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Harmonic Indices of Some Classes of Graphs

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Abstract

The harmonic index H(G) of a graph G is defined as the sum of the weights $\frac{2}{d_u + d_v}$ of all edges uv of G, where d_u denotes the degree of a vertex u in G. We compute the Harmonic index of hanging star graphs and hanging flower graphs.

Keywords: Harmonic index; hanging stars; hanging flower graphs; Helm graphs

1. Introduction

Let $G = (V, E)$ be a simple connected graph, the harmonic index H(G)[2] of a graph G is defined as the sum of the weights $\frac{2}{d_u + d_v}$ of all edges uv of G, where d_u denotes the degree of a vertex u in G.

Various topological indices were defined and studied in detail. Finding a formula of various indices for a family of graphs is interesting. The Augmented Eccentric Connectivity Index of some thorn graphs was computed in [4]. The Eccentricity Connectivity Index of V-phenylenic nanotubes was given in [7]. Padmakar[5,6] et. al estimated the PI index of Polyacenes and Nanostructure and PI index of H-phenylenic Nanotubes and Nano tori was calculated in [1]. Tomislav Doslic [8] et al calculated the Eccentric Connectivity Index of Hexagonal Belts and Chains. Mohammad Ali Irammanesh[3] et al gave a formula to compute the Eccentricity Connectivity Index of some special graphs. In this paper, we give a formula to compute the harmonic index of hanging star graphs and hanging flower graphs.

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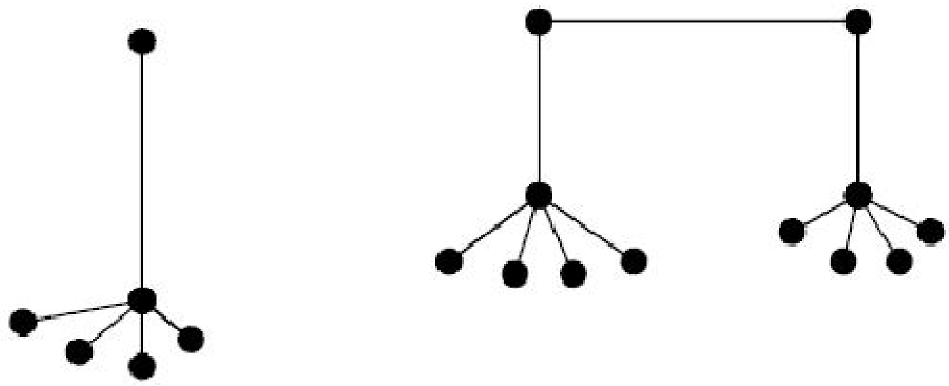
2. Harmonic index of hanging star graphs and hanging flower graphs

We now define hanging star graph and hanging flower graph. Let P_n be a path on n vertices. Let S_m be the bipartite graph $K_{1,m}$ called a star. The vertex with degree m is called the center of the star. We now take n copies of star S_m. To every vertex of the path, we associate a star by making the center of a star adjacent to a vertex of the path. The graph so obtained is called hanging star graph denoted by H_nS_m.

Example: The graph H_1S_4 and H_2S_4 are given below.



It is easy to calculate the Harmonic index of H_1S_m as 2(m+1)/(m+2).

Theorem 2.1: The Harmonic index of $H_2 S_m$ is $\frac{4(m^2+4m+2)}{(m+2)(m+3)}+\frac{1}{2}$.

Proof .

$$H(H_2S_m) = 2\left[m\left(\frac{2}{m+2}\right)+\frac{2}{m+3}\right]+\frac{2}{4}$$

$$= 4\left[\frac{m^2+4m+2}{(m+2)(m+3)}\right]+\frac{1}{2}$$ □

Corollary 2.2: The Harmonic index of P_6 is 17/6.

Proof: Substituting m=1 in the above theorem, we get $H(P_6)$=17/6. □

Theorem 2.3: The Harmonic index of H_nS_m, n>2 is equal to

$$H(H_2S_m)+\left(\frac{n-2}{3}\right)\left(\frac{7m^2+36m+20}{(m+2)(m+4)}\right)-\frac{1}{30}.$$

Proof: The edges of the star at the extreme contribute $2\left(\frac{2}{m+3}+\frac{2m}{m+2}\right)$ to the Harmonic index. The edges in the remaining (n-2) stars contribute $\frac{2m}{m+2}+\frac{2}{m+4}$ to the Harmonic index. Two end edges of the path contribute 2(2/5) and the remaining edges in the path contribute (n-3)(2/6) to the Harmonic index. Therefore,

$$H(H_nS_m) = 2\left[\frac{2}{m+3}+\frac{2m}{m+2}\right]+2\left(\frac{2}{5}\right)+(n-3)\left(\frac{2}{6}\right)+(n-2)\left[\frac{2m}{m+2}+\frac{2}{m+4}\right]$$

$$=2\left[\frac{2}{m+3}+\frac{2m}{m+2}\right]+\frac{4}{5}+(n-2)\left[\frac{2m}{m+2}+\frac{2}{m+4}+\frac{1}{3}\right]-\frac{1}{3}$$

$$=4\left[\frac{m^2+4m+2}{(m+2)(m+3)}\right]+\left(\frac{n-2}{3}\right)\left(\frac{7m^2+36m+20}{(m+2)(m+4)}\right)+\frac{7}{15}$$

$$=H(H_2S_m)-\frac{1}{2}+\left(\frac{n-2}{3}\right)\left(\frac{7m^2+36m+20}{(m+2)(m+4)}\right)+\frac{7}{15}$$

$$=H(H_2S_m)+\left(\frac{n-2}{3}\right)\left(\frac{7m^2+36m+20}{(m+2)(m+4)}\right)-\frac{1}{30}$$ □

Replacing n with n+1, we get a recurrence relation for the harmonic index of hanging stars.

Corollary 2.4: $H(H_{n+1}S_m)=H(H_nS_m)+\left(\frac{1}{3}\right)\left(\frac{7m^2+36m+20}{(m+2)(m+4)}\right)$.

We now define the Helm graph H_n with n >1 is defined to be the graph obtained from a wheel graph $W_{1,n}$ by attaching a pendant edge at each vertex of the n-cycle. A flower graph Fl_m with m>1 is defined to be the graph obtained from a Helm graph H_m by joining each pendant vertex to the central vertex of the Helm graph.

Let P_n be a path on n vertices. Take n copies of Fl_m. To every vertex of the path, we associate a flower by making the centre of the flower adjacent to a vertex of the path. The graph obtained is called hanging flower graph and is denoted by H_nFl_m.

Theorem 2.5: The Harmonic index of H_1Fl_m is $\frac{m(2m+11)(14m+27)}{12(2m+3)(2m+5)}+\frac{1}{m+1}$.

Proof: $H(H_1Fl_m)=\frac{2m}{2m+5}+\frac{2m}{2m+3}+\frac{2m}{6}+\frac{2m}{8}+\frac{2}{2m+2}$

$$=2m\left(\frac{1}{2m+5}+\frac{1}{2m+3}+\frac{1}{6}+\frac{1}{8}\right)+\frac{1}{m+1}$$

$$= \frac{m(2m+11)(14m+27)}{12(2m+3)(2m+5)} + \frac{1}{m+1}.$$ □

Let us denote

$$A = \frac{m(2m+11)(14m+27)}{12(2m+3)(2m+5)}.$$

Theorem 2.6: The Harmonic index of H_nS_m is recursively given by

$$H(H_{n+1}S_m) - H(H_nS_m) = A + \frac{1}{m+2} + \frac{1}{3}.$$

Proof: As calculated in theorem 2.5 each flower in the graph contribute $A = \frac{m(2m+11)(14m+27)}{12(2m+3)(2m+5)}$ to the Harmonic index of the graph. Let the edges of the path be $e_1, e_2, ..., e_n$ be the edges which join a vertex of the path and center of a flower. The edges e_1 and e_n contribute $\frac{2}{2m+3}$ to the harmonic index and the remaining edges contribute $\frac{2(n-2)}{2m+4}$ to the harmonic index. Two edges of the path contribute (2/5) each to the harmonic index and the remaining edges in the path contribute (n-3)(2/6).

$$H(H_nFl_m) = nA + 2\left(\frac{2}{2m+3}\right) + (n-2)\left(\frac{2}{2m+4}\right) + 2\left(\frac{2}{5}\right) + (n-3)\left(\frac{2}{6}\right)$$

$$= n\left(A + \frac{1}{m+2} + \frac{1}{3}\right) + \frac{4}{2m+3} - \frac{4}{2m+4} + \frac{4}{5} - 1.$$

Replacing n with n+1 in the above and calculating $H_{n+1}S_m$, we get

$$H(H_{n+1}Fl_m) - H(H_nFl_m) = A + \frac{1}{m+2} + \frac{1}{3}.$$ □

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