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# Two Warehouse Inventory Model for Deteriorating Items with Linear Trend in Demand, Shortages, Time Varying Holding Cost Under Inflationary Conditions and Permisibble Delay In Payments

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**Abstract**

A two-warehouse inventory model for deteriorating items with linear trend in demand with time varying holding cost and inflationary conditions under permissible delay in payments is developed. Shortages are allowed and completely backlogged. A rented warehouse (RW) is used to store the excess units over the capacity of the own warehouse. Numerical examples are provided to illustrate the model and sensitivity analysis is also carried out for parameters.

**Key Words:** Inventory model, Two-warehouse, Deterioration, Inflation, Shortages, Permissible delay in payment

## 1. INTRODUCTION:

The existing literature on classical inventory model generally deal with single storage facility with the assumption that the available warehouse of the organization has unlimited capacity. But in actual practice many times the supplier provide price discounts for bulk purchases and the retailer may purchase more goods than can be stored in single warehouse (own warehouse). Therefore a rented warehouse (RW) is used to store the excess units over the fixed capacity W of the own warehouse. The rented warehouse is charged higher unit holding cost then the own warehouse, but offers a better preserving facility with a lower rate of deterioration.

A two-warehouse inventory model was first developed by Hartley [5]. An inventory model with infinite rate of replenishment with two-warehouse was considered by Sarma [12]. Pakkala and Achary [10] extended the two-warehouse inventory model for deteriorating items with finite rate of replenishment and shortages. Related work is also find in (Benkherouf [1], Bhunia and Maiti [2], Kar et al. [7], Chung and Huang [3], Rong et al. [11]).

Madhavilata et al. [9] have developed a deterministic inventory model for a single item having two levels of storage. Demand was assumed to be exponentially increasing function of time. Tyagi and Singh [13] considered a two warehouse inventory model with time dependent demand, varying rate of deterioration and variable holding cost. Yang [15] considered a two warehouse inventory problem for deteriorating items with constant rate of demand under inflation in two alternatives when shortages are completely backordered. An order level inventory model with two levels of storage for deteriorating items was developed by Ghosh and Chakrabarty [4]. Demand was assumed to be time dependent and shortages were allowed and completely backlogged. Jaggi and Verma [6] developed a two warehouse inventory model with linear trend in demand under inflationary conditions. Shortages were considered and were completely backlogged.

Liang and Zhou [8] developed a two-warehouse inventory model for deteriorating items with constant rate of demand under conditionally permissible delay in payments. Yadav and Swami [14] studied the effect of permissible delay on two warehouse inventory model for deteriorating items with shortages.

In this paper we have developed a two-warehouse inventory model for deteriorating items with linear trend in demand with time varying holding cost under permissible delay in payments. Shortages are allowed and are completely backlogged under inflation. Numerical examples are provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.

## 2. ASSUMPTIONS AND NOTATIONS:

**NOTATIONS:**

The following notations are used for the development of the model:

| | |
|---|---|
| D(t) | : Demand rate is a linear function of time t ($a+bt$, $a>0$, $0<b<1$) |
| A | : Replenishment cost per order for two warehouse system |
| c | : Purchasing cost per unit |
| p | : Selling price per unit |
| $c_2$ | : Shortage cost per unit |
| HC(OW) | : Holding cost per unit time is a linear function of time t ($x_1+y_1t$, $x_1>0$, $0<y_1<1$) in OW |
| HC(RW) | : Holding cost per unit time is a linear function of time t ($x_2+y_2t$, $x_2>0$, $0<y_2<1$) in RW |
| $I_e$ | : Interest earned per year |
| $I_p$ | : Interest charged per year |
| M | : Permissible period of delay in settling the accounts with the supplier |

| | |
|---|---|
| T | : Length of inventory cycle |
| I(t) | : Inventory level at any instant of time t, $0 \leq t \leq T$ |
| W | : Capacity of owned warehouse |
| $I_0(t)$ | : Inventory level in OW at time t |
| $I_r(t)$ | : Inventory level in RW at time t |
| $Q_1$ | : Inventory level initially |
| $Q_2$ | : Shortage of inventory |
| Q | : Order quantity |
| R | : Inflation rate |
| $t_r$ | : Time at which the inventory level reaches zero in RW in two warehouse system |
| $\theta_1$ | : Deterioration rate in OW, $0< \theta_1<1$ |
| $\theta_2$ | : Deterioration rate in RW, $0< \theta_2<1$ |
| $TC_i$ | : Total relevant cost per unit time (i=1, 2, 3) |

**ASSUMPTIONS:**

The following assumptions are considered for the development of two warehouse model.

- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and completely backlogged.
- OW has a fixed capacity W units and the RW has unlimited capacity.
- The goods of OW are consumed only after consuming the goods kept in RW.
- The unit inventory costs per unit in the RW are higher than those in the OW.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

## 3. THE MATHEMATICAL MODEL AND ANALYSIS:

At time t=0, a lot size of certain units enter the system. W units are kept in OW and the rest is stored in RW. The items of OW are consumed only after consuming the goods kept in RW. In the interval $[0, t_r]$, the inventory in RW gradually decreases due to demand and deterioration and it reaches to zero at $t=t_r$. In OW, however, the inventory W decreases during the interval $[0, t_r]$ due to deterioration only, but during $[t_r, t_1]$, the inventory is depleted due to both demand and deterioration. By the time to $t_1$, both warehouses are empty. Shortages occur during $(t_1, T)$ of size $Q_2$ units. The figure describes the behaviour of inventory system.

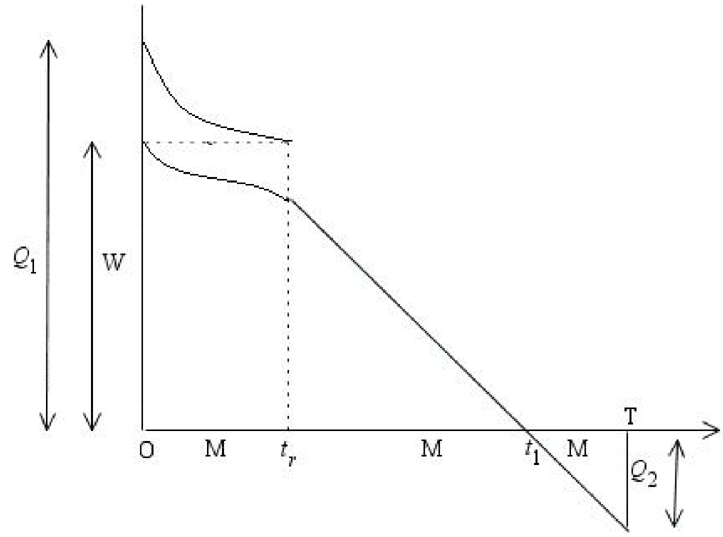



**Figure 1**

Hence, the inventory level at time t at RW and OW are governed by the following differential equations:

$$\frac{dI_r(t)}{dt} + \theta_2 I_r(t) = -(a+bt),\ \ 0 \le t \le t_r \tag{1}$$

with boundary conditions $I_r(t_r) = 0$ and

$$\frac{dI_0(t)}{dt} + \theta_1 I_0(t) = 0,\ 0 \le t \le t_r \tag{2}$$

with initial condition $I_0(0) = W$, respectively.

While during the interval $(t_r, t_1)$, the inventory in OW reduces to zero due to the combined effect of demand and deterioration both. So the inventory level at time t at OW, $I_0(t)$, is governed by the following differential equation:

$$\frac{dI_0(t)}{dt} + \theta_1 I_0(t) = -(a+bt),\ t_r \le t \le t_1 \tag{3}$$

with the boundary condition $I_0(t_1)=0$.

Similarly during $(t_1, T)$ the shortage level at time t, $I_s(t)$ is governed by the following differential equation:

$$\frac{dI_s(t)}{dt} = -(a+bt),\ \ t_1 \le t \le T, \tag{4}$$

with the boundary condition $I_s(t_1)=0$.

The solutions to equations (1) to (4) are given by:

$$I_r(t) = \begin{bmatrix} a\left(t_r - t\right) + \frac{1}{2}b\left(t_r^2 - t^2\right) + \frac{a\theta_2}{2}\left(t_r^2 - t^2\right) \\ + \frac{b\theta_2}{3}\left(t_r^3 - t^3\right) - a\theta_2 t\left(t_r - t\right) - \frac{b\theta_2 t}{2}\left(t_r^2 - t^2\right) \end{bmatrix},\ 0 \le t \le t_r \quad (5)$$

$$I_o(t) = W\left(1 - \theta_1 t\right),\ 0 \le t \le t_r \quad (6)$$

$$I_o(t) = \begin{bmatrix} a\left(t_1 - t\right) + \frac{1}{2}b\left(t_1^2 - t^2\right) + \frac{1}{2}a\theta_1\left(t_1^2 - t^2\right) \\ + \frac{1}{3}b\theta_1\left(t_1^3 - t^3\right) - a\theta_1 t\left(t_1 - t\right) - \frac{1}{2}b\theta_1 t\left(t_1^2 - t^2\right) \end{bmatrix},\ t_r \le t \le t_1 \quad (7)$$

$$I_s(t) = -\left[a(t - t_1) + \frac{1}{2}b(t^2 - t_1^2)\right],\ t_1 \le t \le T, \quad (8)$$

(by neglecting higher powers of $\theta_1$, $\theta_2$)

Using the condition $I_r(t) = Q_1 - W$ at $t=0$ in equation (5), we have

$$Q_1 - W = \left[at_r + \frac{1}{2}bt_r^2 + \frac{1}{2}a\theta_2 t_r^2 + \frac{1}{3}b\theta_2 t_r^3\right],$$

$$\therefore\ Q_1 - W = \left[at_r + \frac{1}{2}bt_r^2 + \frac{1}{2}a\theta_2 t_r^2 + \frac{1}{3}b\theta_2 t_r^3\right]. \quad (9)$$

Using the condition $I_s(t) = Q - Q_1$ at $t=T$ in equation (8), we have

$$Q - Q_1 = -\left[a(T - t_1) + \frac{1}{2}b(T^2 - t_1^2)\right]$$

$$\therefore\ Q - Q_1 = -\left[a(T - t_1) + \frac{1}{2}b(T^2 - t_1^2)\right]. \quad (10)$$

Based on the assumptions and descriptions of the model, the total annual relevant costs TC, include the following elements:

(i) Ordering cost (OC) = A (11)

(ii) $HC(RW) = \int_0^{t_r} (x_2 + y_2 t) I_r(t) e^{-Rt} dt$

$$= \int_0^{t_r} (x_2 + y_2 t)\begin{bmatrix} a\left(t_r - t\right) + \frac{1}{2}b\left(t_r^2 - t^2\right) + \frac{a\theta_2}{2}\left(t_r^2 - t^2\right) \\ + \frac{b\theta_2}{3}\left(t_r^3 - t^3\right) - a\theta_2 t\left(t_r - t\right) - \frac{b\theta_2 t}{2}\left(t_r^2 - t^2\right) \end{bmatrix} e^{-Rt} dt$$

$$= -\frac{1}{60}\left[\begin{array}{l}\frac{5}{6}y_2Rb\theta_2t_r^4 + \left(\left(2x_2Rb + y_2\left(-2b + Ra\right)\right)\theta_2 + 4y_2Rb\right)t_r^3 \\ + \left(\begin{array}{l}\left(\left(\left(-\frac{15}{2}b + \frac{5}{2}Ra\right)x_2 - \frac{5}{2}ay_2\right)\theta_2\right. \\ \left. + \frac{15}{2}x_2Rb + 5\left(-\frac{3}{2}b + Ra\right)y_2\right)\end{array}\right)t_r^2 \\ + \left(-10x_2a\theta_2 + \left(-20b + 10Ra\right)x_2 - 10y_2a\right)t_r - 30x_2a\end{array}\right]t_r^2 \qquad (12)$$

(by neglecting higher powers of R)

(iii) $HC(OW) = \int_0^{t_1}(x_1+y_1t)I_0(t)\,e^{-Rt}dt = \int_0^{t_r}(x_1+y_1t)I_0(t)e^{-Rt}dt + \int_{t_r}^{t_1}(x_1+y_1t)I_0(t)e^{-Rt}dt$

$$= \int_0^{t_r}(x_1+y_1t)W\left(1 - \theta_1t\right)e^{-Rt}dt$$

$$+ \int_{t_r}^{T}(x_1+y_1t)\left[\begin{array}{l}a\left(T - t\right) + \frac{1}{2}b\left(T^2 - t^2\right) + \frac{1}{2}a\theta_1\left(T^2 - t^2\right) \\ + \frac{1}{3}b\theta_1\left(T^3 - t^3\right) - a\theta_1t\left(T - t\right) - \frac{1}{2}b\theta_1t\left(T^2 - t^2\right)\end{array}\right]e^{-Rt}dt$$

$$= \frac{1}{36}y_1Rb\theta_1t_r^6 + \frac{1}{360}\left[\left(12x_1Rb + 36y_1\left(Ra - \frac{1}{3}b\right)\right)\theta_1 - 36y_1Rb\right]t_r^5$$

$$-\frac{1}{360}\left[\begin{array}{l}-45y_1Rb\theta_1t_1^2 - 90y_1Ra\theta_1t_1 + \\ \left(\left(-15b + 45Ra\right)x_1 + 90y_1\left(-\frac{1}{2}a + WR\right)\right)\theta_1 \\ -45x_1Rb - 90\left(-\frac{1}{2}b + Ra\right)y_1\end{array}\right]t_r^4$$

$$+\frac{1}{360}\begin{bmatrix} 40y_1Rb\theta_1 t_1^3 + \begin{pmatrix} \left(-60x_1Rb+60y_1\left(Ra+b\right)\right)\theta_1 \\ +60y_1Rb \end{pmatrix} t_1^2 \\ -120a\left(\left(x_1R-y_1\right)\theta_1 - y_1R\right)t_1 \\ +\left(\left(-60a+120WR\right)x_1-120Wy_1\right)\theta_1 \\ +\left(60b-120Ra\right)x_1 - 120y_1\left(WR-a\right)\end{bmatrix} t_r^3$$

$$+\frac{1}{360}\begin{bmatrix} 60b\theta_1\left(x_1R-y_1\right)t_1^3 + \begin{pmatrix} \left(\left(90b+90Ra\right)x_1-90y_1a\right)\theta_1 \\ +90b\left(x_1R-y_1\right)\end{pmatrix} t_1^2 \\ +180a\left(-y_1+x_1R+x_1\theta_1\right)t_1 - 180Wx_1\theta_1 \\ +\left(-180WR+180a\right)x_1+180Wy_1 \end{bmatrix} t_r^2$$

$$+x_1\left(-\frac{1}{3}b\theta_1 t_1^3 + \left(-\frac{1}{2}b-\frac{1}{2}a\theta_1\right)t_1^2 - at_1 + W\right)t_r$$

$$-\frac{1}{60}\begin{bmatrix} \frac{5}{6}y_1Rb\theta_1 t_1^2 + \left(\left(2x_1Rb+y_1\left(Ra-2b\right)\right)\theta_1 + 4y_1Rb\right)t_1^3 \\ +\begin{pmatrix} \left(\left(-\frac{15}{2}b+\frac{5}{2}Ra\right)x_1 - \frac{5}{2}y_1a\right)\theta_1 \\ +\frac{15}{2}x_1Rb + 5y_1\left(-\frac{3}{2}b+Ra\right) \end{pmatrix} t_1^2 \\ +\left(-10x_1a\theta_1 + \left(-20b+10Ra\right)x_1 - 10y_1a\right)t_1 - 30x_1a \end{bmatrix} t_1^2 \qquad (13)$$

(iv) Shortage cost:

$$SC = -c_2\int_{t_1}^{T} I(t)e^{-Rt}dt = -c_2\int_{t_1}^{T}\left[a\left(t_1-t\right)+\frac{b}{2}\left(t_1^2-t^2\right)\right]e^{-Rt}dt$$

$$= -\frac{1}{8}\left(T-t_1\right)^2 c_2\begin{bmatrix} Rbt_1^2 + \left(2TRb+\frac{4}{3}Ra-\frac{8}{3}b\right)t_1 \\ +RbT^2 + \left(-\frac{4}{3}b+\frac{8}{3}Ra\right)T - 4a \end{bmatrix} \qquad (14)$$

(v) Deterioration cost

The amount of deterioration in both RW and OW during $[0, t_1]$ are:

$$\theta_2\int_0^{t_r} I_r(t)dt \text{ and } \theta_1\int_0^{t_1} I_0(t)dt$$

So deterioration cost

$$DC = c\left[\theta_2\int_0^{t_r} I_r(t)e^{-Rt}dt + \theta_1\int_0^{t_1} I_0(t)e^{-Rt}dt\right]$$

$$= c\left[\theta_2\int_0^{t_r} I_r(t)e^{-Rt}dt + \theta_1\int_0^{t_r} I_0(t)e^{-Rt}dt + \theta_1\int_{t_r}^{t_1} I_0(t)e^{-Rt}dt\right]$$

$$= \frac{1}{3}c\left[\begin{array}{l} \left(\begin{array}{l} \frac{1}{10}Rbt_r^2 + \left(-\frac{1}{8}b + \frac{3}{8}Ra\right)t_r^4 + \left(-Rat_1 - \frac{1}{2}Rbt_1^2 - \frac{1}{2}a + WR\right)t_r^3 \\ + \left(\frac{1}{2}Rbt_1^2 + \left(\frac{3}{4}b + \frac{3}{4}Ra\right)t_1^2 + \frac{3}{2}at_1 - \frac{3}{2}W\right)t_r^2 \\ + \left(-bt_1^3 - \frac{3}{2}at_1^2\right)t_r - \frac{1}{8}t_1^3\left(\frac{4}{5}Rbt_1^2 + \left(-3b + Ra\right)t_1 - 4a\right) \end{array}\right)\theta_1^2 \\ + \left(\begin{array}{l} -\frac{3}{8}t_r^4Rb + \left(\frac{1}{2}b - Ra\right)t_r^3 + \left(\frac{3}{2}Rat_1 - \frac{3}{2}WR + \frac{3}{4}Rbt_1^2 + \frac{3}{2}a\right)t_r^2 \\ + \left(-3at_1 + 3W - \frac{3}{2}bt_1^2\right)t_r - \frac{1}{2}t_1^2\left(\frac{3}{4}Rbt_1^2 + \left(-2b + Ra\right)t_1 - 3a\right) \end{array}\right)\theta_1 \\ - \frac{1}{8}\left(\begin{array}{l} \frac{4}{5}Rb\theta_2t_r^3 + \left(\left(a\theta_2 + 3b\right)R - 3b\theta_2\right)t_r^2 \\ + \left(-4a\theta_2 + 4Ra - 8b\right)t_r - 12a \end{array}\right)t_r^2\theta_2 \end{array}\right] \tag{15}$$

(vi) Interest Earned: There are two cases:

**Case I : M ≤ T:**

In this case interest earned is:

$$IE_1 = pI_e\int_0^M \left(a + bt\right)te^{-Rt}dt$$

$$= pI_e\left[-\frac{1}{4}bRM^4 + \frac{1}{3}\left(-Ra + b\right)M^3 + \frac{1}{2}aM^2\right] \tag{16}$$

**Case II : M > T:**

In this case interest earned is:

$$IE_2 = pI_e\left(\int_0^{t_1}\left(a+bt\right)te^{-Rt}dt + \left(a + bt_1\right)t_1\left(M - t_1\right)\right)$$

$$= pI_e\left[-\frac{1}{4}bRt_1^2 + \frac{1}{3}\left(-Ra + b\right)t_1^3 + \frac{1}{2}at_1^2 + \left(a+bt_1\right)t_1\left(M-t_1\right)\right] \tag{17}$$

(vii) Interest Payable: There are three cases described as in figure:

**Case I : $M \le t_r \le T$:**

In this case, annual interest payable is:

$$IP_1 = cI_p\left[\int_M^{t_r} I_r(t)e^{-Rt}dt + \int_M^{t_r} I_0(t)e^{-Rt}dt + \int_{t_r}^{t_1} I_0(t)e^{-Rt}dt\right]$$

$$= \frac{cI_p}{30}\left[\begin{array}{l} -Rb(\theta_2 - \theta_1)t_r^5 + \left(\begin{array}{l}\left(\left(\frac{15}{4}\theta_1 - \frac{5}{4}\theta_2\right)a - \frac{15}{2}b\right)R \\ + \frac{15}{4}\left(\theta_2 - \frac{1}{3}\theta_1\right)b\end{array}\right)t_r^4 \\ +\left(\begin{array}{l}\left((-15 - 10\theta_1 t_1)a + \left(-5\theta_1 t_1^2 + 5M^2\theta_2\right)b + 10W\theta_1\right)R \\ + (-5\theta_1 + 5\theta_2)a + (-10\theta_2 M + 15)b\end{array}\right)t_r^3\end{array}\right]$$

$$+ \frac{cI_p}{30}\left[+\left(\begin{array}{l}\left(\begin{array}{l}\left(15t_1 + \frac{15}{2}\theta_1 t_1^2 + \frac{15}{2}M^2\theta_2\right)a + \\ \left(\frac{15}{2}M^2 + \frac{15}{2}t_1^2 - 5M^3\theta_2 + 5\theta_1 t_1^3\right)b - 15W\end{array}\right)R \\ +(15\theta_1 t_1 + 30 - 15\theta_2 M)a + \\ \left(\frac{15}{2}\theta_1 t_1^2 + \frac{15}{2}M^2\theta_2 - 15M\right)b - 15W\theta_1\end{array}\right)t_r^2\right]$$

$$+ \frac{cI_p}{30}\left[\left(\begin{array}{l}(15M^2 - 10M^3\theta_2)aR + \\ \left(-15\theta_1 t_1^2 - 30t_1 - 30M + 15M^2\theta_2\right)a \\ +\left(-15t_1^2 - 10\theta_1 t_1^3\right)b + 30W\end{array}\right)t_r\right]$$

$$+ \frac{cI_p}{30}\left[\begin{array}{l}\left(\begin{array}{l}\left(-10M^3 - 5t_1^3 - \frac{5}{4}t_1^4\theta_1 + \frac{15}{4}M^4\theta_2\right)a \\ + \left(-\frac{15}{4}t_1^2 - \frac{15}{4}M^4 + \theta_2 M^5 - \theta_1 t_1^5\right)b \\ + 15WM^2 - 10W\theta_1 M^3\end{array}\right)R \\ +\left(-5M^3\theta_2 + 15M^2 + 5\theta_1 t_1^2 + 15t_1^2\right)a \\ + \left(10t_1^3 + 5M^3 + \frac{15}{4}t_1^4\theta_1 - \frac{5}{4}M^4\theta_2\right)b \\ +15MW(-2 + M\theta_1)\end{array}\right] \tag{18}$$

**Case II : $t_r \le M \le T$:**

In this case interest payable is:

$$IP_2 = cI_p \int_M^{t_1} I_0(t)e^{-Rt}dt$$

$$= \frac{cI_p}{30}\left[\begin{array}{l} -Rb\theta_1 t_1^3 + \left(-2MRb\theta_1 + \left(\frac{15}{4}b - \frac{5}{4}Ra\right)\theta_1 - \frac{15}{4}Rb\right)t_1^2 \\ + \left(\begin{array}{l} 2M^2Rb\theta_1 + \left(\left(-\frac{5}{2}Ra - \frac{5}{2}b\right)\theta_1 - \frac{15}{2}Rb\right)M \\ + 5a\theta_1 - 5Ra + 10b \end{array}\right)t_1 \\ + Rb\theta_1 M^3 + \left(\left(-\frac{5}{4}b + \frac{15}{4}Ra\right)\theta_1 - \frac{15}{4}Rb\right)M^2 \\ + \left(5b - 10Ra - 5a\theta_1\right)M + 15a \end{array}\right]\left(M - t_1\right)^2 \quad (19)$$

**Case III : M > T:**

In this case, no interest charges are paid for the item. So,

$$IP_3 = 0. \quad (20)$$

The retailer's total cost during a cycle, $TC_i(t_r, T)$, i=1, 2, 3 consisted of the following:

$$TC_i = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + IP_i - IE_i\right] \quad (21)$$

Substituting values from equations (11) to (15) and equations (16) to (20) in equation (21), total costs for the three cases will be as under:

$$TC_1 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + IP_1 - IE_1\right] \quad (22)$$

$$TC_2 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + IP_2 - IE_2\right] \quad (23)$$

$$TC_3 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC - IE_2\right] \quad (24)$$

The optimal value of $t_r = t_r^*$, $T=T^*$ (say), which minimizes $TC_i(t_r, T)$ can be obtained by solving equation (22), (23) and (24) by differentiating it with respect to $t_r$ and T and equate it to zero

i.e. $$\frac{\partial TC_i(t_r,T)}{\partial t_r} = 0, \quad \frac{\partial TC_i(t_r,T)}{\partial T} = 0, \quad (25)$$

provided it satisfies the condition

$$\frac{\partial^2 TC_i(t_r,T)}{\partial t_r^2} > 0,\ \frac{\partial^2 TC_i(t_r,T)}{\partial T^2} > 0 \text{ and}$$

$$\left[\left(\frac{\partial^2 TC_i(t_r,T)}{\partial t_r^2}\right)\left(\frac{\partial^2 TC_i(t_r,T)}{\partial T^2}\right) - \left(\frac{\partial^2 TC_i(t_r,T)}{\partial t_r \partial T}\right)^2\right] > 0,\ (i=1, 2, 3). \qquad (26)$$

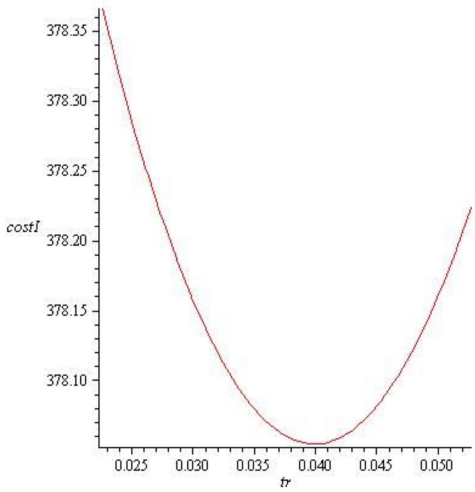
# 4. NUMERICAL EXAMPLES:

- Case I: Considering A= Rs.150, W = 100, a = 200, b=0.05, c=Rs. 10, p= Rs. 15, $\theta_1$=0.1, $\theta_2$ =0.06, $x_1$ = Rs. 1, $y_1$=0.05, $x_2$= Rs. 3, $y_2$=0.06, Ip= Rs. 0.15, Ie= Rs. 0.12, R = 0.06, M=0.01 year, in appropriate units. The optimal value of $t_r^*$ =0.0399, $t_1^*$=0.5774, T*=0.8260 and $TC_1^*$= Rs. 378.0545.
- Case II: Considering A= Rs.150, W = 100, a = 200, b=0.05, c = Rs. 10, p= Rs. 15, $\theta_1$=0.1, $\theta_2$ =0.06, $x_1$= Rs. 1, $y_1$=0.05, $x_2$= Rs. 3, $y_2$=0.06, Ip= Rs. 0.15, Ie = Rs. 0.12, M=0.55 year, in appropriate units. The optimal value of $t_r^*$=0.0475, $t_1^*$=0.6147, T*=0.7773 and $TC_2^*$ = Rs. 248.1514.
- Case III: Considering A= Rs.150, W = 100, a = 200, b=0.05, c = Rs. 10, p= Rs. 15, $\theta_1$=0.1, $\theta_2$ =0.06, $x_1$= Rs. 1, $y_1$=0.05, $x_2$= Rs. 3, $y_2$=0.06, Ip= Rs. 0.15, Ie= Rs. 0.12, M = 0.7 year, in appropriate units. The optimal value of $t_r^*$ =0.0464, $t_1^*$=0.6120, T*=0.7464 and $TC_1^*$ = Rs. 205.4638.

The second order conditions given in equation (26) are also satisfied. The graphical representation of the convexity of the cost functions for the three cases are also given.

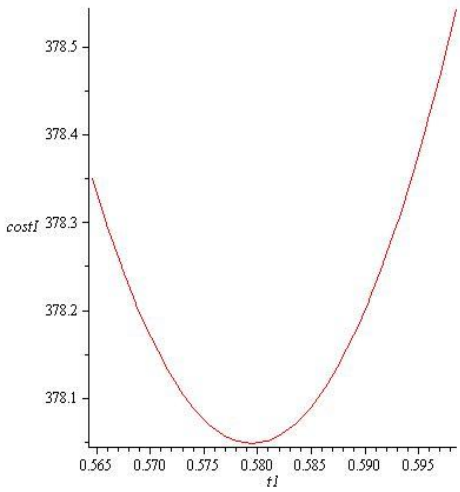
**Case I**

$t_r$ and cost



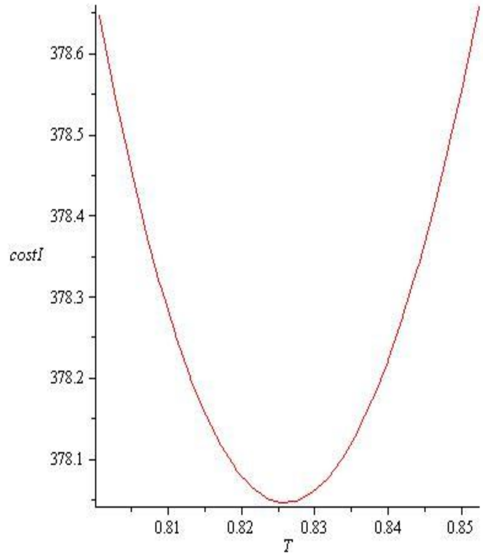

Graph 1

$t_1$ and cost



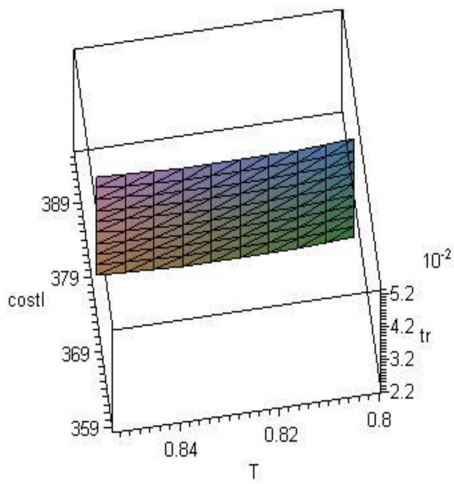

Graph 2

T and cost




Graph 3

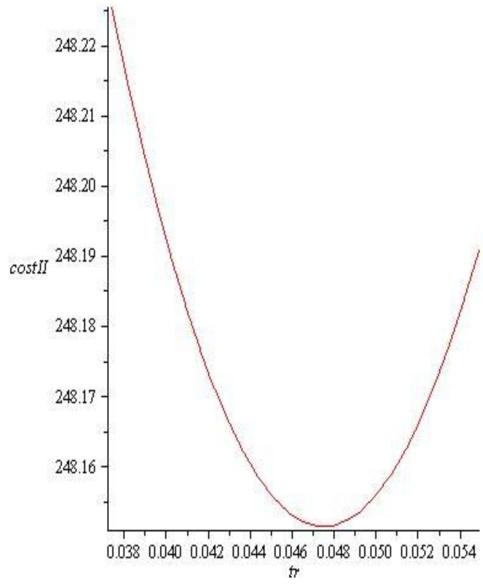
T, $t_r$ and cost




Graph 4

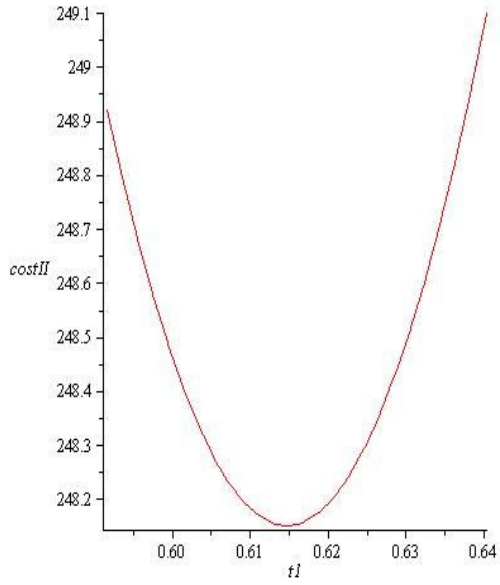
**Case II**

$t_r$ and cost



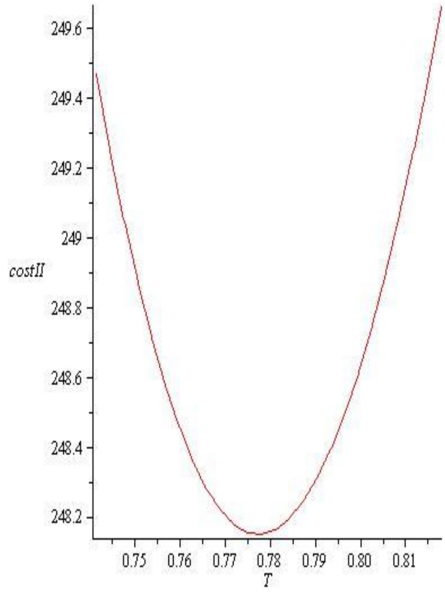

Graph 5

$t_1$ and cost



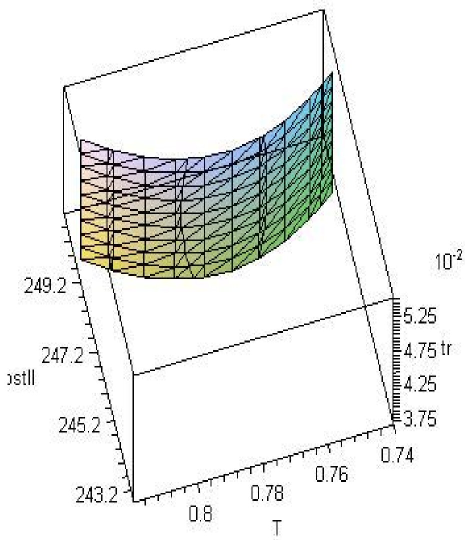

Graph 6

T and cost




Graph 7

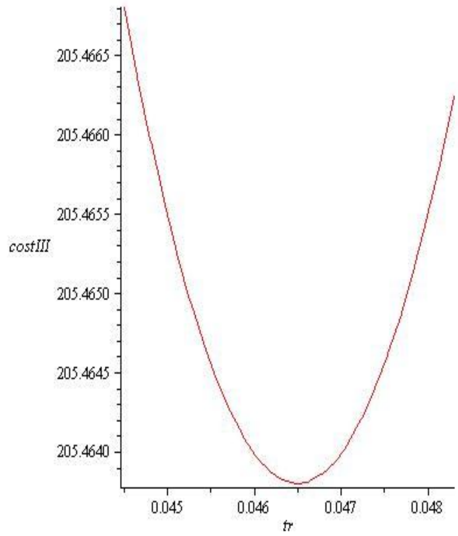
T, $t_r$ and cost




Graph 8

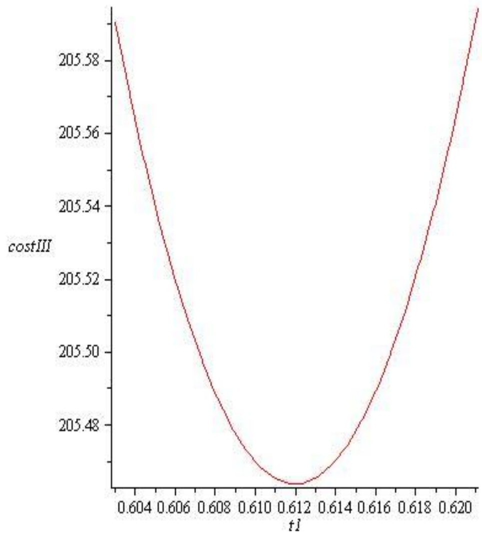
Case III

$t_r$ and cost



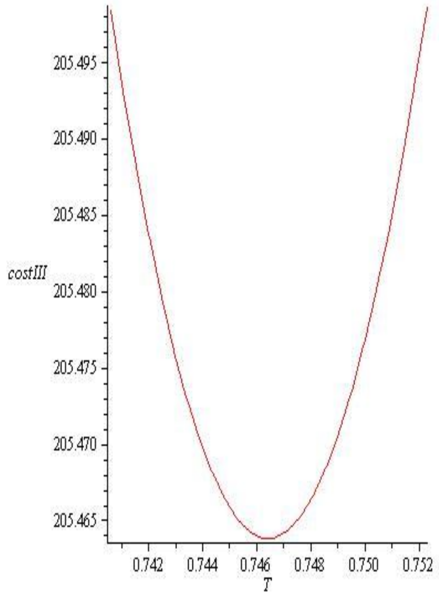

Graph 9

$t_1$ and cost



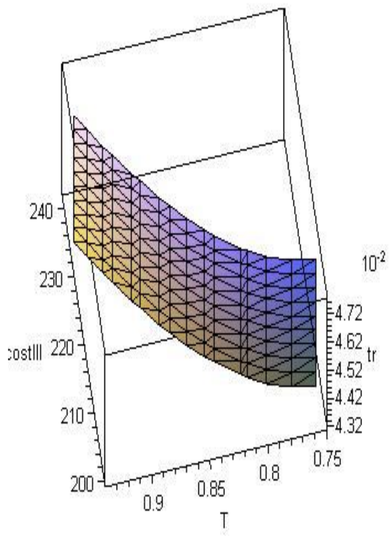

Graph 10

T and cost




Graph 11

T, $t_r$ and cost




Graph 12

## 5. SENSITIVITY ANALYSIS:

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

**Table 1**
**Sensitivity Analysis**
**Case I ($M \le t_r \le T$)**

| Parameter | % | $t_r$ | $t_1$ | T | Cost |
|---|---|---|---|---|---|
| a | +10% | 0.0527 | 0.5646 | 0.8006 | 395.6044 |
| | +5% | 0.0468 | 0.5710 | 0.8132 | 386.9648 |
| | -5% | 0.0319 | 0.5837 | 0.8391 | 368.8166 |
| | -10% | 0.0226 | 0.5895 | 0.8524 | 359.2408 |
| $x_1$ | +10% | 0.0338 | 0.5619 | 0.8128 | 381.9759 |
| | +5% | 0.0368 | 0.5696 | 0.8194 | 380.0277 |
| | -5% | 0.0429 | 0.5853 | 0.8326 | 378.0627 |
| | -10% | 0.0459 | 0.5932 | 0.8393 | 374.0916 |
| $x_2$ | +10% | 0.0376 | 0.5752 | 0.8237 | 378.0833 |
| | +5% | 0.0397 | 0.5768 | 0.8255 | 378.0749 |
| | -5% | 0.0411 | 0.5787 | 0.8272 | 378.0469 |
| | -10% | 0.0425 | 0.5800 | 0.8285 | 377.9845 |
| $\theta_1$ | +10% | 0.0341 | 0.5612 | 0.8122 | 382.1886 |
| | +5% | 0.0370 | 0.5693 | 0.8191 | 380.1345 |

| | | | | | |
|---|---|---|---|---|---|
| | -5% | 0.0427 | 0.5857 | 0.8329 | 375.9236 |
| | -10% | 0.0455 | 0.5940 | 0.8399 | 373.7668 |
| $\theta_2$ | +10% | 0.0394 | 0.5770 | 0.8255 | 378.0576 |
| | +5% | 0.0397 | 0.5772 | 0.8258 | 378.0519 |
| | -5% | 0.0401 | 0.5777 | 0.8263 | 378.0402 |
| | -10% | 0.0404 | 0.5779 | 0.8265 | 378.0343 |
| R | +10% | 0.0405 | 0.5789 | 0.8286 | 377.6117 |
| | +5% | 0.0402 | 0.5782 | 0.8273 | 377.8293 |
| | -5% | 0.0396 | 0.5762 | 0.8247 | 378.2622 |
| | -10% | 0.0393 | 0.5760 | 0.8235 | 378.4775 |

| Parameter | | $t_r$ | $t_1$ | T | Cost |
|---|---|---|---|---|---|
| A | 150 | 0.0399 | 0.5774 | 0.8260 | 378.0545 |
| | 200 | 0.0989 | 0.7154 | 1.0032 | 432.7213 |
| | 250 | 0.1492 | 0.8331 | 1.1546 | 479.0672 |
| | 300 | 0.1938 | 0.9373 | 1.2894 | 519.9834 |

**Table 2**
**Sensitivity Analysis**
**Case II ($t_r \le M \le T$)**

| Parameter | % | $t_r$ | $t_1$ | T | Cost |
|---|---|---|---|---|---|
| a | +10% | 0.0549 | 0.5916 | 0.7414 | 251.8684 |
| | +5% | 0.0514 | 0.6028 | 0.7588 | 250.1521 |
| | -5% | 0.0428 | 0.6271 | 0.7970 | 245.8443 |
| | -10% | 0.0374 | 0.6403 | 0.8179 | 243.2054 |
| $x_1$ | +10% | 0.0447 | 0.6038 | 0.7695 | 252.8647 |
| | +5% | 0.0462 | 0.6092 | 0.734 | 250.5213 |
| | -5% | 0.0487 | 0.6202 | 0.7813 | 248.1650 |
| | -10% | 0.0499 | 0.6257 | 0.7853 | 243.3312 |
| $x_2$ | +10% | 0.0445 | 0.6130 | 0.7757 | 248.2331 |
| | +5% | 0.0459 | 0.6138 | 0.7765 | 248.1936 |
| | -5% | 0.0491 | 0.6156 | 0.7782 | 248.1064 |
| | -10% | 0.0509 | 0.6166 | 0.7792 | 248.0581 |
| $\theta_1$ | +10% | 0.0453 | 0.6034 | 0.7692 | 253.0196 |
| | +5% | 0.0461 | 0.6112 | 0.7605 | 250.8226 |
| | -5% | 0.0485 | 0.6204 | 0.7814 | 245.6790 |
| | -10% | 0.0494 | 0.6261 | 0.7856 | 243.1806 |
| $\theta_2$ | +10% | 0.0468 | 0.6143 | 0.7770 | 248.1687 |
| | +5% | 0.0471 | 0.6145 | 0.7771 | 248.1602 |
| | -5% | 0.0478 | 0.6148 | 0.7775 | 248.1426 |
| | -10% | 0.0481 | 0.6150 | 0.7777 | 248.1337 |
| R | +10% | 0.0479 | 0.6157 | 0.7791 | 248.0616 |

| | | | | | |
|---|---|---|---|---|---|
| | +5% | 0.0477 | 0.6152 | 0.7782 | 248.1068 |
| | -5% | 0.0473 | 0.6142 | 0.7764 | 248.1956 |
| | -10% | 0.0471 | 0.6136 | 0.7755 | 248.2394 |

| Parameter | | $t_r$ | $t_1$ | T | Cost |
|---|---|---|---|---|---|
| A | 150 | 0.0475 | 0.6147 | 0.7773 | 248.1514 |
| | 200 | 0.0884 | 0.7233 | 0.9264 | 306.8496 |
| | 250 | 0.1235 | 0.8168 | 1.0552 | 357.3152 |
| | 300 | 0.1549 | 0.9002 | 1.1705 | 402.2452 |

**Table 3**
**Sensitivity Analysis**
**Case III ($t_r \le T \le M$)**

| Parameter | % | $t_r$ | $t_1$ | T | Cost |
|---|---|---|---|---|---|
| a | +10% | 0.0547 | 0.5911 | 0.7123 | 204.0605 |
| | +5% | 0.0509 | 0.6013 | 0.7288 | 204.9113 |
| | -5% | 0.0453 | 0.6278 | 0.7536 | 205.3275 |
| | -10% | 0.0355 | 0.6351 | 0.7849 | 205.5768 |
| $x_1$ | +10% | 0.0445 | 0.6030 | 0.7406 | 210.3414 |
| | +5% | 0.0455 | 0.6074 | 0.7434 | 207.9144 |
| | -5% | 0.0474 | 0.6165 | 0.7493 | 202.9893 |
| | -10% | 0.0483 | 0.6211 | 0.7523 | 200.4909 |
| $x_2$ | +10% | 0.0436 | 0.6105 | 0.7450 | 205.5454 |
| | +5% | 0.0450 | 0.6112 | 0.7457 | 205.5058 |
| | -5% | 0.0481 | 0.6127 | 0.7472 | 205.4189 |
| | -10% | 0.0497 | 0.6136 | 0.7479 | 205.3709 |
| $\theta_1$ | +10% | 0.0423 | 0.6011 | 0.7400 | 210.5079 |
| | +5% | 0.0457 | 0.6073 | 0.7433 | 207.9939 |
| | -5% | 0.0471 | 0.6167 | 0.7494 | 202.9113 |
| | -10% | 0.0477 | 0.6214 | 0.7525 | 200.3363 |
| $\theta_2$ | +10% | 0.0458 | 0.6117 | 0.7461 | 205.4810 |
| | +5% | 0.0462 | 0.6118 | 0.7462 | 205.4724 |
| | -5% | 0.0468 | 0.6121 | 0.7465 | 205.4550 |
| | -10% | 0.0471 | 0.6123 | 0.7497 | 205.4460 |
| R | +10% | 0.0463 | 0.6116 | 0.7466 | 205.4774 |
| | +5% | 0.0464 | 0.6118 | 0.7465 | 205.4707 |
| | -5% | 0.0465 | 0.6121 | 0.7462 | 205.4565 |
| | -10% | 0.0466 | 0.6124 | 0.7461 | 205.4491 |

| Parameter | | $t_r$ | $t_1$ | T | Cost |
|---|---|---|---|---|---|
| A | 150 | 0.0464 | 0.6120 | 0.7464 | 205.4638 |
| | 200 | 0.0826 | 0.7082 | 0.8842 | 266.7904 |
| | 250 | 0.1138 | 0.7910 | 1.0036 | 319.7602 |
| | 300 | 0.1416 | 0.8648 | 1.1105 | 367.4811 |

From the table we observe that as parameter a increases/ decreases average total cost increases/ decreases in case I and case II, whereas there is decrease/ increase in average total cost with increase/ decreases in parameters a in case III.

Also, we observe that with increase and decrease in the value of $x_2$ and $\theta_2$, there is corresponding very slight increase/ decrease in total cost for case I, case II and case III. From the table we observe that with increase and decrease in parameters $x_1$, there is corresponding increase/ decrease in total cost for case I, case II and case III.

We observe that with increase and decrease in parameter $\theta_1$, there is corresponding increase/ decrease in total cost for cases I, II and III.

Moreover, we observe that with increase and decrease in the value of A, there is corresponding increase/ decrease in total cost in cases I, II and III.

## 6. CONCLUSION:

In this paper, we have developed a two warehouse inventory model for deteriorating items with linear demand and shortages under inflationary conditions and permissible delay in payments. It is assume that rented warehouse holding cost is greater than own warehouse holding cost but provides a better storage facility and there by deterioration rate is low in rented warehouse. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of cost.

## REFERENCES

[1] Benkherouf, L. (1997): A deterministic order level inventory model for deteriorating items with two storage facilities; International J. Production Economics; Vol. 48, pp. 167-175.

[2] Bhunia, A.K. and Maiti, M. (1998): A two-warehouse inventory model for deteriorating items with a linear trend in demand and shortages; J. of Oper. Res. Soc.; Vol. 49, pp. 287-292.

[3] Chung, K.J. and Huang, T.S. (2007): The optimal retailer’s ordering policies for deteriorating items with limited storage capacity under trade credit financing; International J. Production Economics; Vol. 106, pp. 127-145.

[4] Ghosh, S. and Chakrabarty, T. (2007): An order level inventory model under two level storage system with time dependent demand; Opsearch, Vol. 46, pp. 335-344.

[5] Hartley, R.V. (1976): Operations research – a managerial emphasis; Good Year, Santa Monica, CA, Chapter 12, pp. 315-317.

[6] Jaggi, C.K. and Verma, P. (2010): Two warehouse inventory model for deteriorating items with linear trend in demand and shortages under inflationary conditions; International J. of Procurement Management, Vol. 3, pp. 54-71.

[7] Kar, S., Bhunia, A.K. and Maiti, M. (2001): Deterministic inventory model with two levels of storage, a linear trend in demand and a fixed time horizon; Computers and Oper. Res.; Vol. 28, pp. 1315-1331.

[8] Liang, Y. and Zhou, F. (2011): A two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment; Applied Mathematics Modeling, Vol. 35, pp. 2211-2231.

[9] Madhavilata, M., Rao, K.S. and Ravindranath, V. (2011) An order level inventory model under $L_2$-system with exponentially increasing demand; International J. of Engineering Research and Applications, Vol. 1(4), pp. 1744-1750.

[10] Pakkala, T.P.M. and Achary, K.K. (1992): A deterministic inventory model for deteriorating items with two-warehouses and finite replenishment rate; Euro. J. O.R., Vol. 57, pp. 71-76.

[11] Rong, M., Mahapatra, N.K. and Maiti, M. (2008): A two-warehouse inventory model for a deteriorating item with partially/ fully backlogged shortage and fuzzy lead time; Euro. J. of O.R., Vol. 189, pp. 59-75.

[12] Sarma, K.V.S. (1987): A deterministic inventory model for deteriorating items with two storage facilities; Euro. J. O.R., Vol. 29, pp. 70-72.

[13] Tyagi, M. and Singh, S.R. (2013): Two warehouse inventory model with time dependent demand and variable holding cost; International J. of Applications on Innovation in Engineering and Management, Vol. 2, pp. 33-41.

[14] Yadav, A.S. and Swami, A. (2013): Effect of permissible delay on two warehouse inventory model for deteriorating items with shortages; IJAIEM, Vol. 2, pp. 65-71.

[15] Yang, H.L. (2004): Two warehouse inventory models for deteriorating items with shortages under inflation; Euro. J. Oper. Res.; Vol. 157, pp. 344-356.