Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 9, Number 5 (2013), pp. 483-487

http://www.ripublication.com/gjpam.htm

Cartesian Curves Possessing Fixed Points

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Abstract

In this paper we are presenting the curves which possess the fixed points with some finite multiplicity. Also it is discussed in this paper that how fixed points of a curve are important in order to trace the graph of that curve.

Keywords: Fixed points, multiplicity, cartesian curves, curve tracing.

Introduction:

For any continuous mapping there is necessarily some point such that $f(x)=x$. This is indicated by the point where the continuous function crosses the diagonal which represents $x=x$.Graphically we can find all fixed points of a function $f:R\to R$ simply by sketching the curve $y=f(x)$ and observing where it intersects the line $y=x$. At these points of intersection only we have $f(x)=x$. At the beginning we are presenting some curves in cartesian form which posses the fixed points with some finite multiplicity. In this paper we are considering only non zero fixed points.

There are different properties of the curve using which one can trace the graph of given curve. Here we will see how fixed points of a curve are useful while tracing the curve.

Definitions

Fixed points of a curve:

Let $y = f(x)$ *or* $f(x, y) = 0$ be the given curve. Then the fixed points of a given curve are the intersecting points of curve with the line $y = x$.

Finding fixed points of a curve:

To find the fixed points of a curve (in cartesian form) we put $y = x$ in the given equation of a curve and solving it for x, we get the fixed points of a curve with their multiplicity.

Results

1. The tangential fixed point should be counted as double fixed point, two fixed points in one.
2. $f(x) = x$ *and* $f'(x) = 1$ then the degree or multiplicity of such a fixed point is greater than one.
3. If $f(x) = x,\ f'(x) = 1$ *and* $f''(x) \neq 0$ then the multiplicity of the fixed point is two.
4. If $f(x) = x,\ f'(x) = 1$ *and* $f''(x) = 0$ then the multiplicity of the fixed point is three and which is the case of an inflection point.

Curves Possessing Fixed Points with Finite Multiplicity

1) $y^2(2a - x) = x^3$

Putting $y = x$ we get

$$x^3 - 2ax^2 + x^3 = 0$$

that is $$2x^2(a - x) = 0$$

that is $$x = 0 \quad or \quad x = 0 \quad or \quad x = a$$

Therefore $x = a$ is the fixed point of the curve with multiplicity one. For this curve we can also verify that $f(a) = a$.

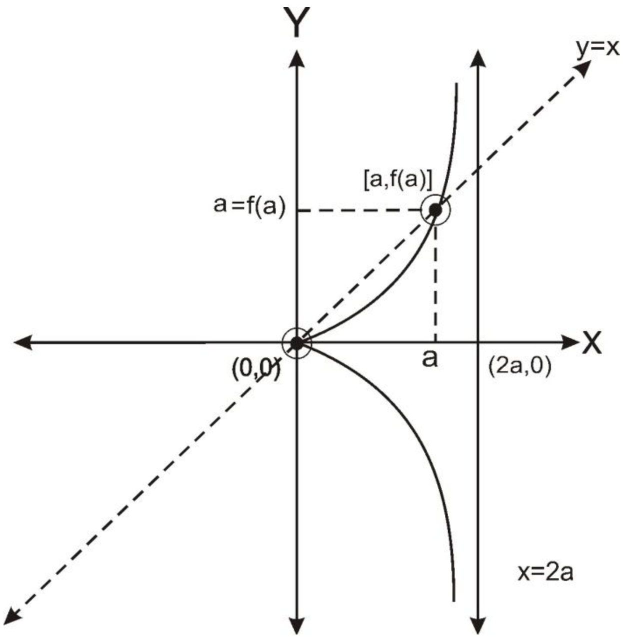

Figure.1

2) $a^2x^2 = y^3(2a - y)$

Putting $y = x$ we get

$$a^2x^2 = x^3(2a - x)$$

that is $x^2(2ax - x^2 - a^2) = 0$

that is $x = 0$ *or* $x = 0$ *or* $x = a$ *or* $x = a$

Therefore $x = a$ is the fixed point of the curve with multiplicity two. Hence this point of the curve should be tangential point.

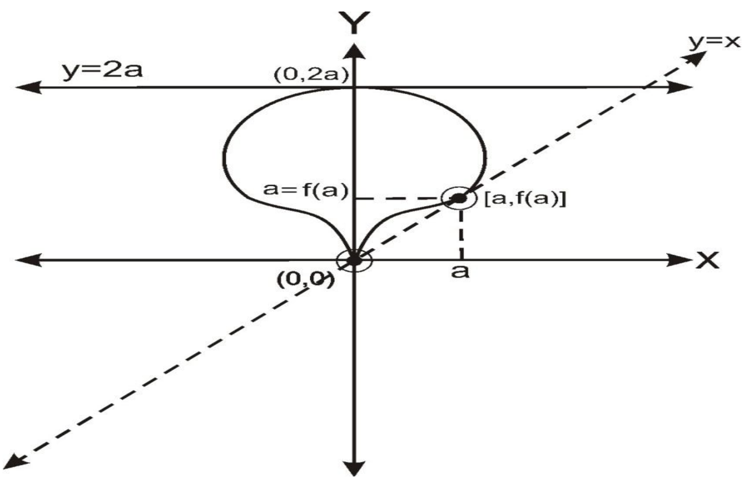

Figure.2

Curve with origin as a fixed point which is a point of inflection

3) $y(1+x^2)=x$

Putting $y=x$ we get

$$x+x^3-x=0$$

that is $x^3=0$

that is $x=0$ *or* $x=0$ *or* $x=0$

Therefore $x=0$ is the fixed point of the curve with multiplicity three.

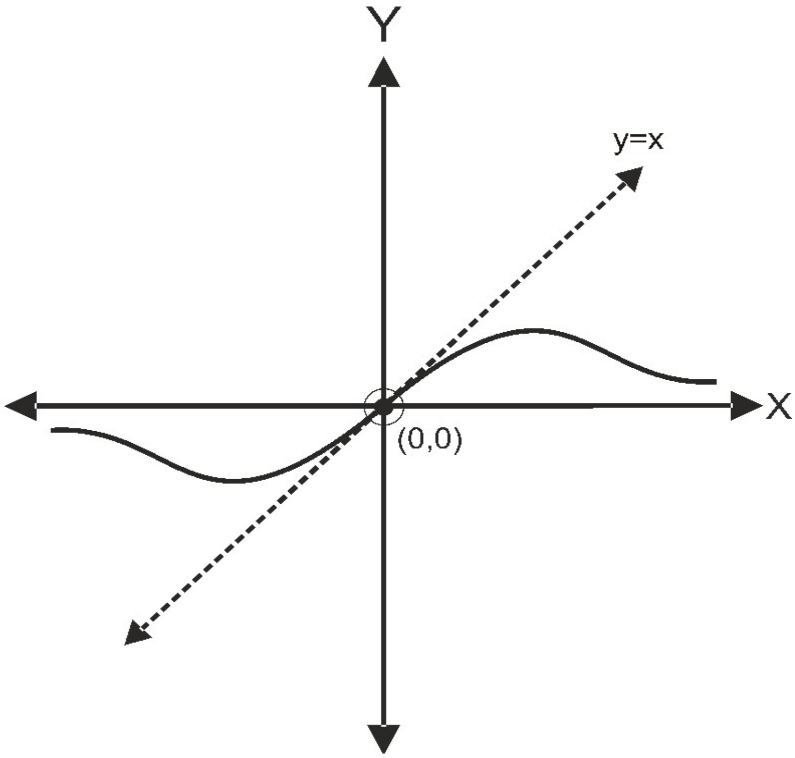

Figure.3

With $f(x)=\dfrac{x}{1+x^2}$ we can observe that $f(0)=0$, $f'(0)=1$ *and* $f''(0)=0$, hence $x=0$ is point of inflection of the curve.

Importance of Fixed points in Curve tracing

Now we will see how the fixed points of a curve are important in curve tracing.

Let us see the curve $y^2(2a-x)=x^3$.

For this curve we know that:

i. curve is symmetrical about X-axis.
ii. it passes through origin which is a cusp at which X-axis is the tangent.
iii. $x = 2a$ is an asymptote.
iv. curve exists only between $x = 0$ and $x = 2a$.

With this knowledge if someone tries to trace the curve then there is little bit confusion with possibilities like below.

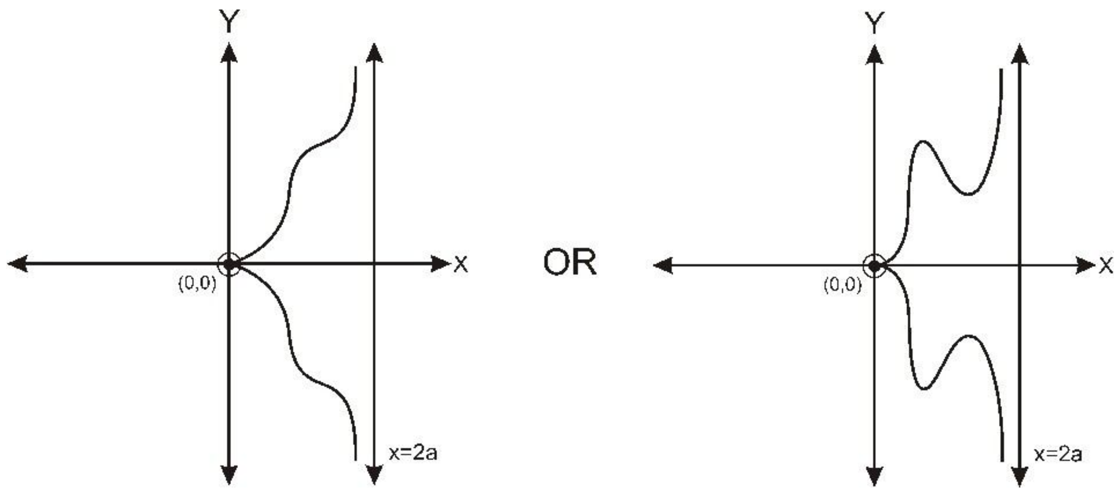

Figure.4

In addition to above four properties if we think about the fixed points of given curve then we can easily trace the exact graph of the curve as below

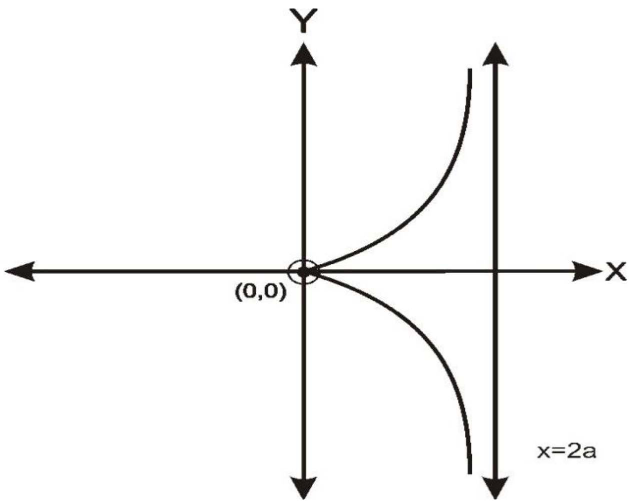

Figure.5