Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 9, Number 3 (2013), pp. 277–286

http://www.ripublication.com/gjpam.htm

Quantum Algorithm for Clique Problem by Numbering Method

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Abstract

A quantum algorithm for the clique problem by a numbering method and its example are reported. When k persons of n persons form the clique, these members are decided. A computational complexity of a classical computation is $n!/((n-k)!k!)$. In the quantum algorithm by using quantum phase inversion gates, quantum inversion about mean gates and the numbering method, its computational complexity is about $(2k + 3log_2n)k$. Therefore, a polynomial time process becomes possible.

AMS subject classification: Primary 81-08; Secondary 68R10, 68W40.
Keywords: Quantum algorithm, clique problem, numbering method, computational complexity, polynomial time.

1. Introduction

Haroche and Wineland [1] developed methods for the very first steps towards building a quantum computer. Deutsch-Jozsa's algorithm for the rapid solution [2–4], Shor's algorithm for the factorization [3–5], Grover's algorithms for the database search [3,6,7] and so on are known. A quantum algorithm for the vertex coloring problem by the central limit theorem has recently been reported by Fujimura [8]. Its computational complexity becomes a polynomial time. The clique problem [9,10] is examined this time. Therefore, its result is reported.

2. Clique Problem

When k persons of n persons form the clique, these members are decided. A computational complexity of a classical computation is $n!/((n-k)!k!)$ because a number of the combinations chooses k from n, and its order is unrelated.

3. Quantum Algorithm

3.1. Premise

It is assumed that k persons of n persons form the clique, where k and n are integers of 1 or more. When i and j have a connection, and $x_{i,j}[=x_{j,i}]$ is 1, and when they don't have it, $x_{i,j}[=x_{j,i}]$ is 0, where i and j are integers with $0 \leq i \neq j \leq n-1$. Next, a situation division is done according to a value of k.

(I) At $k = 1$ or n : It isn't necessary to calculate because there is no clique at $k = 1$, and all persons are the members of the clique at $k = n$.

(II) In $2 \leq k \leq n/2$: When k persons compose the clique, a value of the product of all combinations of x_{a_p,a_q} that p and q are integers with $1 \leq p < q \leq k$ becomes 1, where a_p and a_q correspond to i and j, respectively. In a word, the value of $x_{a_1,a_2}x_{a_1,a_3}\cdots x_{a_1,a_k}x_{a_2,a_3}x_{a_2,a_4}\cdots x_{a_2,a_k}\cdots x_{a_{k-1},a_k}$ becomes 1.

(III) In $n/2 < k \leq n-2$: A range of k' is $2 \leq k' < n/2$ when it is assumed that k' is $n-k$. When the value of $x'_{a'_1,a'_2}x'_{a'_1,a'_3}\cdots x'_{a'_1,a'_{k'}}x'_{a'_2,a'_3}x'_{a'_2,a'_4}\cdots x'_{a'_2,a'_{k'}}\cdots x'_{a'_{k'-1},a'_{k'}}$ is 1 in the combination of k' nonmembers except it of k, k persons are the members of the clique. Therefore, this computational complexity is equivalent to it that exchanged k for k' in (II).

(IV) At $k = n-1$: When it is assumed that k' is $n-k$, k' is 1. Then, the value of $x_{a_1,a_2}x_{a_1,a_3}\cdots x_{a_1,a_k}x_{a_2,a_3}x_{a_2,a_4}\cdots x_{a_2,a_k}\cdots x_{a_{k-1},a_k}$ is 1 in the combination of k except k' nonmember is 1.

(1) The number of the repeated permutation of n persons is n^n.

(2) The number of the permutation of n persons is $n!$.

When there are n persons, $a_1n^{n-1}+a_2n^{n-2}+\cdots+a_{n-1}n^1+a_nn^0 = U$ is the numbering datum from 0 to n^n-1 [The 0-th datum is $0, 0, \cdots, 0$ and 0. The (n^n-1)-th datum is $(n-1), (n-1), \cdots, (n-1)$ and $(n-1)$.] in (1). When, in (2), it is assumed that the first datum is $0, 1, \cdots, n-1$, and the $n!$-th datum is $(n-1), (n-2), \cdots, 0$, the V-th datum is obtained from $v_1(n-1)! + v_2(n-2)! + \cdots + v_{n-1}1!$. Each of φ_i [$1 \leq i \leq n$. i is the integer.] is 1 piece of permutation from 0 to $n-1$. When v_i is 0 from $i = 1$ to $i = n-2$ sequentially, φ_i is the minimum number in remained numbers. When v_i isn't 0 from $i = 1$ to $i = n-2$ sequentially, and $v_{i+1}, v_{i+2}, \cdots, v_{n-2}$ and v_{n-1} are 0, φ_i is the v_i-th small number in remained numbers, and $\varphi_{i+1} > \varphi_{i+2} > \cdots > \varphi_{n-1} > \varphi_n$ is selected in remained numbers. When v_i isn't 0 from $i = 1$ to $i = n-2$ sequentially, and there are $v_{i+1} \neq 0$ or $v_{i+2} \neq 0$ or $\cdots$ or $v_{n-2} \neq 0$ or $v_{n-1} \neq 0$, φ_i is the (v_i+1)-th small number in remained numbers. When v_{n-1} is 1, $\varphi_{n-1} < \varphi_n$ is selected in remained numbers. Therefore, $\varphi_1n^{n-1}+\varphi_2n^{n-2}+\cdots+\varphi_{n-1}n^1+\varphi_nn^0$ is $U(V)$, and then $U_k(V)$ becomes $\sum_{i=1}^{k}\varphi_in^{n-i}$. This method is named a numbering method for this problem. g

is the minimum integer that follows $n!/((n-k)!k!) \leq 2^{2g} = 4^g$. $U_k(V=1), U_k(V=(n!/4)-(n-k)!k!), U_k(V=(n!/4^2)-(n-k)!k!), \cdots, U_k(V=(n!/4^{g-2})-(n-k)!k!)$ and $U_k(V=(n!/4^{g-1})-(n-k)!k!)$ are computated.

Next, a quantum algorithm is shown as the following.

3.2. Case of $2 \leq k \leq n/2$

First of all, quantum registers $|a_1>, |a_2>, \cdots, |a_k>, |b>, |c_1>, |c_2>, |d>$ and $|e>$ are prepared. When α is the minimum integer that is $log_2 n$ or more, each of $|a_f>$ that f is an integer from 1 to k is consisted of α quantum bits [=qubits]. States of $|a_1>, |a_2>, \cdots, |a_k>, |b>, |c_1>, |c_2>, |d>$ and $|e>$ are $a_1, a_2, \cdots, a_k, b, c_1, c_2, d$ and e, respectively.

Step 1: Each qubit of $|a_1>, |a_2>, \cdots, |a_k>, |b>, |c_2>, |d>$ and $|e>$ is set $|0>$, and a qubit of $|c_1>$ is set $|1>$.

Step 2: The Hadamard gate $\boxed{\text{H}}$ [3,4] acts on each qubit of $|a_1>, |a_2>, \cdots, |a_{k-1}>$ and $|a_k>$. It changes them for entangled states. The total states are $(2^\alpha)^k$.

Step 3: It is assumed that a quantum gate (A) changes $|b>$ for $|1>$ in $a_f < n$, or it changes $|b>$ for $|0>$ in the others of a_f. As a target state for $|b>$ is 1, quantum phase inversion gates (PI) and quantum inversion about mean gates (IM) [3,6,7] act on $|b>$. When β is the minimum even integer that is $(2^\alpha/n)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is β, because they are a couple. Next, an observation gate (OB) observes $|b>$. These actions are repeated sequentially from $|a_1>$ to $|a_k>$. Therefore, each state of $|a_f>$ is $0, 1, \cdots, n-2$ and $n-1$, and the total states become $n^k[=M_0]$.

Step 4: It is assumed that a quantum gate (B) changes $|b>, |c_1>$ and $|c_2>$ for $|1>$, $|c_1 \times x_{a_1,a_2}>$ and $|c_2 + (a_1 n^{n-1} + a_2 n^{n-2})/(k-1)>$, respectively, at $a_1 \neq a_2$, or it changes $|b>, |c_1>$ and $|c_2>$ for $|0>, |c_1 \times 0>$ and $|c_2 + 0>$, respectively, at $a_1 = a_2$. As the target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When γ_{a_1,a_2} is the minimum even integer that is $(M_0/M_{a_1,a_2})^{1/2} = (n^k/(n^{k-1}(n-1)))^{1/2} = (n/(n-1))^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is γ_{a_1,a_2}. Next, (OB) observes $|b>$, and the data of $M_{a_1,a_2} = n^{k-1}(n-1)$ remain. (B) changes $|b>, |c_1>$ and $|c_2>$ for $|1>, |c_1 \times x_{a_1,a_3}>$ and $|c_2 + (a_1 n^{n-1} + a_3 n^{n-3})/(k-1)>$, respectively, at $a_1 \neq a_3$, or it changes $|b>, |c_1>$ and $|c_2>$ for $|0>, |c_1 \times 0>$ and $|c_2 + 0>$, respectively, at $a_1 = a_3$. As the target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When γ_{a_1,a_3} is the minimum even integer that is $(M_{a_1,a_2}/M_{a_1,a_3})^{1/2} = (n^{k-1}(n-1)/(n^{k-2}(n-1)^2))^{1/2} = (n/(n-1))^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is γ_{a_1,a_3}. Next, (OB) observes $|b>$, and the data of $M_{a_1,a_3} = n^{k-2}(n-1)^2$ remain. Similarly, (B) changes $|b>, |c_1>$ and $|c_2>$ for $|1>, |c_1 \times x_{a_p,a_q}>$ and $|c_2 + (a_p n^{n-p} + a_q n^{n-q})/(k-1)>$, respectively, at $a_p \neq a_q$ [$1 \leq p < q \leq k$. p and q are integers.], or it changes $|b>, |c_1>$ and

$|c_2 >$ for $|0 >, |c_1 \times 0 >$ and $|c_2 + 0 >$, respectively, at $a_p = a_q$. As the target state for $|b >$ is 1, (PI) and (IM) act on $|b >$. When γ_{a_p,a_q} is the minimum even integer that is $(M^{(pre)}_{a_p,a_q}/M_{a_p,a_q})^{1/2}$ [$M^{(pre)}_{a_p,a_q}$ is the predatum of M_{a_p,a_q}.] or more, the total number that (PI) and (IM) act on $|b >$ is γ_{a_p,a_q}. Next, (OB) observes $|b >$, and the data of M_{a_p,a_q} remain. These actions are repeated sequentially from $|a_1 >$ to $|a_k >$, where M_0 is n^k, $M_{a_1,a_{(m_1"+1)}}$ is $n^{k-m_1}(n-1)^{m_1}$ [$1 \leq m_1 \leq k-1$. m_1 is an integer.], $M_{a_2,a_{(m_2+2)}}$ is $n(n-1)^{(k-1)-m_2}(n-2)^{m_2}$ [$1 \leq m_2 \leq k-2$. m_2 is an integer.], $M_{a_3,a_{(m_3+3)}}$ is $n(n-1)(n-2)^{(k-2)-m_3}(n-3)^{m_3}$ [$1 \leq m_3 \leq k-3$. m_3 is an integer.], $\cdots$ and M_{a_{k-1},a_k} is $n!/(n-k)!$. After all, $|c_1 >, |c_2 >$ and the total states become $|x_{a_1,a_2}x_{a_1,a_3} \cdots x_{a_1,a_k}x_{a_2,a_3}x_{a_2,a_4} \cdots x_{a_2,a_k} \cdots x_{a_{k-1},a_k} >$, $|a_1 n^{n-1} + a_2 n^{n-2} + \cdots + a_k n^{n-k} >$ and $n!/(n-k)![= W_0]$, respectively.

Step 5: It is assumed that a quantum gate (C) doesn't change $|d >$ at $c_1 = 1$, or it changes $|d >$ for $|d + c_2 >$ in the others of c_1 from $|c_1 >$ and $|c_2 >$.

Step 6: It is assumed that a quantum gate (D_1) changes $|e >$ for $|1 >$ in $d = 0$ and $U_k(V = 1) \leq d \leq U_k(V = (n!/4) - (n-k)!k!)$, or it changes $|e >$ for $|0 >$ in the others of d. As the target state for $|e >$ is 1, (PI) and (IM) act on $|e >$. The number of the data that is included in $d = 0$ and $U_k(V = 1) \leq d \leq U_k(V = (n!/4) - (n-k)!k!)$ is $W_1 \approx n!/(4(n-k)!)$. When δ_1 is the minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|e >$ is $\delta_1 \approx 2$. Next, (OB) observes $|e >$, and the data of W_1 remain. Similarly, (D_i) [$2 \leq i \leq g-1$. i is the integer.] changes $|e >$ for $|1 >$ in $d = 0$ and $U_k(V = 1) \leq d \leq U_k(V = (n!/4^i) - (n-k)!k!)$, or it changes $|e >$ for $|0 >$ in the others of d. As the target state for $|e >$ is 1, (PI) and (IM) act on $|e >$. The number of the data that is included in $d = 0$ and $U_k(V = 1) \leq d \leq U_k(V = (n!/4^i) - (n-k)!k!)$ is $W_i \approx n!/(4^i(n-k)!)$. When δ_i is the minimum even integer that is $(W_{i-1}/W_i)^{1/2}$ or more, the total number that (PI) and (IM) act on $|e >$ is $\delta_i \approx 2$. Next, (OB) observes $|e >$, and the data of W_i remain. These actions are repeated sequentially from 2 to $g-1$ at i. (D_g) changes $|e >$ for $|1 >$ at $d = 0$, or it changes $|e >$ for $|0 >$ in the others of d. As the target state for $|e >$ is 1, (PI) and (IM) act on $|e >$. The number of the data that is included at $d = 0$ is $W_g \approx k!$. When δ_g is the minimum even integer that is $(W_{g-1}/W_g)^{1/2}$ or more, the total number that (PI) and (IM) act on $|e >$ is $\delta_g \approx 2$. Next, (OB) observes $|a_1 >, |a_2 >, \cdots, |a_k >, |b >, |c_1 >, |c_2 >, |d >$ and $|e >$, and one of the data of W_g remains. Therefore, one example of orders that are $d = 0$ is obtained.

3.3. Case of $n/2 < k \leq n-2$

When the range of k is $n/2 < k \leq n-2$, it is assumed that k' is $n-k$. The quantum registers of $|a >$-type consists of $|a'_1 >, |a'_2 >, \cdots, |a'_{k'-1} >$ and $|a'_{k'} >$. The value of $|x'_{a'_1,a'_2}x'_{a'_1,a'_3} \cdots x'_{a'_1,a'_{k'}}x'_{a'_2,a'_3}x'_{a'_2,a'_4} \cdots x'_{a'_2,a'_{k'}} \cdots x'_{a'_{k'-1},a_{k'}} >$ of the combinations of k'

except those of k is calculated at (B), where, when $x_{i,j}$ is 0, $x'_{i,j}$ is 1, and when $x_{i,j}$ is 1, $x'_{i,j}$is 0. At the last (OB), states of the combination of k' and the state of the calculation value of $|c_1>$ are obtained. The combination of k except it of k' becomes it of the members of the clique that is requested.

3.4. Case of $k = n - 1$ $(k' = n - k = 1)$

It is assumed that the one person alone isn't a member of the clique among n persons. In $x_{i,j}$ [$0 \le i \ne j \le n-1$. i and j are integers.], when the R-th [$0 \le R \le n-1$. R is an integer.] isn't contained in i or j, $x_{i,j}$ is 1, and when the R-th is contained in i or j, $x_{i,j}$ is 0. It is assumed that there are α [$\ge log_2 n$] and g [$n \le 4^g$].

$|a'_1>, |b>, |c_1>, |c_2>, |d>$ and $|e>$ are set $|0>, |0>, |1>, |0>, |0>$ and $|0>$, respectively. The state of $|a'_1>$ is a'_1. And, $\boxed{\text{H}}$ is made to act on $|a'_1>$. Therefore, the total number becomes α [$\ge log_2 n$].

At (A), a state of $|a'_1>$ is compared with n. At $a'_1 < n$, $|b>, |c_1>$ and $|c_2>$ are changed for $|1>$, $|c_1 \times x_{a_1,a_2} x_{a_1,a_3} \cdots x_{a_1,a_k} x_{a_2,a_3} x_{a_2,a_4} \cdots x_{a_2,a_k} \cdots x_{a_{k-1},a_k}>$ and $|c_2 + a'_1>$, respectively. At the others of a'_1, $|b>, |c_1>$ and $|c_2>$ are changed for $|0>, |c_1 \times 0>$ and $|c_2 + 0>$, respectively. Then, (PI) and (IM) act on $|b>$, where a target state is 1. The frequency is the minimum even integer β that is $(2^\alpha/n)^{1/2} \le \beta$. Then, when only $|b>$ is observed by (OB), states of $|b>$ remain only 1. The number of states of $|a'_1>$ becomes $n[= W_0]$. Here, the action of (B) is unnecessary.

(C) doesn't change $|d>$ at $c_1 = 1$, or it changes $|d>$ for $|d+1+c_2>$ in the others of c_1 from $|c_1>$ and $|c_2>$. (D_1) changes $|e>$ for $|1>$ in $d = 0$ and $1 \le d \le (n/4)-1$, or it changes $|e>$ for $|0>$ in the others of d. As the target state for $|e>$ is 1, (PI) and (IM) act on $|e>$. The number of the data that is included in $d = 0$ and $1 \le d \le (n/4)-1$ is $W_1 \approx n/4$. When δ_1 is the minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|e>$ is $\delta_1 \approx 2$. Next, (OB) observes $|e>$, and the data of W_1 remain. Similarly, (D_i) [$2 \le i \le g-1$. i is the integer.] changes $|e>$ for $|1>$ in $d = 0$ and $1 \le d \le (n/4^i)-1$, or it changes $|e>$ for $|0>$ in the others of d. As the target state for $|e>$ is 1, (PI) and (IM) act on $|e>$. The number of the data that is included in $d = 0$ and $1 \le d \le (n/4^i)-1$ is $W_i \approx n/4^i$. When δ_i is the minimum even integer that is $(W_{i-1}/W_i)^{1/2}$ or more, the total number that (PI) and (IM) act on $|e>$ is $\delta_i \approx 2$. Next, (OB) observes $|e>$, and the data of W_i remain. These actions are repeated sequentially from 2 to $g-1$ at i. (D_g) changes $|e>$ for $|1>$ at $d = 0$, or it changes $|e>$ for $|0>$ in the others of d. As the target state for $|e>$ is 1, (PI) and (IM) act on $|e>$. The number of the data that is included at $d = 0$ is $W_g \approx 1$. When δ_g is the minimum even integer that is $(W_{g-1}/W_g)^{1/2}$ or more, the total number that (PI) and (IM) act on $|e>$ is $\delta_g \approx 2$. Next, (OB) observes $|a'_1>, |b>, |c_1>, |c_2>, |d>$ and $|e>$, and one datum remains. Therefore, the $n-1$ except the 1 nonmember are the members of the clique.

4. Numerical Computation

It describes as follows as examples in case of $k = 16, 17$, and 32 at $n = 33$. The cases of $k = 16$ and 17 become the maximum computational complexity at $n = 33$. And the case of $k = 32$ ($k' = n - k = 33 - 32 = 1$) becomes the minimum computational complexity at $n = 33$.

4.1. Case of $k = 16$ at $n = 33$

16 of the 0th, 3rd, 5th, 6th, 8th, 10th, 13th, 16th, 17th, 20th, 21st, 22nd, 23rd, 26th, 27th and 30th are the members of the clique among 33 from the 0th to the 32nd. When i and j that are integers with $0 \le i \ne j \le 32$ have the connection, $x_{i,j}[= x_{j,i}]$ is 1, and when they don't have the connection, $x_{i,j}[= x_{j,i}]$ is 0. It is assumed that there are $g = 16[n!/((n-k)!k!) = 33!/(17!16!) \approx 8.683 \times 10^{36}/7.441 \times 10^{27} \approx 1.167 \times 10^9 \le 4^{16} \approx 4.295 \times 10^9]$, $U_{16}(V = 1) = \sum_{i=1}^{16}(i-1)33^{33-i}$, $U_{16}(V = (33!/4) - 17!16! \approx 2.171 \times 10^{36}) \approx \sum_{i=1}^{16}\varphi_i 33^{33-i}$ [for example, $\varphi_1 = 8, \varphi_2 = 9, \varphi_3 = 1, \varphi_4 = 20, \varphi_5 = 2, \varphi_6 = 11, \varphi_7 = 17, \varphi_8 = 3, \varphi_9 = 32, \varphi_{10} = 0, \varphi_{11} = 4, \varphi_{12} = 15, \varphi_{13} = 28, \varphi_{14} = 18, \varphi_{15} = 13, \varphi_{16} = 24, \varphi_{17} = 27, \cdots, \varphi_{32} = 16$ and $\varphi_{33} = 5$. $V = \sum_{i=1}^{32} v_i(33-i)!$. $v_1 = 8, v_2 = 8, v_3 = 1, v_4 = 17, v_5 = 1, v_6 = 7, v_7 = 12, v_8 = 1, v_9 = 24, v_{10} = 0, v_{11} = 0, v_{12} = 7, v_{13} = 17, v_{14} = 8, v_{15} = 5, v_{16} = 11, v_{17} = 13, \cdots, v_{31} = 0$ and $v_{32} = 0$.], $U_{16}(V = (33!/4^2) - 17!16! \approx 5.427 \times 10^{35}), \cdots, U_{16}(V = (33!/4^{14}) - 17!16! \approx 2.491 \times 10^{28})$ and $U_{16}(V = (33!/4^{15}) - 17!16! \approx 6.44 \times 10^{26})$.

First of all, $|a_1>, |a_2>, \cdots, |a_{16}>, |b>, |c_1>, |c_2>, |d>$ and $|e>$ are prepared. When α is the minimum integer that is $log_2 n = log_2 33 \approx 5.047 \le 6 = \alpha$, each of $|a_f>$ that f is the integer from 1 to 16 is consisted of 6 qubits. States of $|a_1>, |a_2>, \cdots, |a_{16}>, |b>, |c_1>, |c_2>, |d>$ and $|e>$ are $a_1, a_2, \cdots, a_{16}, b, c_1, c_2, d$ and e, respectively.

Step 1: Each qubit of $|a_1>, |a_2>, \cdots, |a_{16}>, |b>, |c_2>, |d>$ and $|e>$ is set $|0>$, and the qubit of $|c_1>$ is set $|1>$.

Step 2: $\boxed{\text{H}}$ acts on each qubit of $|a_1>, |a_2>, \cdots, |a_{15}>$ and $|a_{16}>$. It changes them for entangled states. The total states are $(2^6)^{16}$.

Step 3: (A) changes $|b>$ for $|1>$ in $a_f < 33$, or it changes $|b>$ for $|0>$ in the others of a_f. As a target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When β is the minimum even integer that is $(2^6/33)^{1/2} = (64/33)^{1/2} \approx 1.393 \le 2 = \beta$, the total number that (PI) and (IM) act on $|b>$ is β. Next, (OB) observes $|b>$. These actions are repeated sequentially from $|a_1>$ to $|a_{16}>$. Therefore, each state of $|a_f>$ is $0, 1, \cdots, 31$ and 32, and the total states become $33^{16}[= M_0]$.

Step 4: (B) changes $|b>, |c_1>$ and $|c_2>$ for $|1>, |c_1 \times x_{a_1,a_2}>$ and $|c_2+(a_1 33^{33-1}+a_2 33^{33-2})/15>$, respectively, at $a_1 \neq a_2$, or it changes $|b>, |c_1>$ and $|c_2>$ for $|0>, |c_1 \times 0>$ and $|c_2+0>$, respectively, at $a_1 = a_2$. As the target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When γ_{a_1,a_2} is the minimum even integer that is $(M_0/M_{a_1,a_2})^{1/2} = (33^{16}/(33^{15} \cdot 32))^{1/2} = (33/32)^{1/2} \approx 1.016 \leq 2 = \gamma_{a_1,a_2}$, the total number that (PI) and (IM) act on $|b>$ is γ_{a_1,a_2}. Next, (OB) observes $|b>$, and the data of $M_{a_1,a_2} = 33^{15} \cdot 32$ remain. (B) changes $|b>, |c_1>$ and $|c_2>$ for $|1>, |c_1 \times x_{a_1,a_3}>$ and $|c_2 + (a_1 33^{33-1} + a_3 33^{33-3})/15>$, respectively, at $a_1 \neq a_3$, or it changes $|b>, |c_1>$ and $|c_2>$ for $|0>, |c_1 \times 0>$ and $|c_2 + 0>$, respectively, at $a_1 = a_3$. As the target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When γ_{a_1,a_3} is the minimum even integer that is $(M_{a_1,a_2}/M_{a_1,a_3})^{1/2} = (33^{15} \cdot 32/(33^{14} \cdot 33^2))^{1/2} = (33/32)^{1/2} \approx 1.016 \leq 2 = \gamma_{a_1,a_3}$, the total number that (PI) and (IM) act on $|b>$ is γ_{a_1,a_3}. Next, (OB) observes $|b>$, and the data of $M_{a_1,a_3} = 33^{14} \cdot 32^2$ remain. Similarly, (B) changes $|b>, |c_1>$ and $|c_2>$ for $|1>, |c_1 \times x_{a_p,a_q}>$ and $|c_2 + (a_p n^{n-p} + a_q n^{n-q})/(k-1)>$, respectively, at $a_p \neq a_q$ [$1 \leq p < q \leq 16$. p and q are integers.], or it changes $|b>, |c_1>$ and $|c_2>$ for $|0>, |c_1 \times 0>$ and $|c_2 + 0>$, respectively, at $a_p = a_q$. As the target state for $|b>$ is 1, (PI) and (IM) act on $|b>$. When γ_{a_p,a_q} is the minimum even integer that is $(M^{(pre)}_{a_p,a_q}/M_{a_p,a_q})^{1/2} \approx 2 \leq 2 = \gamma_{a_p,a_q}$, the total number that (PI) and (IM) act on $|b>$ is γ_{a_p,a_q}. Next, (OB) observes $|b>$, and the data of M_{a_p,a_q} remain. These actions are repeated sequentially from $|a_1>$ to $|a_{16}>$, where M_0 is 33^{16}, $M_{a_1,a_{(m_1"+1)}}$ is $33^{16-m_1}32^{m_1}$ [$1 \leq m_1 \leq 15$. m_1 is the integer.], $M_{a_2,a_{(m_2+2)}}$ is $33 \cdot 32^{15-m_2}31^{m_2}$ [$1 \leq m_2 \leq 14$. m_2 is the integer.], $M_{a_3,a_{(m_3+3)}}$ is $33 \cdot 32 \cdot 31^{14-m_3}30^{m_3}$ [$1 \leq m_3 \leq 13$. m_3 is the integer.], $\cdots$ and $M_{a_{15},a_{16}}$ is $33!/17!$. After all, the total states become $33!/17![= W_0]$.

Step 5: (C) doesn't change $|d>$ at $c_1 = 1$, or it changes $|d>$ for $|d + c_2>$ in the others of c_1 from $|c_1>$ and $|c_2>$.

Step 6: (D_1) changes $|e>$ for $|1>$ in $d = 0$ and $U_{16}(V = 1) \leq d \leq U_{16}(V = (33!/4) - 17!16! \approx 2.171 \times 10^{36})$, or it changes $|e>$ for $|0>$ in the others of d. As the target state for $|e>$ is 1, (PI) and (IM) act on $|e>$. The number of the data that is included in $d = 0$ and $U_{16}(V = 1) \leq d \leq U_{16}(V \approx 2.171 \times 10^{36})$ is $W_1 \approx 33!/(4 \cdot 17!)$. When δ_1 is the minimum even integer that is $(W_0/W_1)^{1/2} \approx 2 \leq 2 = \delta_1$, the total number that (PI) and (IM) act on $|e>$ is $\delta_1 \approx 2$. Next, (OB) observes $|e>$, and the data of W_1 remain. Similarly, (D_i) [$2 \leq i \leq 15$. i is the integer.] changes $|e>$ for $|1>$ in $d = 0$ and $U_{16}(V = 1) \leq d \leq U_{16}(V = (33!/4^i) - 17!16!)$, or it changes $|e>$ for $|0>$ in the others of d. As the target state for $|e>$ is 1, (PI) and (IM) act on $|e>$. The number of the data that is included in $d = 0$ and $U_{16}(V = 1) \leq d \leq U_{16}(V = (33!/4^i) - 17!16!)$ is $W_i \approx 33!/(4^i \cdot 17!)$. When δ_i is the minimum even integer that is $(W_{i-1}/W_i)^{1/2} \approx 2 \leq 2 = \delta_i$, the total number that (PI) and (IM) act on $|e>$ is $\delta_i \approx 2$. Next, (OB) observes $|e>$, and the data of W_i remain. These actions are repeated sequentially from 2 to 15 at i. (D_{16}) changes $|e>$ for $|1>$ at $d = 0$, or it changes $|e>$ for $|0>$ in the others of d. As the target state for $|e>$ is 1, (PI) and (IM) act on $|e>$. The number of the data that is included at $d = 0$ is $W_{16} \approx 16!$. When δ_{16} is the minimum even integer that is $(W_{15}/W_{16})^{1/2} \approx 2 \le 2 = \delta_{16}$, the total number that (PI) and (IM) act on $|e>$ is $\delta_{16} \approx 2$. Next, (OB) observes $|a_1>, |a_2>, \cdots, |a_{16}>, |b>, |c_1>, |c_2>, |d>$ and $|e>$, and one of the data of W_{16} remains. Therefore, $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, b, c_1, c_2, d$ and e are 0, 17, 16, 20, 8, 5, 26, 6, 27, 10, 23, 13, 21, 30, 3, 22, 1, 1, $\sum_{i=1}^{16} a_i 33^{33-i}$ $[= U_{16}\ (V = (33!/4^3) - 17!16! \approx 1.357 \times 10^{35})], 0$ and 1, respectively. As a result, a combination of the clique that is the 0th, 3rd, 5th, 6th, 8th, 10th, 13th, 16th, 17th, 20th, 21st, 22nd, 23rd, 26th, 27th and 30th is obtained.

4.2. Case of $k = 17$ $(k' = 33 - 17 = 16)$ at $n = 33$

When the range of $k = 17$ is $33/2 < k \le 31$, it is assumed that k' is $n - k = 33 - 17 = 16$. The quantum registers of $|a>$-type consists of $|a'_1>, |a'_2>, \cdots, |a'_{15}>$ and $|a'_{16}>$. The value of $|x'_{a'_1,a'_2} x'_{a'_1,a'_3} \cdots x'_{a'_1,a'_{16}} x'_{a'_2,a'_3} x'_{a'_2,a'_4} \cdots x'_{a'_2,a'_{16}} \cdots x'_{a'_{15},a'_{16}}>$ of the combinations of $k' = 16$ except those of $k = 17$ is calculated at (B), where, when $x_{i,j}$ is 0, $x'_{i,j}$ is 1, and when $x_{i,j}$ is 1, $x'_{i,j}$is 0. At the last (OB), states of the combination of $k' = 16$ and the state of the calculation value of $|c_1>$ are obtained. The combination of $k = 17$ except it of $k' = 16$ becomes it of the members of the clique that is requested.

4.3. Case of $k = 32$ $(k' = 33 - 32 = 1)$ at $n = 33$

It is assumed that the 5th person alone isn't a member of the clique among 33 from the 0th to the 32nd. In $x_{i,j}$ [$0 \le i \ne j \le 32$. i and j are integers.], when the 5th isn't contained in i or j, $x_{i,j}$ is 1, and when the 5th is contained in i or j, $x_{i,j}$ is 0. It is assumed that there are $\alpha = 6$ [$log_2 33 \approx 5.047 \le 6 = \alpha$] and $g = 3$ [$33 \le 4^g = 4^3 = 64$].

$|a'_1>, |b>, |c_1>, |c_2>, |d>$ and $|e>$ are set $|0>, |0>, |1>, |0>, |0>$ and $|0>$, respectively. The state of $|a'_1>$ is a'_1. And, $\boxed{\text{H}}$ is made to act on $|a'_1>$. Therefore, the total number becomes $\alpha = 6$.

At (A), the state of $|a'_1>$ is compared with $n = 33$. At $a'_1 < 33$, $|b>, |c_1>$ and $|c_2>$ are changed for $|1>, |c_1 \times x_{a_1,a_2} x_{a_1,a_3} \cdots x_{a_1,a_{32}} x_{a_2,a_3} x_{a_2,a_4} \cdots x_{a_2,a_{32}} \cdots x_{a_{31},a_{32}}>$ and $|c_2 + a'_1>$, respectively. At the others of a'_1, $|b>, |c_1>$ and $|c_2>$ are changed for $|0>, |c_1 \times 0>$ and $|c_2 + 0>$, respectively. Then, (PI) and (IM) act on $|b>$, where the target state is 1. The frequency is the minimum even integer β that is $(2^6/33)^{1/2} \approx 1.393 \le 2 = \beta$. Then, when only $|b>$ is observed by (OB), states of $|b>$ remain only 1. The number of states of $|a'_1>$ becomes $33[= W_0]$. Here, the action of (B) is unnecessary.

(C) doesn't change $|d>$ at $c_1 = 1$, or it changes $|d>$ for $|d + 1 + c_2>$ in the others of c_1 from $|c_1>$ and $|c_2>$. (D_1) changes $|e>$ for $|1>$ in $d = 0$ and $1 \le d \le (33/4) - 1$, or it changes $|e>$ for $|0>$ in the others of d. As the target state for $|e>$ is 1, (PI) and (IM) act on $|e>$. The number of the data that is included in $d = 0$ and $1 \le d \le (33/4) - 1$ is $W_1 \approx 33/4$. When δ_1 is the minimum even integer that is $(W_0/W_1)^{1/2} \approx 2 \le 2 = \delta_1$, the total number that (PI) and (IM) act on $|e>$ is $\delta_1 \approx 2$. Next, (OB) observes $|e>$, and the data of W_1 remain. (D_2) changes $|e>$ for $|1>$ in $d = 0$ and $1 \le d \le (33/4^2) - 1$, or it changes $|e>$ for $|0>$ in the others of d. As the target state for $|e>$ is 1, (PI) and (IM) act on $|e>$. The number of the data that is included in $d = 0$ and $1 \le d \le (33/4^2) - 1$ is $W_2 \approx 33/4^2$. When δ_2 is the minimum even integer that is $(W_1/W_2)^{1/2} \approx 2 \le 2 = \delta_2$, the total number that (PI) and (IM) act on $|e>$ is $\delta_2 \approx 2$. Next, (OB) observes $|e>$, and the data of W_2 remain. (D_3) changes $|e>$ for $|1>$ at $d = 0$, or it changes $|e>$ for $|0>$ in the others of d. As the target state for $|e>$ is 1, (PI) and (IM) act on $|e>$. The number of the data that is included at $d = 0$ is $W_3 \approx 1$. When δ_3 is the minimum even integer that is $(W_2/W_3)^{1/2} \approx 2 \le 2 = \delta_3$, the total number that (PI) and (IM) act on $|e>$ is $\delta_3 \approx 2$. Next, (OB) observes $|a_1'>, |b>, |c_1>, |c_2>, |d>$ and $|e>$, and one of the W_3 remains. Therefore, the 32 except the 1 nonmember as the 5th are the members of the clique.

5. Discussion and Summary

The computational complexity of the quantum algorithm is generally as follows.

(I) For $2 \le k \le n/2$: In the order of the actions, frequencies are αk at $\boxed{\text{H}}$, k at (A), $\beta k \approx 2k$ at (PI) and (IM), k at (OB), $k(k-1)/2$ at (B), $\sum_{1 \le p < q \le k} \gamma_{a_p,a_q} \approx k(k-1)$ at (PI) and (IM), $k(k-1)/2$ at (OB), 2 at (C), g at (D_i) [$1 \le i \le g$. i is the integer.], $\sum_{i=1}^{g} \delta_i \approx 2g$ at (PI) and (IM), and g at (OB). Therefore, total S_1 is $2k^2 + (\alpha + 2)k + 2 + 4g$. S_1 is 706 in the section 4.1 for $n = 33$ and $k = 16$. In the classical computation, total Z is about 1.167×10^9.

(II) For $n/2 < k \le n - 2$ $(k' = n - k, 2 \le k' < n/2)$: In the above-mentioned example, a frequency of case of k is same as that of k'. Therefore, in the section 4.2 for $n = 33$ and $k = 17$ $(k' = 16)$, S_1 is 706. In the classical computation, Z is about 1.167×10^9.

(III) For $k = n - 1$ $(k' = n - k = 1)$: In the order of the actions, frequencies are α at $\boxed{\text{H}}$, once at (A) gate, $\beta \approx 2$ at (PI) and (IM), once at (OB), 2 at (C), g at (D_i), $\sum_{i=1}^{g} \delta_i \approx 2g$ at (PI) and (IM), and g at (OB). Therefore, total S_2 is $\alpha + 6 + 4g$. S_2 is 24 in the section 4.3 for $n = 33$ and $k = 32$ $(k' = 1)$. In the classical computation, Z is 33.

When n is large enough, S_1 and S_2 become about $(2k + 3log_2 n)k$ and $2(log_2 n)n$, respectively, where α is about $log_2 n$, g is about $(1/2)log_2 n!$, and $n!$ is about $n^n e^{-n}(2\pi n)^{1/2}$

[Stirling's formula]. For example, as for $n = 100$ and $k = 50$, S_1/Z is about $(2k + 3log_2 n)k/(n!/((n-k)!k!)) = (2 \cdot 50 + 3log_2 100)50/\ (100!/(50!50!)) \approx 1/10^{25}$.

Therefore, the polynomial time process becomes possible.

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