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Quantum Algorithm for Timetabling Problem by Numbering Method

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Abstract

A quantum algorithm for the timetabling problem by a numbering method and its example are reported. When n lectures are given at k rooms, an optimal timetable is decided. It is considered this time that the number of persons admitted for each room, the number of participants for each lecture, and connected lectures. A computational complexity of a classical computation is k^n. The computational complexity becomes about n^2 by the quantum algorithm that uses quantum phase inversion gates, quantum inversion about mean gates and the numbering method. Therefore, a polynomial time process becomes possible.

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1. Introduction

Haroche and Wineland [1] developed methods for measuring and manipulating individual quantum particles, which were the very first steps towards building a quantum computer. The algorithms of the quantum computer by Deutsch-Jozsa [2−4], Shor [3−5], Grover [3, 6, 7] and so on are known. A quantum algorithm for the traveling salesman problem by a numbering method has recently been reported by Fujimura [8]. Its computational complexity becomes a polynomial time. The timetabling problem [9, 10] is examined by the numbering method this time. Therefore, its result is reported.

2. Timetabling Problem

When lectures are given at rooms, an optimal timetable is decided. It is considered that time periods, rooms and other conditions.

3. Quantum Algorithm

When n lectures that have time periods R_t $[0 \leq t \leq n-1$. t is an integer.], numbers of participants P_t, and connected lectures are given at k rooms that have numbers of persons admitted M_u $[0 \leq u \leq k-1$. u is an integer.], an optimal timetable is decided. When there is an overlap between the time period of the i-th lecture R_i and it of the j-th lecture R_j, $x_{i,j}$ $[0 \leq i < j \leq n-1$. i and j are integers.] is 1, and when there isn't an overlap between these lectures, it is 0. $\sum_{j=1 \to n-1, i<j} \sum_{i=0 \to n-2} x_{i,j}$ is m. When P_t is M_u or less, $y_{t,u}$ is 1, and when P_t is larger than M_u, $y_{t,u}$ is 0. When the i-th lecture connects the j-th lecture, $z_{i,j}$ $[0 \leq i < j \leq n-1$. i and j are integers.] is 0, and when it doesn't connect the j-th lecture, $z_{i,j}$ is 1. Now, it is assumed that a_i $[0 \leq i \leq n-1$. i is the integer.] is 0 or 1 or ⋯ or $k-1$. When the number of the n times repeated permutation of 0, 1, ⋯, $k-2$ and $k-1$ is k^n, $a_0 k^{n-1} + a_1 k^{n-2} + \cdots + a_{n-1} k^0 = \sum_{i=0 \to n-1} a_i k^{n-1-i} = U$ is the numbering datum from 0 to $k^n - 1$ [The 0-th datum is 0, 0, ⋯, 0 and 0. The $(k^n - 1)$-th datum is $(k-1)$, $(k-1)$, ⋯, $(k-1)$ and $(k-1)$.]. This method is named the numbering method for this problem. g is the minimum integer that follows $k^n/1 \leq 4^g = 2^{2g}$, because a number of combinations of an answer is 1 at least. α

First of all, quantum registers $|a_0>$, $|a_1>$, ⋯, $|a_{n-1}>$, $|b_1>$, $|b_2>$, $|c>$ and $|d>$ are prepared. When α is the minimum integer that is $log_2 k$ or more, each of $|a_f>$ that f is an integer from 0 to $n-1$ is consisted of α quantum bits [=qubits]. States of $|a_f>$, $|b_1>$, $|b_2>$, $|c>$ and $|d>$ are a_f, b_1, b_2, c and d, respectively.

Step 1: Each qubit of $|a_f>$, $|b_1>$, $|b_2>$, $|c>$ and $|d>$ is set $|0>$.

Step 2: The Hadamard gate $\boxed{\text{H}}$ [3, 4] acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^{\alpha})^n$.

Step 3: It is assumed that a quantum gate (A) changes $|b_1>$ for $|1>$ in $a_f < k$, or it changes $|b_1>$ for $|0>$ in the others of a_f. As a target state for $|b_1>$ is 1, quantum phase inversion gates (PI) and quantum inversion about mean gates (IM) [3, 6, 7] act on $|b_1>$. When β is the minimum even integer that is $(2^{\alpha}/k)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b_1>$ is β, because they are a couple. Next, an observation gate (OB) observes $|b_1>$. These actions are repeated sequentially from $|a_0>$ to $|a_{n-1}>$. Therefore, each state of $|a_f>$ is 0, 1, ⋯, $k-2$ and $k-1$, and the total states become k^n [$= W_0$].

Step 4: It is assumed that a quantum gate ($B_{i,j}$) [$0 \le i < j \le n - 1$. i and j are integers.] changes $|b_1>$ for $|b_1 + x_{i,j}\, y_{i,ai}\, y_{j,aj}\, z_{i,j}>$ at $a_i \ne a_j$, or it doesn't change $|b_1>$ at $a_i = a_j$, and it changes $|b_2>$ for $|b_2 + (a_i\, k^{n-1-i} + a_j\, k^{n-1-j}) / (n-1)>$ from a_i and a_j. These actions are repeated sequentially at i and j. Therefore, $|b_1>$ becomes from $|0>$ to $|x_{0,1}\, y_{0,a0}\, y_{1,a1}\, z_{0,1} + x_{0,2}\, y_{0,a0}\, y_{2,a2}\, z_{0,2} + \cdots + x_{n-2,n-1}\, y_{n-2,a(n-2)}\, y_{n-1,a(n-1)}\, z_{n-2,n-1}>$, and $|b_2>$ becomes $|a_0\, k^{n-1} + a_1\, k^{n-2} + \cdots + a_{n-1}\, k^0>$.

Step 5: It is assumed that a quantum gate (C) doesn't change $|c>$ at $b_1 = m$, or it changes $|c>$ for $|c + 1 + b_2>$ in the others of b_1.

Step 6: It is assumed that a quantum gate (D_1) changes $|d>$ for $|1>$ in $0 \le c \le (k^n/4) - 1$, or it changes $|d>$ for $|0>$ in the others of c. As the target state for $|d>$ is 1, (PI) and (IM) act on $|d>$. The number of the data that is included in $0 \le c \le (k^n/4) - 1$ is $W_1 \approx k^n/4$. When γ_1 is the minimum even integer that is $(W_0/W_1)^{1/2} \approx (k^n/(k^n/4))^{1/2}$ or more, the total number that (PI) and (IM) act on $|d>$ is $\gamma_1 \approx 2$. Next, (OB) observes $|d>$, and the data of W_1 remain.

Similarly, (D_i) [$2 \le i \le g - 1$. i is the integer.] changes $|d>$ for $|1>$ in $0 \le c \le (k^n/4^i) - 1$, or it changes $|d>$ for $|0>$ in the others of c. As the target state for $|d>$ is 1, (PI) and (IM) act on $|d>$. The number of the data that is included in $0 \le c \le (k^n/4^i) - 1$ is $W_i \approx k^n/4^i$. When γ_i is the minimum even integer that is $(W_{i-1}/W_i)^{1/2} \approx ((k^n/4^{i-1})/(k^n/4^i))^{1/2}$ or more, the total number that (PI) and (IM) act on $|d>$ is $\gamma_i \approx 2$. Next, (OB) observes $|d>$, and the data of W_i remain.

(D_g) changes $|d>$ for $|1>$ at $c = 0$, or it changes $|d>$ for $|0>$ in the others of c. As the target state for $|d>$ is 1, (PI) and (IM) act on $|d>$. The number of the data that is included at $c = 0$ is $W_g \approx 1 \approx k^n/4^g$. When γ_g is the minimum even integer that is $(W_{g-1}/W_g)^{1/2} \approx ((k^n/4^{g-1})/(k^n/4^g))^{1/2}$ or more, the total number that (PI) and (IM) act on $|d>$ is $\gamma_g \approx 2$. Next, (OB) observes $|a_f>$, $|b_1>$, $|b_2>$, $|c>$ and $|d>$, and one of the data of W_g remains. Therefore, one example of combinations that are $b_1 = m$ is obtained.

4. Numerical Computation

It is assumed that there are $n = 10$, $R_0 = (9{:}00 \rightarrow 11{:}50)$, $R_1 = (13{:}00 \rightarrow 15{:}50)$, $R_2 = (14{:}30 \rightarrow 17{:}20)$, $R_3 = (13{:}00 \rightarrow 17{:}20)$, $R_4 = (9{:}00 \rightarrow 10{:}20)$, $R_5 = (9{:}00 \rightarrow 10{:}20)$, $R_6 = (10{:}30 \rightarrow 11{:}50)$, $R_7 = (10{:}30 \rightarrow 11{:}50)$, $R_8 = (13{:}00 \rightarrow 14{:}20)$, $R_9 = (16{:}00 \rightarrow 17{:}20)$, $P_0 = 100$, $P_1 = 50$, $P_2 = 100$, $P_3 = 50$, $P_4 = 60$, $P_5 = 40$, $P_6 = 70$, $P_7 = 30$, $P_8 = 100$, $P_9 = 50$, $M_0 = 100$, $M_1 = 80$, $M_2 = 50$, $k = 3$, $z_{0,8} = z_{1,4} = 0$, and (the others of $z_{i,j}$ [$0 \le i < j \le 9$]) = 1. Therefore, it is set that there are $x_{0,4} = x_{0,5} = x_{0,6} = x_{0,7} = x_{1,2} = x_{1,}$

$_3 = x_{1,8} = x_{2,3} = x_{2,9} = 1$, (the others of $x_{i,j}$ $[0 \leq i < j \leq 9]$) = 0, $m = 9$, $y_{0,1} = y_{2,1} = y_{8,1} = y_{0,2} = y_{2,2} = y_{4,2} = y_{6,2} = y_{8,2} = 0$, (the others of $y_{t,u}$ $[0 \leq t \leq 9, 0 \leq u \leq 2]$) = 1, and $g = 8$ $[k^n/1 = 3^{10}/1 \leq 4^g = 4^8 = 65536]$.

First of all, $|a_0>$, $|a_1>$, ⋯, $|a_9>$, $|b_1>$, $|b_2>$, $|c>$ and $|d>$ are prepared. When α is the minimum integer that is $log_2 3 \approx 1.6 \leq 2 = \alpha$, each of $|a_f>$ that f is the integer from 0 to 9 is consisted of 2 qubits. States of $|a_f>$, $|b_1>$, $|b_2>$, $|c>$ and $|d>$ are a_f, b_1, b_2, c and d, respectively.

Step 1: Each qubit of $|a_f>$, $|b_1>$, $|b_2>$, $|c>$ and $|d>$ is set $|0>$.

Step 2: $\boxed{\text{H}}$ acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^{\alpha})^n = (2^2)^{10}$.

Step 3: (A) changes $|b_1>$ for $|1>$ in $a_f < 3$, or it changes $|b_1>$ for $|0>$ in the others of a_f. As the target state for $|b_1>$ is 1, (PI) and (IM) act on $|b_1>$. When β is the minimum even integer that is $(2^{\alpha}/k)^{1/2} = (2^2/3)^{1/2} \approx 1.2 \leq 2 = \beta$, the total number that (PI) and (IM) act on $|b_1>$ is $\beta \approx 2$. Next, (OB) observes $|b_1>$. These actions are repeated sequentially from $|a_0>$ to $|a_9>$. Therefore, each state of $|a_f>$ is 0, 1 and 2, and the total states become $k^n = 3^{10}$ [$= W_0$].

Step 4: ($B_{i,j}$) $[0 \leq i < j \leq 9$. i and j are integers.] changes $|b_1>$ for $|b_1 + x_{i,j}\, y_{i,ai}\, y_{j,aj}\, z_{i,j}>$ at $a_i \neq a_j$, or it doesn't change $|b_1>$ at $a_i = a_j$, and it changes $|b_2>$ for $|b_2 + (a_i\, 3^{9-i} + a_j\, 3^{9-j})/9>$ from a_i and a_j. These actions are repeated sequentially at i and j. Therefore, $|b_1>$ becomes from $|0>$ to $|x_{0,1}\, y_{0,a0}\, y_{1,a1}\, z_{0,1} + x_{0,2}\, y_{0,a0}\, y_{2,a2}\, z_{0,2} + \cdots + x_{8,9}\, y_{8,a8}\, y_{9,a9}\, z_{8,9}>$, and $|b_2>$ becomes $|a_0\, 3^9 + a_1\, 3^8 + \cdots + a_9\, 3^0>$.

Step 5: (C) doesn't change $|c>$ at $b_1 = m = 9$, or it changes $|c>$ for $|c + 1 + b_2>$ in the others of b_1.

Step 6: (D_1) changes $|d>$ for $|1>$ in $0 \leq c \leq (3^{10}/4) - 1$, or it changes $|d>$ for $|0>$ in the others of c. As the target state for $|d>$ is 1, (PI) and (IM) act on $|d>$. The number of the data that is included in $0 \leq c \leq (3^{10}/4) - 1$ is $W_1 \approx 3^{10}/4$. When γ_1 is the minimum even integer that is $(W_0/W_1)^{1/2} \approx (3^{10}/(3^{10}/4))^{1/2} \approx 2 \leq 2 = \gamma_1$, the total number that (PI) and (IM) act on $|d>$ is $\gamma_1 \approx 2$. Next, (OB) observes $|d>$, and the data of W_1 remain.

Similarly, (D_i) $[2 \leq i \leq 7$. i is the integer.] changes $|d>$ for $|1>$ in $0 \leq c \leq (3^{10}/4^i) - 1$, or it changes $|d>$ for $|0>$ in the others of c. As the target state for $|d>$ is 1, (PI) and

(*IM*) act on $|d>$. The number of the data that is included in $0 \leq c \leq (3^{10}/4^{i}) - 1$ is $W_i \approx 3^{10}/4^{i}$. When γ_i is the minimum even integer that is $(W_{i-1}/W_i)^{1/2} \approx ((3^{10}/4^{i-1})/(3^{10}/4^{i}))^{1/2} \approx 2 \leq 2 = \gamma_i$, the total number that (*PI*) and (*IM*) act on $|d>$ is $\gamma_i \approx 2$. Next, (*OB*) observes $|d>$, and the data of W_i remain.

(D_8) changes $|d>$ for $|1>$ at $c = 0$, or it changes $|d>$ for $|0>$ in the others of c. As the target state for $|d>$ is 1, (*PI*) and (*IM*) act on $|d>$. The number of the data that is included at $c = 0$ is $W_8 \approx 1 \approx 3^{10}/4^{8}$. When γ_8 is the minimum even integer that is $(W_7/W_8)^{1/2} \approx ((3^{10}/4^{7})/(3^{10}/4^{8}))^{1/2} \approx 2 \leq 2 = \gamma_8$, the total number that (*PI*) and (*IM*) act on $|d>$ is $\gamma_8 \approx 2$. Next, (*OB*) observes $|a_f>$, $|b_1>$, $|b_2>$, $|c>$ and $|d>$, and one of the data of W_8 remains. For example, a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, b_1, b_2, c and d are 0, 1, 0, 2, 1, 2, 1, 2, 0, 1, 9, $\sum_{i=0 \to 9} (a_i 3^{9-i})$, 0 and 1, respectively. Therefore, an answer is 0, 1, 0, 2, 1, 2, 1, 2, 0 and 1.

5. Discussion and Summary

The computational complexity of this quantum algorithm [= S] becomes the following. In the order of the actions by the gates, the number of them is αn at $\boxed{H}$, n at (A), $\beta n \approx 2n$ at (*PI*) and (*IM*), n at (*OB*), $n(n-1)$ at ($B_{i,j}$) [$0 \leq i < j \leq n-1$. i and j are integers.], 2 at (C), g at (D_i) [$1 \leq i \leq g$. i is the integer.], $\sum_{i=1 \to g} \gamma_i \approx 2g$ at (*PI*) and (*IM*), and g at (*OB*). Therefore, S becomes $n^2 + (\alpha + 3)n + 2 + 4g$. In the example of the section 4, S is 184. The computational complexity of the classical computation [= Z] is $k^n = 3^{10} = 59049$. After all, S/Z becomes about 1/320. When n is large enough, S becomes about n^2. And then S/Z is about n^2/k^n. For example, as for $n = 100$ and $k = 3$, S/Z is about $1/(5 \times 10^{43})$.

Therefore, the polynomial time process becomes possible.

References

[1] Kungl. Vetenskapsakademien (The Royal Swedish Academy of Sciences), The Nobel Prize in Physics 2012, [On line], Available: http: //www. kva. se/en /pressroom /Press-releases-2012 /The-Nobel-Prize-in-Physics-2012/, 2012.

[2] Deutsch D., and Jozsa R., Rapid solution of problems by quantum computation, *Proc. Roy. Soc. Lond. A*, 439:553-558, 1992.

[3] Takeuchi S., Ryoshi Konpyuta (Quantum Computer), Kodansha, Tokyo, Japan [in Japanese], 2005.

[4] Miyano K., and Furusawa A., Ryoshi Konpyuta Nyumon (An Introduction to Quantum Computation), Nippon Hyoron sha, Tokyo, Japan [in Japanese], 2008.

[5] Shor P.W., Algorithms for quantum computation: discrete logarithms and factoring, *Proc. 35th Annu. Symp. Foundations of Computer Science*, IEEE, pp.124-134, 1994.
[6] Grover L.K., A fast quantum mechanical algorithm for database search, *Proc. 28th Annu. ACM Symp. Theory of Computing*, pp.212-219, 1996.
[7] Grover L.K., A framework for fast quantum mechanical algorithms, *Proc. 30th Annu. ACM Symp. Theory of Computing*, pp.53-62, 1998.
[8] Fujimura T., Quantum algorithm for traveling salesman problem by numbering method, *Glob. J. Pure Appl. Math.*, 9, 2013, [to be published].
[9] Willemen R.J., School timetable construction, algorithms and complexity, [On line], Available: http: // alexandria. tue. nl/ extra2/ 200211248. pdf, 2002.
[10] Fujimura T., Quantum algorithm for timetabling problem, *Glob. J. Pure Appl. Math.*, 7:371-374, 2011.