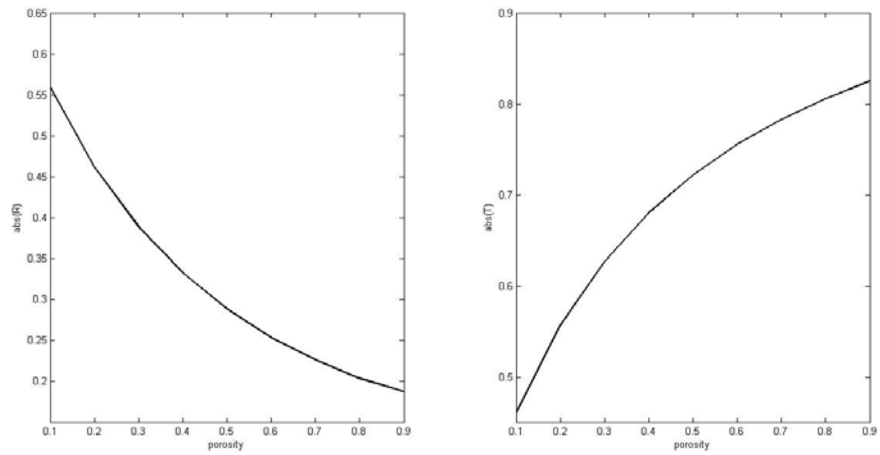

Figure 5 Behaviour of absolute value of reflection and transmission coefficients with the porosity.

4. Conclusion

The reflection and transmission phenomenon in porous piezoelectric plates is an important problem in underwater acoustics, non destructive evaluation and surface acoustic wave devices. The behaviour of reflection and transmission coefficients relative to frequency, angle of incidence, layer thickness, porosity is observed. The reflection and transmission coefficients are found to be very sensitive in respect of the frequency and incidence angle. Less amount of incident energy gets reflected back as the porosity increases.

Appendix

$c_1 = \beta_1 y_1 - \beta_5 y_{10} + \beta_9 y_{17} - \beta_{13} y_{26}$,

$c_2 = \beta_2 y_1 + \beta_1 y_2 - \beta_6 y_{10} - \beta_5 y_{11} + \beta_{10} y_{17} + \beta_9 y_{18} - \beta_{14} y_{26} - \beta_{13} y_{27}$,

$c_3 = \beta_3 y_1 + \beta_2 y_2 + \beta_1 y_3 - \beta_7 y_{10} - \beta_6 y_{11} + \beta_{11} y_{17} + \beta_{10} y_{18} + \beta_9 y_{19} - \beta_{15} y_{26} - \beta_{14} y_{27} - \beta_{13} y_{28}$,

$c_4 = \beta_4 y_1 + \beta_3 y_2 + \beta_2 y_3 - \beta_8 y_{10} - \beta_7 y_{11} + \beta_{12} y_{17} + \beta_{11} y_{18} + \beta_{10} y_{19} - \beta_{16} y_{26} - \beta_{15} y_{27} - \beta_{14} y_{28}$,

$c_5 = \beta_4 y_2 + \beta_3 y_3 + \beta_{12} y_{18} + \beta_{11} y_{19} - \beta_8 y_{11} - \beta_{16} y_{27} - \beta_{15} y_{28}$,

$c_6 = \beta_4 y_3 + \beta_{12} y_{19} - \beta_{16} y_{28}$.

$\beta_1 = q_1 y_{12} - q_4 y_{14} + q_7 y_{15}$; $\beta_2 = q_2 y_{12} + q_1 y_{13} - q_5 y_{14} + q_8 y_{15} + q_7 y_{16}$;

$\beta_3 = q_3 y_{12} + q_2 y_{13} - q_6 y_{14} + q_9 y_{15} + q_8 y_{16}$; $\beta_4 = q_3 y_{13} + q_9 y_{16}$; $\beta_5 = q_1 y_4 - q_4 y_6 + q_7 y_8$;

$\beta_6 = q_2 y_4 + q_1 y_5 - q_5 y_6 - q_4 y_7 + q_8 y_8 + q_7 y_9$; $\beta_7 = q_3 y_4 + q_2 y_5 - q_6 y_6 - q_5 y_7 + q_9 y_8 + q_8 y_9$;

$\beta_8 = q_3 y_5 - q_6 y_7 + q_9 y_9$; $\beta_9 = r_1 y_4 - r_4 y_6 + r_7 y_8$; $\beta_{10} = r_2 y_4 + r_1 y_5 - r_5 y_6 - r_4 y_7 + r_8 y_8 + r_7 y_9$;

$\beta_{11} = r_3 y_4 + r_2 y_5 - r_6 y_6 - r_5 y_7 + r_9 y_8 + r_8 y_9$; $\beta_{12} = r_3 y_5 - r_6 y_7 + r_9 y_9$; $\beta_{13} = s_1 y_4 - s_4 y_6 + s_7 y_8$

$\beta_{14} = s_2 y_4 + s_1 y_5 - s_5 y_6 - s_4 y_7 + s_8 y_8 + s_7 y_9$; $\beta_{15} = s_3 y_4 + s_2 y_5 - s_6 y_6 - s_5 y_7 + s_9 y_8 + s_8 y_9$

$\beta_{16} = s_3 y_5 - s_6 y_7 + s_9 y_9$;

$q_1 = y_{22} y_{33} - y_{24} y_{31}$; $q_2 = y_{22} y_{34} + y_{23} y_{33} - y_{25} y_{31} - y_{24} y_{32}$; $q_3 = y_{23} y_{34} - y_{25} y_{32}$;

$q_4 = y_{20} y_{33} - y_{24} y_{29}$; $q_5 = y_{20} y_{34} + y_{21} y_{33} - y_{24} y_{30} - y_{25} y_{29}$; $q_6 = y_{21} y_{34} - y_{25} y_{30}$;

$q_7 = y_{20} y_{31} - y_{22} y_{29}$; $q_8 = y_{20} y_{32} + y_{21} y_{31} - y_{23} y_{29} - y_{22} y_{30}$; $q_9 = y_{21} y_{32} - y_{23} y_{30}$;

$r_1 = y_{14} y_{33} - y_{15} y_{31}$; $r_2 = y_{14} y_{34} - y_{16} y_{31} - y_{15} y_{32}$; $r_3 = -y_{16} y_{32}$; $r_4 = y_{12} y_{33} - y_{15} y_{29}$;

$r_5 = y_{12} y_{34} + y_{13} y_{33} - y_{16} y_{29} - y_{15} y_{30}$; $r_6 = y_{13} y_{34} - y_{16} y_{30}$; $r_7 = y_{12} y_{31} - y_{14} y_{29}$;

$r_8 = y_{12} y_{32} - y_{14} y_{30} + y_{13} y_{31}$; $r_9 = y_{13} y_{32}$;

$s_1 = y_{14} y_{24} - y_{15} y_{22}$; $s_2 = y_{14} y_{25} - y_{16} y_{22} - y_{15} y_{23}$; $s_3 = -y_{16} y_{23}$; $s_4 = y_{12} y_{24} - y_{15} y_{20}$

$s_5 = y_{12} y_{25} + y_{13} y_{24} - y_{16} y_{20} - y_{15} y_{21}$; $s_6 = y_{13} y_{25} - y_{16} y_{21}$; $s_7 = y_{12} y_{22} - y_{14} y_{20}$;

$s_8 = y_{12} y_{23} - y_{14} y_{21} + y_{13} y_{22}$; $s_9 = y_{13} y_{23}$;

$y_1 = e_{33} x_1 + \zeta_{33} x_6$; $y_2 = (e_{15} x_1 + \zeta_{15} x_6)/c^2 + (c_{44} + c_{13})/c + e_{33} x_2 + \zeta_{33} x_7$;

$y_3 = (e_{15} x_2 + \zeta_{15} x_7)/c^2$; $y_4 = e_{33} x_3 + \zeta_{33} x_8 + c_{33}$; $y_5 = (e_{15} x_3 + \zeta_{15} x_8 + c_{44})/c^2 - \bar{\rho}_{33}^{11}$;

$y_6 = e_{33} x_4 + \zeta_{33} x_9 + m_{33}/c$; $y_7 = (e_{15} x_4 + \zeta_{15} x_9)/c^2$; $y_8 = e_{33} x_5 + \zeta_{33} x_{10} + m_{33}$;

$y_9 = (e_{15} x_5 + \zeta_{15} x_{10})/c^2 - \bar{\rho}_{33}^{12}$; $y_{10} = \tilde{\zeta}_3 x_1 + e_3^* x_6$; $y_{11} = m_{11}/c + \tilde{\zeta}_3 x_2 + e_3^* x_7$;

$y_{12} = m_{33} + \tilde{\zeta}_3 x_3 + e_3^* x_8$; $y_{13} = -\bar{\rho}_{33}^{12}$; $y_{14} = R/c + \tilde{\zeta}_3 x_4 + e_3^* x_9$; $y_{15} = R + \tilde{\zeta}_3 x_5 + e_3^* x_{10}$;

$y_{16} = -\bar{\rho}_{33}^{22}$; $y_{17} = -x_1 \xi_{33} - x_6 A_{33}$; $y_{18} = (-x_1 \xi_{11} - x_6 A_{11})/c^2 + (e_{15} + e_{31})/c - \xi_{33} x_2 - A_{33} x_7$;

$y_{19} = -(x_2 \xi_{11} + x_7 A_{11})/c^2$; $y_{20} = e_{33} - x_3 \xi_{33} - x_8 A_{33}$; $y_{21} = (e_{15} - x_3 \xi_{11} - x_8 A_{11})/c^2$;

$y_{22} = \tilde{\zeta}_3/c - A_{33} x_9 - \xi_{33} x_4$; $y_{23} = -(x_4 \xi_{11} + x_9 A_{11})/c^2$; $y_{24} = -x_{10} A_{33} - x_5 \xi_{33} + \tilde{\zeta}_3$;

$y_{25} = -(x_5\xi_{11} + x_{10}A_{11})/c^2$; $y_{26} = -x_1A_{33} - x_6\xi_{33}^*$;
$y_{27} = (-x_1A_{11} - x_6\xi_{11}^*)/c^2 + (\zeta_{15} + \zeta_{31})/c - x_2A_{33} - x_7\xi_{33}^*$; $y_{28} = -(x_2A_{11} + x_7\xi_{11}^*)/c^2$;
$y_{29} = \zeta_{33} - x_3A_{33} - x_8\xi_{33}^*$; $y_{30} = (\zeta_{15} - x_3A_{11} - x_8\xi_{11}^*)/c^2$; $y_{31} = e_3^*/c - \xi_{33}^*x_9 - A_{33}x_4$;
$y_{32} = -(x_4A_{11} + x_9\xi_{11}^*)/c^2$; $y_{33} = -x_{10}\xi_{33}^* - x_5A_{33} + e_3^*$; $y_{34} = -(x_5A_{11} + x_{10}\xi_{11}^*)/c^2$;
$x_1 = -c_{44}\,d\,c$;
$x_2 = -(c_{11}/c^2 - \bar{\rho}_{11}^{11})\,d\,c - (m_{11}/c^2 - \bar{\rho}_{11}^{12})\,c/\tilde{\zeta}_3 + (m_{11}/c^2 - \bar{\rho}_{11}^{12})(e_{15} + e_{31})\,d\,c/\tilde{\zeta}_3$;
$x_3 = (m_{33}(e_{15} + e_{31})d - m_{33})/\tilde{\zeta}_3 - (c_{13} + c_{44})\,d$
$x_4 = -(m_{11}/c^2 - \bar{\rho}_{11}^{12})\,d\,c - (R/c^2 - \bar{\rho}_{11}^{22})\,c/\tilde{\zeta}_3 + (R/c^2 - \bar{\rho}_{11}^{22})(e_{15} + e_{31})\,d\,c/\tilde{\zeta}_3$;
$x_5 = (R(e_{15} + e_{31})d - R)/\tilde{\zeta}_3 - m_{11}\,d$; $x_6 = c_{44}\,\tilde{\zeta}_3\,d\,c/e_3^*$;
$x_7 = (c_{11}/c^2 - \bar{\rho}_{11}^{11})\,d\,c\,\tilde{\zeta}_3/e_3^* - (m_{11}/c^2 - \bar{\rho}_{11}^{12})(e_{15} + e_{31})\,d\,c/e_3^*$;
$x_8 = (c_{13} + c_{44})\,d\,\tilde{\zeta}_3/e_3^* - m_{33}(e_{15} + e_{31})\,d/e_3^*$;
$x_9 = (m_{11}/c^2 - \bar{\rho}_{11}^{12})\,d\,c\,\tilde{\zeta}_3/e_3^* - (R/c^2 - \bar{\rho}_{11}^{22})(e_{15} + e_{31})\,d\,c/e_3^*$;
$x_{10} = m_{11}\,d\,\tilde{\zeta}_3/e_3^* - R(e_{15} + e_{31})\,d/e_3^*$
$d = e_3^*/(e_3^*(e_{15} + e_{31}) - \tilde{\zeta}_3(\zeta_{15} + \zeta_{31}))$.

$d_{1i} = c_{55}\,p_i + (c_{55}\,r_{1i} + e_{15}\,r_{4i} + \zeta_{15}\,r_{5i})/c$,
$d_{2i} = (c_{33}\,r_{1i} + m_{33}\,r_{3i} + e_{33}r_{4i} + \zeta_{33}r_{5i})\,p_i + (c_{31} + m_{33}\,r_{2i})/c$,
$d_{3i} = (m_{33}\,r_{1i} + R\,r_{3i} + \tilde{\zeta}_3\,r_{4i} + e_3^*\,r_{5i})\,p_i + (m_{11} + R\,r_{2i})/c$,
$d_{4i} = (e_{33}\,r_{1i} + \tilde{\zeta}_3\,r_{3i} - \xi_{33}\,r_{4i} - a_{33}\,r_{5i})\,p_i + (e_{31} + \tilde{\zeta}_3\,r_{2i})/c$,
$d_{5i} = (\zeta_{33}\,r_{1i} + e_3^*\,r_{3i} - a_{33}\,r_{4i} - \xi_{33}^*\,r_{5i})\,p_i + (\zeta_{31} + e_3^*\,r_{2i})/c$.

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