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Einstein- Pauli-Yukawa Paradox: Our view

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Abstract

A review on the application of Schrödinger wave equation to a particle confined in one-dimensional square well of infinite depth is made. An expression of the solution in momentum probability distribution in ground state is made and different expressions by the trio of Einstein, Pauli, and Yukawa were mentioned and our position on the debate made.
Keywords: Momentum Distribution, Ground State, Schrödinger Wave Equation, Square Well of Infinite Depth.

Introduction

In his doctoral dissertation in 1926, Erwin Schrödinger made public his famous equation, the time dependent and time independent Schrödinger wave equation through a plausibility argument based upon an analogy with other wave phenomena that are already familiar. The equation has been extremely successful in explaining the behaviour of atomic and nuclear systems. As explained in many textbooks of Quantum Mechanics (Ghatak and Lokanathan (2002)), Ni and Chen (2002), if the time independent Schrödinger wave equation is applied to simple system of a particle confined in one-dimensional square well of infinite depth, the expression of the momentum distribution probability in ground state and that separately obtained by the trio of Einstein, Pauli, and Yukawa are different. This resulted into confusion and many scholars contributed to the debate as to which group is correct or wrong. Our aim is to extend the debate further and possibly take a position and add to existing literatures.

Formalism

We assume a particle of mass m in a one-dimensional square well potential of infinite depth characterized by the potential energy variation of the form

$$V(x) = \begin{matrix} 0, for\, 0<x<a \\ \infty, for\, 0>x>a \end{matrix} \qquad (1)$$

Method of Solution

The energy eigenvalues of stationary states and corresponding stationary eigen functions of ground state as reported by Ghatak and Lokanathan(2002) and Ni and Chen (2002) are respectively

$$E_n = \frac{n^2\pi^2\hbar^2}{8ma^2} \quad (n = 1,2....) \qquad (3)$$

and

$$\psi_1(x) = \sqrt{\frac{1}{a}} Cos\frac{\pi x}{2a} \quad 0 < x < a \qquad (4)$$

where n is the quantum numbers and $\hbar$ is Planck's constant divided by 2π .

If we represent equation (4) in terms of momentum, then Landau and Lifshitz (1977) have it as

$$\varphi_1(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \varphi_1(x) e^{\frac{-ipx}{\hbar} dx} \qquad (5)$$

Also the momentum distribution probability in ground state becomes

$$p_1(p) = |\varphi(p)|^2 = \frac{\pi\hbar^3}{2a^3} Cos^2\left(\frac{pa}{\hbar}\right)\left[p^2 - \left(\frac{\pi\hbar}{2a}\right)^2\right]^{-2} \qquad (6)$$

However, Tao (1998), pointed out that there are alternative representation of (6) which culminated independently into the work of Einsten (1953), Pauli, (1973) and Yukawa (1978) that in the ground state, there are only two discrete momenta $p' = \pm\frac{\pi\hbar}{2a}$ with probability $\frac{1}{2}$ respectively

$$p_1'\left(p' = \frac{\pi\hbar}{2a}\right) = p_1'\left(p' = -\frac{\pi\hbar}{2a}\right) = \frac{1}{2} \qquad (7)$$

The paradox

It is common knowledge that equation (6) and equation (7) is supposed to give the same result but they differ dramatically. The question is which one of the two expressions is correct or wrong? or are they both correct or wrong?

Marhley (1972), said equation (6) is correct whereas equation (7) not correct.

Domingos and Caldeira(1972) and Guan (1998) postulated that equation (7) is the appropriate expression while equation (6) is faulty.

Tao (1998), reported that both equations are correct but incorrect as well and the conflict between them actually reflects the logical inconsistency of present framework in quantum theory.

Ni and Chen(2002), also added to the debate in their point of view that both equations are correct because they are involved in different kinds of momenta.

Our point of view

The light – intensity distribution observed in a single – slit diffraction experiment resembles a graph of equation (6) and this is likened to an experiment in which a particle in a box is liberated along z direction to the screen of the single – slit diffraction of optical experiment. This implies that the particle distribution will reflect $p_1(p)$. This implies that the momentum p in equations (5) and (6) in x space is boundless and $\psi_1(x)$ and $\varphi_1(p)$ of equation (4) and equation (5) relates via Fourier transform. Also equation (7) is seen as a direct consequence of equation (4) with the momentum operator

$$p = -i\hbar\frac{\partial}{\partial x} \quad (-\infty < x < \infty) \tag{8}$$

as

$$\psi_1(x) = \frac{1}{2}\sqrt{\frac{1}{a}}\left[\exp\left(i\frac{\pi x}{2a}\right) + \exp\left(-i\frac{\pi x}{2a}\right)\right] \tag{9}$$

in conjunction with

$$p' = -i\hbar\frac{\partial}{\partial x} \quad (-a < x < a) \tag{10}$$

Now, the operator p' can also be realized in an infinite matrix while p fails to have such kind of matrix representation but there exist a momentum conservation law relevant to p an uncertainty relation

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{11}$$

By contrast, p' only has conservation law in discrete form modified with momentum exchange with lattice hence no uncertainty relation like equation (11) for p'. We could therefore posit that equation (6) and (7) may be all correct since they are involved in different kinds of momenta. However, like many other paradoxes in physics, it reflects some fundamental concepts either overlooked or misunderstood by physicists in the formulation of the concept of quantum theory.

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