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Quantum Algorithm for Eulerian Graph Problem by Numbering Method

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Abstract

A quantum algorithm for the Eulerian graph problem by a numbering method and its example are reported. When nodes are connected by edges [=n], a route that traverses each edge just for once is obtained. When it is assumed that a computational complexity of a classical computation is $O(n!)$, a computational complexity of the quantum algorithm by using quantum phase inversion gates, quantum inversion about mean gates and the numbering method is $O(n)$. Therefore, a polynomial time process becomes possible.

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Keywords: Quantum algorithm, Eulerian graph problem, numbering method, computational complexity, polynomial time.

1. Introduction

Haroche and Wineland [1] made the very first steps towards building a quantum computer. Deutsch-Jozsa's algorithm for the rapid solution [2–4], Shor's algorithm for the factorization [3–5], Grover's algorithms for the database search [3,6,7] and so on are known. A quantum algorithm for the vertex coloring problem by the central limit theorem has recently been reported by Fujimura [8]. Its computational complexity becomes a polynomial time. The Eulerian graph problem [9,10] is examined this time. Therefore, its result is reported.

2. Eulerian Graph Problem

When nodes are connected by edges, a route that traverses each edge just for once is obtained. There are a unicursal route and a traversable route.

3. Quantum Algorithm

3.1. Premise

A graph is assumed that k nodes connected by n edges. There are $h_{u,v}$ edges between the u-th node and the v-th node $[0 \leq u < v \leq k-1]$. Where, there aren't any edges from a node to same node directly. It is assumed that t is $k + \sum_{v=1,u<v}^{k-1} \sum_{u=0}^{k-2} (h_{u,v} - 1)$, and m is $n + \sum_{v=1,u<v}^{k-1} \sum_{u=0}^{k-2} (h_{u,v} - 1)$. When the w-th node $[w = 0, \cdots, t-1]$ has Q_w edges, R_w is the minimum integer that is $Q_w/2$ or more. Where, $\sum_{w=0}^{t-1} R_w$ is a necessary number of nodes that the graph changes a line that may be contained same nodes. Therefore, the graph has 3 cases.

(I) A case of $\sum_{w=0}^{t-1} R_w = m$. This case is a unicursal route.

(II) A case of $\sum_{w=0}^{t-1} R_w = m + 1$. This case is a traversable route.

(III) A case of $\sum_{w=0}^{t-1} R_w > m + 1$. This case isn't the Eulerian graph. Therefore, it isn't necessary that we calculate this case.

A length of the δ-th edge $[\delta = 0, \cdots, h_{u,v} - 1]$ between the i-th node and the j-th node $[0 \leq i < j \leq t-1]$ is $L_{i,j}^{(\delta)}$. Where, $L_{(p,q)}^{(0)}$ [p and q are integers.] is $L_{p,q}^{(0)}$ at $p < q$, or $L_{q,p}^{(0)}$ at $q < p$. A total length of edges on the Eulerian graph is L^*.

At (I), L^* is L_{uni}^*, and at (II), it is L_{tra}^*. One edge of $h_{u,v}$ is $L_{u,v}^{(0)}$, and remains $L_{u,v}^{(1)}, \cdots, L_{u,v}^{(h_{u,v}-2)}$ and $L_{u,v}^{(h_{u,v}-1)}$ are $L_{u,g_{u,v}^{(1)}}^{(0)} + L_{v,g_{u,v}^{(1)}}^{(0)}, \cdots, L_{u,g_{u,v}^{(h_{u,v}-2)}}^{(0)} + L_{v,g_{u,v}^{(h_{u,v}-2)}}^{(0)}$ and $L_{u,g_{u,v}^{(h_{u,v}-1)}}^{(0)} + L_{v,g_{u,v}^{(h_{u,v}-1)}}^{(0)}$, respectively. Where, $g_{u,v}^{(1)}, \cdots, g_{u,v}^{(h_{u,v}-2)}$ and $g_{u,v}^{(h_{u,v}-1)}$ are middle nodes. When there aren't edges between the i-th node and the j-th node, $L_{i,j}^{(0)}$ is assumed $2L^*$. Moreover, $L_{i,i}^{(0)}$ and $L_{j,j}^{(0)}$ are assumed $2L^*$. These edges that are assumed $2L^*$ are excluded by gate actions.

(1) The number of the repeated permutation of n points is n^n.

(2) The number of the permutation of n points is $n!$.

When there are n points, $a_0 n^{n-1} + a_1 n^{n-2} + \cdots + a_{n-2} n^1 + a_{n-1} n^0 = U$ is the numbering datum from 0 to $n^n - 1$ [The 0-th datum is $0, 0, \cdots, 0$ and 0. The $(n^n - 1)$-th datum is $(n-1), (n-1), \cdots, (n-1)$ and $(n-1)$.] in (1). When, in (2), it is assumed that the first datum is $0, 1, \cdots, (n-2)$ and $(n-1)$, and the $n!$-th datum is $(n-1), (n-2), \cdots, 1$ and 0, the V-th datum is obtained from $v_1(n-1)! + v_2(n-2)! + \cdots + v_{n-1}1!$. Each of φ_i [$1 \leq i \leq n$. i is an integer.] is 1 piece of permutation from 0 to $n-1$. When v_i is 0 from $i = 1$ to $i = n-2$ sequentially, φ_i is the minimum number in remained numbers. When v_i isn't 0 from $i = 1$ to $i = n-2$ sequentially, and $v_{i+1}, v_{i+2}, \cdots, v_{n-2}$ and v_{n-1} are 0, φ_i is the v_i-th small number in remained numbers, and $\varphi_{i+1} > \varphi_{i+2} > \cdots > \varphi_{n-1} > \varphi_n$ is selected in remained numbers. When v_i isn't 0 from $i = 1$ to $i = n-2$ sequentially, and there are $v_{i+1} \neq 0$ or $v_{i+2} \neq 0$ or $\cdots$ or $v_{n-2} \neq 0$ or $v_{n-1} \neq 0$, φ_i is the $(v_i + 1)$-th small number in remained numbers. When v_{n-1} is 1, $\varphi_{n-1} < \varphi_n$ is selected in remained numbers. Therefore, $\varphi_1 n^{n-1} + \varphi_2 n^{n-2} + \cdots + \varphi_{n-1} n^1 + \varphi_n n^0$ is $U(V)$. This method is named a numbering method for this problem. g is the minimum integer that follows (a computational complexity of a classical computation) $\leq 2^{2g} = 4^g$. Next, quantum algorithms of the cases of (I) and (II) are as follows.

3.2. Case of $\sum_{w=0}^{t-1} R_w = m$ [Unicursal route]

It is assumed that k nodes are $P_0(x_0, y_0, z_0), P_1(x_1, y_1, z_1), \cdots, P_{k-2}(x_{k-2}, y_{k-2}, z_{k-2})$ and $P_{k-1}(x_{k-1}, y_{k-1}, z_{k-1})$. There are $h_{u,v}$ edges between the u-th node and the v-th node. New nodes are assumed $P_k(x_k, y_k, z_k), \cdots, P_{t-2}(x_{t-2}, y_{t-2}, z_{t-2})$ and $P_{t-1}(x_{t-1}, y_{t-1}, z_{t-1})$. These nodes are $g_{u,v}^{(1)}, \cdots, g_{u,v}^{(h_{u,v}-1)}$ and so on. When P_0 starts and ends, we consider that combinations of t nodes that may be contained same nodes. An answer contains a counter course for a course. Therefore, the final answer has 2 routes at least. After all, the computational complexity of the classical computation of a unicursal route [$= Z_{uni}$] is $(1/2)(m-1)!/((R_0 - 1)!R_1! \cdots R_{t-1}!)$.

First of all, quantum registers $|a_0>, |a_1>, \cdots, |a_{m-2}>, |b>, |c_0>, |c_1>, \cdots, |c_{m-3}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$ are prepared. When α is the minimum integer that is $log_2(m-1)$ or more, each of $|a_f>$ that f is an integer from 0 to $m-2$ is consisted of α quantum bits [=qubits]. States of $|a_f>, |b>, |c_0>, |c_1>, \cdots, |c_{m-3}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$ are $a_f, b, c_0, c_1, \cdots, c_{m-3}, d_1, d_2, e_1$ and e_2, respectively. It is assumed that $0, 1, \cdots, m-3$ and $m-2$ of a_f are $0_{(1)}, 0_{(2)}, \cdots, 0_{(R_0-1)}, 1_{(1)}, 1_{(2)}, \cdots, 1_{(R_1)}, \cdots, (t-1)_{(1)}, (t-1)_{(2)}, \cdots, (t-1)_{(R_{t-1}-1)}$ and $(t-1)_{R_{t-1}}$ of σ_f, respectively. Where, $i_{(j)}$ is i of the j-th number[$1 \leq j$. j is an integer.].

Step 1: Each qubit of $|a_f>, |b>, |c_0>, |c_1>, \cdots, |c_{m-3}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$ is set $|0>$.

Step 2: The Hadamard gate $\boxed{\mathrm{H}}$ [3, 4] acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^{\alpha})^{m-1}$.

Step 3: It is assumed that a quantum gate (A) changes $|b>$ for $|1>$ in $a_f < m-1$, it changes $|b>$ for $|0>$ in the others of a_f, and it changes $|c_i>$ [$0 \leq i \leq m-3$. i is an integer.] for $|c_i+1>$ at $a_f = i$. As a target state for $|b>$ is 1, quantum phase inversion gates (PI) and quantum inversion about mean gates (IM) [3,6,7] act on $|b>$. When β is the minimum even integer that is $(2^{\alpha}/(m-1))^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is β, because they are a couple. Next, an observation gate (OB) observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_{m-2}>$. Therefore, each state of $|a_f>$ is $0, 1, \cdots, m-3$, or $m-2$, and the total states become $(m-1)^{m-1}[=M_0]$.

Step 4: As the target state for $|c_i>$ is 1, (PI) and (IM) act on $|c_i>$. When γ_i is the minimum even integer that is $(M_{i-1}/M_i)^{1/2}$ or more, the total number that (PI) and (IM) act on $|c_i>$ is γ_i. Next, (OB) observes $|c_i>$. These actions are repeated sequentially from $|c_0>$ to $|c_{m-3}>$. Where, M_i is $(m-1)(m-2)\cdots(m-1-i)^{m-1-i}$. Therefore, the total states become $(m-1)![=W_0]$.

Step 5: It is assumed that a quantum gate (B_1) changes $|d_1>$ and $|d_2>$ for $|d_1+L^{(0)}_{(0,\sigma_0)}+L^{(0)}_{(\sigma_0,\sigma_1)}>$ and $|d_2+a_0(m-1)^{m-2}+a_1(m-1)^{m-3}>$, respectively, at $|a_0>$ and $|a_1>$. (B_i) [$2 \leq i \leq m-3$. i is the integer.] changes $|d_1>$ and $|d_2>$ for $|d_1+L^{(0)}_{(\sigma_{i-1},\sigma_i)}>$ and $|d_2+a_i(m-1)^{m-2-i}>$, respectively, at $|a_{i-1}>$ and $|a_i>$. These actions are repeated sequentially from $|a_2>$ to $|a_{m-3}>$. (B_{m-2}) changes $|d_1>$ and $|d_2>$ for $|d_1+L^{(0)}_{(\sigma_{m-3},\sigma_{m-2})}+L^{(0)}_{(\sigma_{m-2},0)}>$ and $|d_2+a_{m-2}(m-1)^0>$, respectively, at $|a_{m-3}>$ and $|a_{m-2}>$.

Step 6: It is assumed that a quantum gate (C) doesn't changes $|e_1>$ at $d_1 = L^*_{uni}$, or it changes $|e_1>$ for $|e_1+d_2>$ in the others of d_1.

Step 7: It is assumed that a quantum gate (D_1) changes $|e_2>$ for $|1>$ in $e_1 = 0$ and $U(V=1) \leq e_1 \leq U(V=((m-1)!/4)-2(R_0-1)!R_1!\cdots R_{t-1}!)$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included in $e_1 = 0$ and $U(V=1) \leq e_1 \leq U(V=((m-1)!/4)-2(R_0-1)!R_1!\cdots R_{t-1}!)$ is $W_1 \approx (m-1)!/4$. When δ_1 is the minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|e_2>$ is $\delta_1 \approx 2$. Next, (OB) observes $|e_2>$, and the data of W_1 remain. Similarly, (D_i) [$2 \leq i \leq g-1$. i is the integer.] changes $|e_2>$ for $|1>$ in $e_1 = 0$ and $U(V=1) \leq e_1 \leq U(V=((m-1)!)/4^i)-2(R_0-1)!R_1!\cdots R_{t-1}!)$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included in $e_1 = 0$ and $U(v=1) \leq e_1 \leq U(V=((m-1)!/4^i)-2(R_0-1)!R_1!\cdots R_{t-1}!)$ is $W_i \approx (m-1)!/4^i$. When δ_i is the minimum even integer that is $(W_{i-1}/W_i)^{1/2}$ or more, the total number that (PI) and (IM) act on $|e_2>$ is $\delta_i \approx 2$. Next,

(OB) observes $|e_2>$, and the data of W_i remain. These actions are repeated sequentially from 2 to $g-1$ at i. (D_g) changes $|e_2>$ for $|1>$ at $e_1=0$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included at $e_1=0$ is $W_g \approx 2(R_0-1)!R_1!\cdots R_{t-1}! \approx (m-1)!/4^g$. When δ_g is the minimum even integer that is $(W_{g-1}/W_g)^{1/2}$ or more, the total number that (PI) and (IM) act on $|e_2>$ is $\delta_g \approx 2$. Next, (OB) observes $|a_f>, |b>, |c_0>, |c_1>, \cdots, |c_{m-3}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$, and one of the data of W_g remains. Therefore, one example of orders that are $e_1=0$ is obtained.

3.3. Case of $\sum_{w=0}^{t-1} R_w = m+1$ [Traversable route]

It is assumed that k nodes are $P_0(x_0, y_0, z_0), P_1(x_1, y_1, z_1), \cdots, P_{k-2}(x_{k-2}, y_{k-2}, z_{k-2})$ and $P_{k-1}(x_{k-1}, y_{k-1}, z_{k-1})$. There are $h_{u,v}$ edges between the u-th node and the v-th node. New nodes are assumed $P_k(x_k, y_k, z_k), \cdots, P_{t-2}(x_{t-2}, y_{t-2}, z_{t-2})$ and $P_{t-1}(x_{t-1}, y_{t-1}, z_{t-1})$. These nodes are $g_{u,v}^{(1)}, \cdots, g_{u,v}^{(h_{u,v}-1)}$ and so on. When a node P_s [$0 \le s \le t-1$. s is an integer.] starts and other node P_e [$0 \le e \le t-1$. e is an integer.] ends, where Q_s and Q_e are odd numbers, we consider that combinations of t nodes that may be contained same nodes. Therefore, the computational complexity of the classical computation of a traversable route [$= Z_{tra}$] is $(m-1)!/(R_0!R_1!\cdots(R_s-1)!\cdots(R_e-1)!\cdots R_{t-1}!)$.

First of all, $|a_0>, |a_1>, \cdots, |a_{m-2}>, |b>, |c_0>, |c_1>, \cdots, |c_{m-3}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$ are prepared. When α is the minimum integer that is $log_2(m-1)$ or more, each of $|a_f>$ that f is the integer from 0 to $m-2$ is consisted of α qubits. States of $|a_f>, |b>, |c_0>, |c_1>, \cdots, |c_{m-3}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$ are $a_f, b, c_0, c_1, \cdots, c_{m-3}, d_1, d_2, e_1$ and e_2, respectively.

It is assumed that $0, 1, \cdots, m-3$ and $m-2$ of a_f are $0_{(1)}, 0_{(2)}, \cdots, 0_{(R_0)}, 1_{(1)}, 1_{(2)}, \cdots, 1_{(R_1)}, \cdots, (t-1)_{(1)}, (t-1)_{(2)}, \cdots, (t-1)_{(R_{t-1})}$ of σ_f, respectively. Where, $i_{(j)}$ is i of the j-th number [$1 \le j$. j is the integer.].

Step 1: Each qubit of $|a_f>, |b>, |c_0>, |c_1>, \cdots, |c_{m-3}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$ is set $|0>$.

Step 2: $\boxed{\text{H}}$ acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^\alpha)^{m-1}$.

Step 3: (A) changes $|b>$ for $|1>$ in $a_f < m-1$, it changes $|b>$ for $|0>$ in the others of $|a_f>$, and it changes $|c_i>$ [$0 \le i \le m-3$. i is the integer.] for $|c_i+1>$ at $a_f = i$. As a target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When β is the minimum even integer that is $(2^\alpha/(m-1))^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is β. Next, (OB) observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_{m-2}>$. Therefore, each state of $|a_f>$ is $0, 1, \cdots, m-3$ or $m-2$, and the total states become $(m-1)^{m-1}$[$= M_0$].

Step 4: As the target state for $|c_i>$ is 1, (PI) and (IM) act on $|c_i>$. When γ_i is the minimum even integer that is $(M_{i-1}/M_i)^{1/2}$ or more, the total number that (PI) and (IM) act on $|c_i>$ is γ_i. Next, (OB) observes $|c_i>$. These actions are repeated sequentially from $|c_0>$ to $|c_{m-3}>$. Where, M_i is $(m-1)(m-2)\cdots(m-1-i)^{m-1-i}$. Therefore, the total states become $(m-1)![=W_0]$.

Step 5: (B_1) changes $|d_1>$ and $|d_2>$ for $|d_1+L^{(0)}_{(s,\sigma_0)}+L^{(0)}_{(\sigma_0,\sigma_1)}>$ and $|d_2+a_0(m-1)^{m-2}+a_1(m-1)^{m-3}>$, respectively, at $|a_0>$ and $|a_1>$. Similarly, (B_i) $[2\leq i\leq m-3$. i is the integer.$]$ changes $|d_1>$ and $|d_2>$ for $|d_1+L^{(0)}_{(\sigma_{i-1},\sigma_i)}>$ and $|d_2+a_i(m-1)^{m-2-i}>$, respectively, at $|a_{i-1}>$ and $|a_i>$. These actions are repeated sequentially from $|a_2>$ to $|a_{m-3}>$. (B_{m-2}) changes $|d_1>$ and $|d_2>$ for $|d_1+L^{(0)}_{(\sigma_{m-3},\sigma_{m-2})}+L^{(0)}_{(\sigma_{m-2},e)}>$ and $|d_2+a_{m-2}(m-1)^0>$, respectively, at $|a_{m-3}>$ and $|a_{m-2}>$.

Step 6: (C) doesn't change $|e_1>$ at $d_1=L^*_{tra}$, or it changes $|e_1>$ for $|e_1+d_2>$ in the others of d_1.

Step 7: (D_1) changes $|e_2>$ for $|1>$ in $e_1=0$ and $U(V=1)\leq e_1\leq U(V=((m-1)!/4)-R_0!R_1!\cdots(R_s-1)!\cdots(R_e-1)!\cdots R_{t-1}!)$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included in $e_1=0$ and $U(V=1)\leq e_1\leq U(V=((m-1)!/4)-R_0!R_1!\cdots(R_s-1)!\cdots(R_e-1)!\cdots R_{t-1}!)$ is $W_1\approx(m-1)!/4$. When δ_1 is the minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|e_2>$ is $\delta_1\approx 2$. Next, (OB) observes $|e_2>$, and the data of W_1 remain. Similarly, (D_i) $[2\leq i\leq g-1$. i is the integer.$]$ changes $|e_2>$ for $|1>$ in $e_1=0$ and $U(V=1)\leq e_1\leq U(V=((m-1)!/4^i)-R_0!R_1!\cdots(R_s-1)!\cdots(R_e-1)!\cdots R_{t-1}!)$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included in $e_1=0$ and $U(V=1)\leq e_1\leq U(V=((m-1)!/4^i)-R_0!R_1!\cdots(R_s-1)!\cdots(R_e-1)!\cdots R_{t-1}!)$ is $W_i\approx(m-1)!/4^i$. When δ_i is the minimum even integer that is $(W_{i-1}/W_i)^{1/2}$ or more, the total number that (PI) and (IM) act on $|e_2>$ is $\delta_i\approx 2$. Next, (OB) observes $|e_2>$, and the data of W_i remain. These actions are repeated sequentially from 2 to $g-1$ at i. (D_g) changes $|e_2>$ for $|1>$ at $e_1=0$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included at $e_1=0$ is $W_g\approx R_0!R_1!\cdots(R_s-1)!\cdots(R_e-1)!\cdots R_{t-1}!\approx(m-1)!/4^g$. When δ_g is the minimum even integer that is $(W_{g-1}/W_g)^{1/2}$ or more, the total number that (PI) and (IM) act on $|e_2>$ is $\delta_g\approx 2$. Next, (OB) observes $|a_f>,|b>,|c_0>,|c_1>,\cdots,|c_{m-3}>,|d_1>,|d_2>,|e_1>$ and $|e_2>$, and one of the data of W_g remains. Therefore, one example of orders that are $e_1=0$ is obtained.

4. Numerical computation

4.1. Case of $\sum_{w=0}^{t-1} R_w = m$ [Unicursal route]

6[$= k$] nodes on a graph are $P_0(0,0,0)$, $P_1(2,0,0)$, $P_2(2,2,1)$, $P_3(1,2,0)$, $P_4(0,\ 2,2)$ and $P_5(1,1,0)$. As for a unicursal route, the length of 12[$= n$] edges on the graph are $L^{(0)}_{0,1} = 2, L^{(0)}_{1,5} = L^{(0)}_{0,5} = 2^{1/2} \approx 1.4, L^{(0)}_{0,2} = 3, L^{(0)}_{2,3} = 2^{1/2} \approx 1.4, L^{(1)}_{2,3} = L^{(2)}_{2,3} = 2, L^{(0)}_{3,4} = 5^{1/2} \approx 2.2, L^{(1)}_{3,4} = 6^{1/2} + 1 \approx 3.4, L^{(0)}_{1,4} = 12^{1/2} \approx 3.5, L^{(0)}_{1,3} = 5^{1/2} \approx 2.2$ and $L^{(0)}_{0,4} = 2 \cdot 2^{1/2} \approx 2.8$. L^*_{uni} is about 27.3. P_2 and P_3 have 3 edges, and P_3 and P_4 have 2 edges. $h_{2,3}$ is 3, and $h_{3,4}$ is 2. Therefore, t is $k + \sum_{v=1,u<v}^{k-1} \sum_{u=0}^{k-2} (h_{u,v} - 1) = 6 + 2 + 1 = 9$, and m is $n + \sum_{v=1,u<v}^{k-1} \sum_{u=0}^{k-2} (h_{u,v} - 1) = 12 + 2 + 1 = 15$. Added 3 nodes are $P_6(2,2,0)[= g^{(1)}_{2,3}]$, $P_7(1,2,1)[= g^{(2)}_{2,3}]$ and $P_8(0,1,2)[= g^{(1)}_{3,4}]$. $L^{(1)}_{2,3}, L^{(2)}_{2,3}$ and $L^{(1)}_{3,4}$ become $L^{(0)}_{2,6}+L^{(0)}_{3,6} = 1+1 = 2, L^{(0)}_{2,7}+L^{(0)}_{3,7} = 1+1 = 2$ and $L^{(0)}_{3,8}+L^{(0)}_{4,8} \approx 2.4+1 = 3.4$, respectively. When there aren't edges between the i-th node and the j-th node, $L^{(0)}_{i,j}$ is assumed $2L^*_{uni}$. Moreover, $L^{(0)}_{i,i}$ and $L^{(0)}_{j,j}$ are assumed $2L^*_{uni}$. These edges that are assumed $2L^*_{uni}$ are excluded by gate actions. The values of $Q_0/2, Q_1/2, \cdots, Q_7/2$ and $Q_8/2$ are $4/2 \le 2 = R_0 = R_1 = R_2, 6/2 \le 3 = R_3, 4/2 \le 2 = R_4$ and $2/2 \le 1 = R_5 = R_6 = R_7 = R_8$, respectively. Therefore, $\sum_{w=0}^{8} R_w$ is 15[$= m$], and it is checked this graph is a unicursal route. It is assumed that P_0 starts and ends. An answer contains a counter course for a course. Therefore, the final answer has 2 routes at least. Z_{uni} is $(1/2)(m-1)!/((R_0-1)!R_1! \cdots !R_8!) = (1/2)(15-1)!/((2-1)!2!2!3!2!1!1!1!1!) \approx 9.1 \times 10^8$. It is assumed that there are $g = 15$ [$Z_{uni} \le 4^g = 4^{15} \approx 1.074 \times 10^9$], $U(V = 1) = \sum_{i=1}^{14} (i-1)14^{14-i}, U(V = (14!/4) - 2 \cdot 1!2!2!3!2!1!1!1!1! = (14!/4) - 96 \approx 2.179 \times 10^{10}) = \sum_{i=1}^{14} \varphi_i 14^{14-i}$ [for example, $\varphi_1 = 3, \varphi_2 = 7, \varphi_3 = 6, \varphi_4 = 12, \varphi_5 = 10, \varphi_6 = 5, \varphi_7 = 13, \varphi_8 = 2, \varphi_9 = 11, \varphi_{10} = 9, \varphi_{11} = 0, \varphi_{12} = 4, \varphi_{13} = 8$ and $\varphi_{14} = 1$. $V = \sum_{i=1}^{13} v_i(14-i)!$. $v_1 = 3, v_2 = 6, v_3 = 5, v_4 = 9, v_5 = 7, v_6 = 4, v_7 = 7, v_8 = 2, v_9 = 5, v_{10} = 4, v_{11} = 0, v_{12} = 2$ and $v_{13} = 0$.], $U(V = (14!/4^2) - 96 \approx 5.449 \times 10^9), \cdots, U(V = (14!/4^{13}) - 96 \approx 1203)$ and $U(V = (14!/4^{14}) - 96 \approx 229)$.

First of all, $|a_0>, |a_1>, \cdots, |a_{13}>, |b>, |c_0>, |c_1>, \cdots, |c_{12}>, |d_1>, |d_2>$, $|e_1>$ and $|e_2>$ are prepared. When α is the minimum integer that is $log_2 14 \approx 3.8 \leq 4 = \alpha$, each of $|a_f>$ that f is the integer from 0 to 13 is consisted of $\alpha = 4$ qubits. States of $|a_f>, |b>, |c_0>, |c_1>, \cdots, |c_{12}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$ are $a_f, b, c_0, c_1, \cdots, c_{12}, d_1, d_2, e_1$ and e_2, respectively. It is assumed that 0, 1, $\cdots$, 12 and 13 of a_f are $0_{(1)}, 1_{(1)}, 1_{(2)}, 2_{(1)}, 2_{(2)}, 3_{(1)}, 3_{(2)}, 3_{(3)}, 4_{(1)}, 4_{(2)}, 5_{(1)}, 6_{(1)}, 7_{(1)}$ and $8_{(1)}$ of σ_f, respectively. Where, $i_{(j)}$ is i of the j-th number [$1 \leq j$. j is the integer.].

Step 1: Each qubit of $|a_f>, |b>, |c_0>, |c_1>, \cdots, |c_{12}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$ is set $|0>$.

Step 2: $\boxed{\mathrm{H}}$ acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^{\alpha})^{m-1} = (2^4)^{14}$.

Step 3: (A) changes $|b>$ for $|1>$ at $a_f < 14$, it changes $|b>$ for $|0>$ in the others of a_f, and it changes $|c_i>$ [$0 \leq i \leq 12$. i is the integer.] for $|c_i + 1>$ at $a_f = i$. As the target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When β is the minimum even integer that is $(2^4/14)^{1/2} \approx 1.1 \leq 2 = \beta$, the total number that (PI) and (IM) act on $|b>$ is $\beta \approx 2$. Next, (OB) observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_{13}>$. Therefore, each state of $|a_f>$ is $0, 1, \cdots, 12$ and 13, and the total states become $14^{14}[= M_0]$.

Step 4: As the target state for $|c_i>$ is 1, (PI) and (IM) act on $|c_i>$. When γ_i is the minimum even integer that is $(M_{i-1}/M_i)^{1/2}$ or more, the total number that (PI) and (IM) act on $|c_i>$ is γ_i. Next, (OB) observes $|c_i>$. These actions are repeated sequentially from $|c_0>$ to $|c_{12}>$. Where, M_i is $14 \cdot 13 \cdots (14-i)^{14-i}$. Therefore, the total states become $14![= W_0]$.

Step 5: (B_1) changes $|d_1>$ and $|d_2>$ for $|d_1 + L^{(0)}_{(0,\sigma_0)} + L^{(0)}_{(\sigma_0,\sigma_1)}>$ and $|d_2 + a_0 14^{13} + a_1 14^{12}>$, respectively, at $|a_0>$ and $|a_1>$. (B_i) [$2 \leq i \leq 12$. i is the integer.] changes $|d_1>$ and $|d_2>$ for $|d_1 + L^{(0)}_{(\sigma_{i-1},\sigma_i)}>$ and $|d_2 + a_i 14^{13-i}>$, respectively, at $|a_{i-1}>$ and $|a_i>$. These actions are repeated sequentially from $|a_2>$ to $|a_{12}>$. (B_{13}) changes $|d_1>$ and d_2 for $|d_1 + L^{(0)}_{(\sigma_{12},\sigma_{13})} + L^{(0)}_{(\sigma_{13},0)}>$ and $|d_2 + a_{13} 14^0>$, respectively, at $|a_{12}>$ and $|a_{13}>$.

Step 6: (C) doesn't change $|e_1>$ at $d_1 = L^*_{uni} \approx 27.3$, or it is changes $|e_1>$ for $|e_1 + d_2>$ in the others of d_1.

Step 7: (D_1) changes $|e_2>$ for $|1>$ in $e_1 = 0$ and $U(V = 1) \leq e_1 \leq U(V = ((m-1)!/4) - 2(R_0 - 1)!R_1! \cdots R_8!) = U(V = (14!/4) - 2 \cdot 1!2!2!3!2!1!1!1!1! \approx (8.718 \times 10^{10}/4) - 96 \approx 2.179 \times 10^{10} - 96 \approx 2.179 \times 10^{10}) = U(V = 2.179 \times 10^{10})$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included in $e_1 = 0$ and $U(V = 1) \leq e_1 \leq U(V = 2.179 \times 10^{10})$ is $W_1 \approx 14!/4$. When δ_1 is the minimum even integer that is $(W_0/W_1)^{1/2} \approx 2 \leq 2 = \delta_1$, the total number that (PI) and (IM) act on $|e_2>$ is $\delta_1 \approx 2$. Next, (OB) observes $|e_2>$, and the data of W_1 remain. Similarly, (D_i) [$2 \leq i \leq g-1 = 15-1 = 14$. i is the integer.] changes $|e_2>$ for $|1>$ in $e_1 = 0$ and $U(V=1) \leq e_1 \leq U(V = (14!/4^i) - 96)$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included in $e_1 = 0$ and $U(V) \leq e_1 \leq U(V = (14!/4^i) - 96)$ is $W_i \approx 14!/4^i$. When δ_i is the minimum even integer that is $(W_{i-1}/W_i)^{1/2} \approx 2 \leq 2 = \delta_i$, the total number that (PI) and (IM) act on $|e_2>$ is $\delta_i \approx 2$. Next, (OB) observes $|e_2>$, and the data of W_i remain. These actions are repeated sequentially from 2 to 14 at i. (D_{15}) changes $|e_2>$ for $|1>$ at $e_1 = 0$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included at $e_1 = 0$ is $W_{15} \approx 96 \approx 14!/4^{15}$. When δ_{15} is the minimum even integer that is $(W_{14}/W_{15})^{1/2} \approx 2 \leq 2 = \delta_{15}$, the total number that (PI) and (IM) act on $|e_2>$ is $\delta_{15} \approx 2$. Next, (OB) observes $|a_f>, |b>, |c_0>, |c_1>, \cdots, |c_{12}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$, and one of the data of W_{15} remains. Therefore, for example, $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, b, c_0, c_1, \cdots, c_{12}, d_1, d_2, e_1$ and e_2 are 1, 10, 0, 3, 5, 8, 2, 6, 11, 4, 12, 7, 13, 9, 1, 1, 1, $\cdots$, 1, $L^*_{uni} = 27.3$, $\sum_{i=0}^{13} a_i 14^{13-i}$, 0 and 1, respectively. As a result, a permutation of the unicursal route that is [P_0 (start) $\rightarrow$] $P_1 \rightarrow P_5 \rightarrow P_0 \rightarrow P_2 \rightarrow P_3 \rightarrow P_4 \rightarrow P_1 \rightarrow P_3 \rightarrow P_6 \rightarrow P_2 \rightarrow P_7 \rightarrow P_3 \rightarrow P_8 \rightarrow P_4$ [$\rightarrow P_0$ (end)] is obtained. And the counter course for this course is another final answer.

4.2. Case of $\sum_{w=0}^{t-1} R_w = m + 1$ [Traversable route]

6[$=k$] nodes on a graph are $P_0(0,0,0)$, $P_1(2,0,0)$, $P_2(2,2,1)$, $P_3(1,2,0)$, $P_4(0,2,2)$ and $P_5(1,1,0)$. As for a traversable route, the length of 11[$=n$] edges on the graph are $L^{(0)}_{0,1} = 2$, $L^{(0)}_{1,5} = L^{(0)}_{0,5} = 2^{1/2} \approx 1.4$, $L^{(0)}_{0,2} = 3$, $L^{(0)}_{2,3} = 2^{1/2} \approx 1.4$, $L^{(1)}_{2,3} = L^{(2)}_{2,3} = 2$, $L^{(0)}_{3,4} = 5^{1/2} \approx 2.2$, $L^{(1)}_{3,4} = 6^{1/2} + 1 \approx 3.4$, $L^{(0)}_{1,4} = 12^{1/2} \approx 3.5$ and $L^{(0)}_{1,3} = 5^{1/2} \approx 2.2$. L^*_{tra} is about 24.5. P_2 and P_3 have 3 edges, and P_3 and P_4 have 2 edges. $h_{2,3}$ is 3, and $h_{3,4}$ is 2. Therefore, t is $k + \sum_{v=1,u<v}^{k-1} \sum_{u=0}^{k-2} (h_{u,v} - 1) = 6 + 2 + 1 = 9$, and m is $n + \sum_{v=1,u<v}^{k-1} \sum_{u=0}^{k-2} (h_{u,v} - 1) = 11 + 2 + 1 = 14$. Added 3 nodes are $P_6(2,2,0)[= g^{(1)}_{2,3}]$, $P_7(1,2,1)[= g^{(2)}_{2,3}]$ and $P_8(0,1,2)[= g^{(1)}_{3,4}]$. $L^{(1)}_{2,3}, L^{(2)}_{2,3}$ and $L^{(1)}_{3,4}$ become $L^{(0)}_{2,6} + L^{(0)}_{3,6} = 1 + 1 = 2$, $L^{(0)}_{2,7} + L^{(0)}_{3,7} = 1 + 1 = 2$ and $L^{(0)}_{3,8} + L^{(0)}_{4,8} \approx 2.4 + 1 = 3.4$, respectively. When there aren't edges between the i-th node and the j-th node, $L^{(0)}_{i,j}$ is assumed $2L^*_{tra}$. Moreover, $L^{(0)}_{i,i}$ and $L^{(0)}_{j,j}$ are assumed $2L^*_{tra}$. These edges that are assumed $2L^*_{tra}$ are excluded by gate actions. The values of $Q_0/2, Q_1/2, \cdots, Q_7/2$ and $Q_8/2$ are $3/2 \leq 2 = R_0$ [P_0 is a starting

point.], $4/2 \leq 2 = R_1 = R_2, 6/2 \leq 3 = R_3, 3/2 \leq 2 = R_4$ [P_4 is an ending point.] and $2/2 \leq 1 = R_5 = R_6 = R_7 = R_8$, respectively. Therefore, $\sum_{w=0}^{8} R_w$ is 15[$= m + 1$], and it is checked this graph is a traversable route.

Z_{tra} is $(m-1)!/((R_0-1)!R_1!\cdots(R_4-1)!\cdots R_8!) = (14-1)!/(1!2!2!3!1!1!1!1!1!) \approx 2.594 \times 10^8$. It is assumed that there are $g = 14$ [$Z_{tra} \leq 4^g = 4^{14} \approx 2.684 \times 10^8$], $U(V = 1) = \sum_{i=1}^{13}(i-1)13^{13-i}$, $U(V = (13!/4) - 1!2!2!3!1!1!1!1!1! = (13!/4) - 24 \approx 1.557 \times 10^9) = \sum_{i=1}^{13} \varphi_i 13^{13-i}$ [for example, $\varphi_1 = 3, \varphi_2 = 4, \varphi_3 = 0, \varphi_4 = 1, \varphi_5 = 9, \varphi_6 = 12, \varphi_7 = 8, \varphi_8 = 5, \varphi_9 = 11, \varphi_{10} = 10, \varphi_{11} = 7, \varphi_{12} = 6$ and $\varphi_{13} = 2$. $V = \sum_{i=1}^{12} v_i(13-i)!$. $v_1 = 3, v_2 = 3, v_3 = v_4 = 0, v_5 = 5, v_6 = 7, v_7 = 4, v_8 = 2$ and $v_9 = v_{10} = v_{11} = v_{12} = 0$.], $U(V = (13!/4^2) - 24 \approx 3.892 \times 10^8), \cdots, U(V = (13!/4^{12}) - 24 \approx 347)$ and $U(V = (13!/4^{13}) - 24 \approx 69)$.

First of all, $|a_0>, |a_1>, \cdots, |a_{12}>, |b>, |c_0>, |c_1>, \cdots, |c_{11}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$ are prepared. When α is the minimum integer that is $log_2 13 \approx 3.701 \leq 4 = \alpha$, each of $|a_f>$ that f is the integer from 0 to 12 is consisted of 4 qubits. States of $|a_f>, |b>, |c_0>, |c_1>, \cdots, |c_{11}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$ are $a_f, b, c_0, c_1, \cdots, c_{11}, d_1, d_2, e_1$ and e_2, respectively. It is assumed that 0, 1, $\cdots$, 11 and 12 of a_f are $0_{(1)}, 1_{(1)}, 1_{(2)}, 2_{(1)}, 2_{(2)}, 3_{(1)}, 3_{(2)}, 3_{(3)}, 4_{(1)}, 5_{(1)}, 6_{(1)}, 7_{(1)}$ and $8_{(1)}$ of σ_f, respectively. Where $i_{(j)}$ is i of the j-th number [$1 \leq j$. j is the integer.].

Step 1: Each qubit of $|a_f>, |b>, |c_0>, |c_1>, \cdots, |c_{11}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$ is set $|0>$.

Step 2: $\boxed{\text{H}}$ acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^\alpha)^{m-1} = (2^4)^{13}$.

Step 3: (A) changes $|b>$ for $|1>$ in $a_f < 13$, or it changes $|b>$ for $|0>$ in the others of a_f, and it changes $|c_i>$ [$0 \leq i \leq 11$. i is the integer.] for $|c_i + 1>$ at $a_f = i$. As the target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When β is the minimum even integer that is $(2^4/13)^{1/2} \approx 1.109 \leq 2 = \beta$, the total number that (PI) and (IM) act on $|b>$ is $\beta \approx 2$. Next, (OB) observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_{12}>$. Therefore, each state of $|a_f>$ is $0, 1, \cdots, 11$ and 12, and the total states become 13^{13}[$= M_0$].

Step 4: As the target state for $|c_i>$ is 1, (PI) and (IM) act on $|c_i>$. When γ_i is the minimum even integer that is $(M_{i-1}/M_i)^{1/2}$ or more, the total number that (PI) and (IM) act on $|c_i>$ is γ_i. Next, (OB) observes $|c_i>$. These actions are repeated sequentially from $|c_0>$ to $|c_{11}>$. Where, M_i is $13 \cdot 12 \cdots (13-i)^{13-i}$. Therefore, the total states become 13![$= W_0$].

Step 5: (B_1) changes $|d_1>$ and $|d_2>$ for $|d_1 + L^{(0)}_{(s,\sigma_0)} + L^{(0)}_{(\sigma_0,\sigma_1)}>$ and $|d_2 + a_0 13^{12} + a_1 13^{11}>$, respectively, at $|a_0>$ and $|a_1>$. (B_i) [$2 \le i \le 11$. i is the integer.] changes $|d_1>$ and $|d_2>$ for $|d_1 + L^{(0)}_{(\sigma_{i-1},\sigma_i)}>$ and $|d_2 + a_i 13^{12-i}>$, respectively, at $|a_{i-1}>$ and $|a_i>$. These actions are repeated sequentially from $|a_2>$ to $|a_{11}>$. (B_{12}) changes $|d_1>$ and $|d_2>$ for $|d_1 + L^{(0)}_{(\sigma_{11},\sigma_{12})} + L^{(0)}_{(\sigma_{12},e)}>$ and $|d_2 + a_{12} 13^0>$, respectively, at $|a_{11}>$ and $|a_{12}>$.

Step 6: (C) doesn't change $|e_1>$ at $d_1 = L^*_{tra} \approx 24.5$, or it changes $|e_1>$ for $|e_1 + d_2>$ in the others of d_1.

Step 7: (D_1) changes $|e_2>$ for $|1>$ in $e_1 = 0$ and $U(V = 1) \le e_1 \le U(V = (13!/4) - 24 \approx 1.557 \times 10^9)$, or it changes $|e_2>$ for $|0>$ in the others of $|e_1>$. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included in $e_1 = 0$ and $U(V = 1) \le e_1 \le U(V \approx 1.557 \times 10^9)$ is $W_1 \approx 13!/4$. When δ_1 is the minimum even integer that is $(W_0/W_1)^{1/2} \approx 2 \le 2 = \delta_1$, the total number that (PI) and (IM) act on $|e_2>$ is $\delta_1 \approx 2$. Next, (OB) observes $|e_2>$, and the data of W_1 remain. Similarly, (D_i) [$2 \le i \le g - 1 = 13$. i is the integer.] changes $|e_2>$ for $|1>$ in $e_1 = 0$ and $U(V = 1) \le e_1 \le U(V = (13!/4^i) - 24)$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included in $e_1 = 0$ and $U(V = 1) \le e_1 \le U(V = (13!/4^i) - 24)$ is $W_i = 13!/4^i$. When δ_i is the minimum even integer that is $(W_{i-1}/W_i)^{1/2} \approx 2 \le 2 = \delta_i$, the total number that (PI) and (IM) act on $|e_2>$ is $\delta_i \approx 2$. Next, (OB) observes $|e_2>$, and the data of W_i remain. These actions are repeated sequentially from 2 to 13 at i. (D_{14}) changes $|e_2>$ for $|1>$ at $e_1 = 0$, or it changes $|e_2>$ for $|0>$ in the others of e_1. As the target state for $|e_2>$ is 1, (PI) and (IM) act on $|e_2>$. The number of the data that is included at $e_1 = 0$ is $W_{14} \approx 24 \approx 13!/4^{14}$. When δ_{14} is the minimum even integer that is $(W_{13}/W_{14})^{1/2} \approx 2 \le 2 = \delta_{14}$, the total number that (PI) and (IM) act on $|e_2>$ is $\delta_{14} \approx 2$. Next, (OB) observes $|a_f>, |b>, |c_0>, |c_1>, \cdots, |c_{11}>, |d_1>, |d_2>, |e_1>$ and $|e_2>$, and one of the data of W_{14} remains. Therefore, for example, $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, b, c_0, c_1, \cdots, c_{11}, d_1, d_2, e_1$ and e_2 are $1, 9, 0, 3, 5, 8, 2, 6, 10, 4, 11, 7, 12, 1, 1, 1, \cdots, 1, L^*_{tra} \approx 24.5, \sum_{i=0}^{12} a_i 13^{12-i}$, 0 and 1, respectively. As a result, a permutation of the traversable route that is [P_0 (start) $\rightarrow$] $P_1 \rightarrow P_5 \rightarrow P_0 \rightarrow P_2 \rightarrow P_3 \rightarrow P_4 \rightarrow P_1 \rightarrow P_3 \rightarrow P_6 \rightarrow P_2 \rightarrow P_7 \rightarrow P_3 \rightarrow P_8$ [$\rightarrow P_4$ (end)] is obtained.

5. Discussion and Summary

The computational complexity of this quantum algorithm [$= S$] becomes the following.

(I) The cace of $\sum_{w=0}^{t-1} R_w = m$. [the unicursal route]: In the order of the actions by the gates, the number of them is $\alpha(m-1)$ at $\boxed{\mathrm{H}}$, $(m-1)$ at (A), $\beta(m-1) \approx 2(m-1)$ at (PI) and (IM), $(m-1)$ at (OB), $\sum_{i=1}^{m-2} \gamma_i \approx 2(m-2)$ at (PI) and (IM), $m-2$ at (OB), $2(m-2)$ at (B_i) [$1 \leq i \leq m-2$. i is the integer.], 2 at (C), g at (D_i) [$1 \leq i \leq g$. i is the integer.], $\sum_{i=1}^{g} \delta_i \approx 2g$ at (IP) and (IM) and g at (OB). Therefore, S_{uni} that is S of the unicursal route becomes $(\alpha+9)(m-1)-3+4g$. In the example of the section 4.1, S_{uni} is 239. S_{uni}/Z_{uni} is about $1/10^7$ because Z_{uni} is about 9.1×10^8.

(II) The cace of $\sum_{w=0}^{t-1} R_w = m+1$. [the traversable route]: In the order of the actions by the gates, the number of them is $\alpha(m-1)$ at $\boxed{\mathrm{H}}$, $(m-1)$ at (A), $\beta(m-1) \approx 2(m-1)$ at (PI) and (IM), $(m-1)$ at (OB), $\sum_{i=1}^{m-2} \gamma_i \approx 2(m-2)$ at (PI) and (IM), $m-2$ at (OB), $2(m-2)$ at (B_i) [$1 \leq i \leq m-2$. i is the integer.], 2 at (C), g at (D_i) [$1 \leq i \leq g$. i is the integer.], $\sum_{i=1}^{g} \delta_i \approx 2g$ at (IP) and (IM) and g at (OB). Therefore, S_{tra} that is S of the traversable route becomes $(\alpha+9)(m-1)-3+4g$. In the example of the section 4.2, S_{tra} is 222. S_{tra}/Z_{tra} is about $1/10^6$ because Z_{tra} is about 2.6×10^8.

When k and n [in other words, t and m] are large enough, S_{uni} or S_{tra} is about $3(n-1)log_2(n-1)$, where α is about $log_2(n-1)$, g is about $(1/2)log_2(n-1)!$, and $(n-1)!$ is about $(n-1)^{n-1}e^{-(n-1)}(2\pi(n-1))^{1/2}$ [Stirling's formura]. For example, as for $k = t = n = m = 100$, $R_0 = R_2 = \cdots = R_{99} = 1$ and $R_1 = 2$, S_{tra}/Z_{tra} is about $1/10^{153}$ because of $\sum_{w=0}^{t-1} R_w = n+1 = m+1$ [in other words, the traversable route] and $Z_{tra} \approx 9.4 \times 10^{155}$.

Therefore, the polynomial time process becomes possible.

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