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MHD Couette-Poiseuille Flow in a Porous Medium

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Abstract

A study on the influence of viscous dissipation and radiation on magnetohydrodynamic plane Couette-Poiseuille flow in a porous medium was carried out. On the basis of certain simplifying assumptions, the fluid equations of continuity, Navier-Stokes and energy were reduced to mathematical terms, and closed-form analytical solutions of the velocity distribution and energy were obtained based on approximations under the considered parameters. These parameters in various degrees increase the velocity profile of the fluid when they are increased except magnetic field and permeability term that decrease the velocity profile of the fluid. While radiation parameter decreases the temperature profile, results also showed that viscous dissipation and Reynolds number increase the temperature profile of the fluid.
Keywords: Magnetohydrodynamic, Couette-Poiseuille flow, Viscous dissipation, Radiation, Porous medium.

Introduction

Maurice Couette studied the flow of fluid with its motion brought about by the relative movement of two parallel plates or surfaces or where one of the plates or surfaces is moving laterally in its own plane. The study was later named after him and defined as two parallel plates moving relative to each other which cause a flow of fluid in between them. The plates could be flat, parallel or two concentric cylinders of varying radii, all generally referred to as plane Couette flow. It is a steady laminar flow of viscous incompressible fluid between two infinite parallel plates separated by a distance (d). Jean Louis Poiseuille a physician interested in the flow of blood also carried out a study of a fluid provoked by a pressure gradient between two rigid parallel plates that are stationary. The fluid is observed as a steady laminar flow that is incompressible. As a pioneer researcher of a flow of fluid in such a configuration the name Plane Poiseuille flow is given. The similarity of the two flows draws the attention of researchers and study on the combination of the two flows under the influence of material parameters in different coordinates and configurations commenced. Corenflos et al (1993) carried out an experimental and numerical study of a plane Couette – Poiseuille flow as a test case for turbulence modeling from experimental data available for both the developing and developed flows. They showed that instability sets in as Re is increased from 3000 when the wall is moving (Couette flow) and when Re is equal to 5000 fixed bottom wall (Poiseuille flow). Spurk (1997) also studied Couette–Poiseuille flow by developing a general solution wherein he distinguished between them by setting the pressure gradient (Kp) to zero for Couette flow, characteristic velocity U=0. $Kp \neq 0$ for Poiseuille flow and $U \neq 0$, $Kp \neq 0$ for Couette–Poiseuille flow in the model formulated using the equation of motion. An investigation on the anchoring distortion coupled with plane Couette–Poiseuille flows of nematic polymers in viscous solvents Morphology in molecular orientation, stress and flow was examined by Zhou and Forest (2006) to model and simulate processing–induced heterogeneity in rigid, rod–like nematic polymers in solvents. Ngiangia (2007) considered the effects of permeability and radiation on the stability of Couette - Poiseuille flow in a porous medium and showed that, on the basis of linear theory using analysis of normal modes both parameters induced instability at high wave numbers and high Reynolds number regime. Most of the previous studies of the same problem neglected viscous dissipation whereas; its effect plays an important role in natural convection flow fields of extremely high or extremely low in various situations prevalent in physiology and engineering as well as its importance in astrophysical and geophysical problems cannot be overemphasized. Permeability which is a measure of the ease with which a formation permits a fluid to flow through it also suffers the same neglect whereas its importance in engineering geology is begging for attention. Heat transfer by radiation, which is significant when one is concerned with space applications, liquid metal fast breeder reactors and higher operating temperature is also few. This present study however, is an attempt to complement the earlier work of Ngiangia (2007) by investigating the simultaneous effects of viscous dissipation and magnetic field to his problem of study. This attempt therefore, widens the applicability of problems of this nature.

Mathematical Formulation of the Physical Problem

We consider plane Couette – Poiseuille flow in an infinite parallel plates (as shown in figure1) with the motion of the fluid provoked by pressure gradient for Poiseuille flow and relative movement of the plates in Couette flow. The basic hydrodynamic equations governing the physics of the problem following the argument of Zheng *et al* (2002), Israel – Cookey *et al* (2003) and Mebine (2007) are

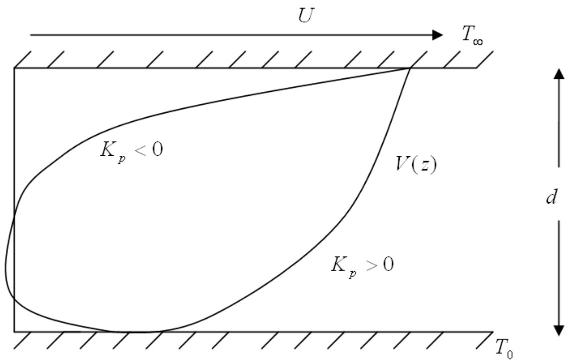

Figure 1: The physical model of the problem.

$$\nabla . V = 0 \tag{2.0}$$

$$\rho\left(\frac{\partial V}{\partial t} + (V.\nabla)V\right) = -\nabla P + \mu\nabla^2 V + \rho g - \frac{\upsilon}{k}V - \frac{\sigma_c \mu^2 H_0 V}{\rho_\infty} \tag{2.2}$$

$$\left(\frac{\partial T}{\partial t} + (V.\nabla)T\right) = a^2\nabla^2 T - \frac{1}{\rho C_p}\nabla . q_z + \frac{\mu}{\rho C_p}\left(\frac{\partial V}{\partial Z}\right)^2 \tag{2.3}$$

where $T, V, \rho, P, \mu, g, \upsilon, k, H_0, \rho_\infty, a, q_z, C_p$ are respectively temperature, fluid velocity, fluid density, fluid pressure, absolute viscosity, acceleration due to gravity, kinematic viscosity, permeability of the medium, magnetic field, porous medium density, thermal diffusivity, radiative term and specific heat capacity at constant pressure and

$$\frac{\partial^2 q_z}{\partial Z^2} - 3\alpha^2 q_z - 16\alpha T_\infty^3 \frac{\partial T}{\partial Z} = 0 \tag{2.4a}$$

With the assumptions that:

(i) The fluid properties are assumed to be constants except for the buoyancy body force term in the Navier–Stokes equation which is approximated by the Boussinesq relation.

(ii) The heat generated by viscous shear(viscous dissipation) is not negligible and the function reduces to $\mu\left(\frac{\partial V}{\partial Z}\right)^2$.

(iii) The flow is fully developed in terms of velocity only.

(iv) The difference in temperature between the plates and that of the fluid is large enough for free convection to flow. This condition may prevail in practice and therefore is physically important.

An exact treatment of radiative transfer in a fluid requires a formulation in terms of integro – differential equations. The solution of these equations is rather complex. Approximation theories have however been developed which permits a formulation involving only differential equation. Though the usual formulation of the problem is well known, its modification by radiative term permit in the spirit of Cogley *et al* (1968) the reduction of (2.4a) to

$$\frac{\partial q_z}{\partial Z} = 4\delta^2 (T - T_\infty) \tag{2.4b}$$

$$\text{where } \delta^2 = \int_0^\infty \left(\alpha_{k^*} \frac{\partial \Lambda}{\partial T} \right) dk^* \tag{2.4c}$$

Λ is the planck's function, α_{k^*} is the absorption coefficient, k^* is the frequency of radiation and T is the temperature. If we put equation (2.4b) in equation (2.3) and under Boussinesq approximation which restricts the effect of variation of density with temperature exclusively to the body forces term. With these assumptions, the flow equations that describes the physical situation are given by

$$\frac{\partial V}{\partial Z} = 0 \quad \text{s} \tag{2.5}$$

$$\rho \frac{\partial V}{\partial t} = -\frac{\partial P}{\partial Z} + \frac{\mu \partial^2 V}{\partial Z^2} + g\rho_0 \xi (T - T_0) - \frac{\upsilon}{k} V - \frac{\sigma_c \mu^2 H_0^2 V}{\rho_\infty} \tag{2.6}$$

$$\frac{\partial T}{\partial t} = \frac{a^2 \partial^2 T}{\partial Z^2} - \frac{4\delta^2 (T - T_0)}{\rho c_p} + \frac{\mu}{\rho c_p} \left(\frac{\partial V}{\partial Z} \right)^2 \tag{2.7}$$

where ξ is coefficient of volume expansion.

A. Non – dimensional analysis

For dimensional homogeneity of the governing hydrodynamic equations, we substitute the following expressions

$$Z = \frac{V't}{d}, P = \frac{P'}{\rho V^2}, \alpha^2 = \frac{4\delta^2 \rho_\infty C_\infty d^2}{\rho C_p \upsilon}, K^a = \frac{\upsilon \mu d^2}{k\rho}$$

$$V = \frac{V'}{U}, \beta^2 = \frac{a^2 \rho t}{T_\infty}, g = \frac{gd}{V^2}, \theta = \frac{T' - T'_\infty}{T - T_\infty}$$

$$\mathrm{Re}^{-1} = \frac{\mu}{Vd\rho}, t' = \frac{Vd}{t}, M^2 = \frac{\sigma_c \mu^2 H_0^1 \upsilon}{\rho_\infty U^2}, Gr = \left[g\zeta \frac{(T - T_0) d^3}{V^2} \right]$$

$$\Pr = \frac{\mu c_p}{a^2 \rho C_v}, Ec = \frac{U^2}{C_p (T - T_0)}$$

having employed the Rayleigh's Technique into equations (2.5), (2.6), and (2.7), which results in

$$\frac{\partial V}{\partial Z} = 0 \tag{2.8}$$

$$\rho \frac{\partial V}{\partial t} = -\frac{\partial P}{\partial Z} + \mathrm{Re}^{-1} \frac{\partial^2 V}{\partial Z^2} + Gr\theta - K^a V - M^2 V \tag{2.9}$$

$$\frac{\partial\theta}{\partial t'} = \beta^2 \frac{\partial^2\theta}{\partial Z^2} - \alpha^2\theta + \Pr Ec\left(\frac{\partial V}{\partial Z}\right)^2 \tag{2.10}$$

where M is dimensionless magnetic field parameter, Pr is prandtl number, Ec is Eckert number and Gr is Grashof number. Equations (2.9) and (2.10) are now subject to the boundary conditions

$$\theta(0) = 1, \theta(\infty) = 0$$

$\mathrm{V}(0) = 0, V(d) = U$ for Couette flow

$V(0) = 0, V(d) = 0$ for Poiseuille flow.

assuming that the fluid velocity at the wall of the plates is equal to the wall velocity (no- slip condition).

Method of solution

The problem posed in equations (2.9) and (2.10) are highly nonlinear equations and generally will involve a step by step numerical integration of the explicit finite difference scheme. However, analytical solution is possible if we assume small Re (Bestman, 1985; Gbadeyan, and Idowu, 2006) and by adopting regular perturbation of the form (Israel- Cookey *et al* 2003)

$$V(Z,t) = V_0(Z) + \mathrm{Re} V_1(Z)e^{i\omega t} \tag{3.1}$$

$$\theta(Z,t) = \theta_0(Z) + \mathrm{Re}\,\theta_1(Z)e^{i\omega t} \tag{3.2}$$

Substituting equations (3.1) and (3.2) into equations (2.9) and (2.10), neglecting $0(\mathrm{Re}^2)$ and simplifying, we obtain the following sequence of approximations after collecting terms of the same order:

$$\mathrm{Re}^{-1} V_0^{11}(Z) + Gr\theta_0(Z) - K^a V_0(Z) - M^2 V_0(Z) - k_p = 0 \tag{3.3}$$

$$\beta^2\theta_0^{11}(Z) - \alpha^2\theta_0(Z) + \Pr Ec V_0^1(Z) = 0 \tag{3.4}$$

subject to

$$\begin{aligned} &\theta_0(0) = 1, \theta_0(\infty) = 0 \\ &V_0(0) = 0, V_0(d) = U \end{aligned} \tag{3.5}$$

for 0(1) equations, and

$$i\omega V_1(Z) = \mathrm{Re}^{-1} V_1^{11}(Z) + Gr\theta_1(Z) - K^a V_1(Z) - M^2 V_1(Z) \tag{3.6}$$

$$i\omega\theta_1(Z) = \beta^2\theta_1^{11}(Z) - \alpha^2\theta_1(Z) + 2\Pr Ec V_0^1(Z) V_1^1(Z) \tag{3.7}$$

subject to

$$\begin{aligned} &V_1(0) = 0, V_1(d) = U \\ &\theta_1(0) = 1, \theta_1(\infty) = 0 \end{aligned} \tag{3.8}$$

for 0(Re) equations.

where, $\mathrm{k}_p = \frac{1}{\rho}\frac{\partial P}{\partial Z}$ is a constant pressure gradient.

To solve the nonlinear- coupled equations (3.3) – (3.8) we further assume that the Eckert number (Ec) is small, (Israel- Cookey *et al* 2003) and therefore, advance an asymptotic expansion for the flow temperature and velocity as follows

$$V_0(Z) = V_{01}(Z) + EcV_{02}(Z) \tag{3.9a}$$

$$\theta_0(Z) = \theta_{01}(Z) + Ec\theta_{02}(Z) \tag{3.9b}$$

$$V_1(Z) = V_{11}(Z) + EcV_{12}(Z) \tag{3.9c}$$

$$\theta_1(Z) = \theta_{11}(Z) + Ec\theta_{12}(Z) \tag{3.9d}$$

Substituting equations (3.9) into equations (3,3) – (3.8), and neglecting squares and products of perturbation quantities, we obtain the following sequence of approximations;

$$\mathrm{Re}^{-1} V_{01}''(Z) + Gr\theta_{01}(Z) - (K^a + M^2)V_{01}(Z) - k_p = 0 \tag{3.10}$$

$$\beta^2\theta_{01}''(Z) - \alpha^2\theta_{01}(Z) = 0 \tag{3.11}$$

$$\mathrm{Re}^{-1} V_{02}''(Z) + Gr\theta_{02}(Z) - (K^a + M^2)V_{02}(Z) = 0 \tag{3.12}$$

$$\beta^2\theta_{02}''(Z) - \alpha^2\theta_{02}(Z) = 0 \tag{3.13}$$

subject to

$$\begin{aligned} &V_{01}(0) = 0, V_{01}(d) = U, V_{02}(0) = 0, V_{02}(d) = U \\ &\theta_{01}(0) = 1, \theta_{01}(\infty) = 0, \theta_{02}(0) = 1, \theta_{02}(\infty) = 0 \end{aligned} \tag{3.14}$$

for 0(1) equations, and

$$i\omega V_{11}'(Z) = \mathrm{Re}^{-1} V_{11}''(Z) + Gr\theta_{11}(Z) - (K^a + M^2)V_{11}(Z) \tag{3.15}$$

$$i\omega\theta_{11}(Z) = \beta^2\theta_{11}''(Z) - \alpha^2\theta_{11}(Z) \tag{3.16}$$

$$i\omega V_{12}'(Z) = \mathrm{Re}^{-1} V_{12}''(Z) + Gr\theta_{12}(Z) - (K^a + M^2)V_{12}(Z) \tag{3.17}$$

$$i\omega\theta_{12}(Z) = \beta^2\theta_{12}''(Z) - \alpha^2\theta_{12}(Z) \tag{3.18}$$

subject to

$$\begin{aligned} &V_{11}(0) = 0, V_{11}(d) = U, V_{12}(0) = 0, V_{12}(d) = U \\ &\theta_{11}(0) = 1, \theta_{11}(\infty) = 0, \theta_{12}(0) = 1, \theta_{12}(\infty) = 0 \end{aligned} \tag{3.19}$$

for 0(Ec) equations.

Solving equation (3.13), we assume a solution of the form

$$\theta_{02}(Z) = e^{\lambda Z} \tag{3.20}$$

Substituting equation (3.20) into equation (3.13) together with the appropriate boundary conditions of equation (3.14), we get

$$\theta_{02}(Z) = e^{m_1 Z} \tag{3.21}$$

If we substitute equation (3.21) into equation (3.12) and simplify, we obtain

$$V_{02}''(Z) - AV_{02}(Z) = -\mathrm{Re}\, Gre^{m_1 Z} \tag{3.22}$$

Solutions to equation (3.22) after the application of the appropriate boundary conditions of equation (3.14), gives

$$V_{02}(Z) = A_1 e^{m_6 Z} + Ue^{-m_6 Z} - A_2 e^{m_1 Z} \tag{3.23}$$

Following the same procedure as in equation (3.23), the solution of equation (3.18) is given by

$$\theta_{12}(Z) = e^{m_2 Z} \qquad (3.24)$$

Substituting equation (3.24) into equation (3.17) and simplifying, results

$$V_{12}''(Z) - A_3 V_{12}'(Z) - AV_{12}(Z) = -A_2 e^{m_2 Z} \qquad (3.25)$$

The complete solution of equation (3.25) is therefore,

$$V_{12}(Z) = (C_1 + U)e^{m_3 Z} + Ue^{m_4 Z} + C_1 e^{m_2 Z} \qquad (3.26)$$

Following the same steps taken in equation (3.20), the solution to equation (3.16) can be written as

$$\theta_{11}(Z) = e^{m_2 Z} \qquad (3.27)$$

Substituting equation (3.27) into equation (3.15) and rearrangement results in

$$V_{11}^{11}(Z) - A_3 V_{11}^{1}(Z) - AV_{11}(Z) = -A_2 e^{m_2 Z} \qquad (3.28)$$

Following the approach in determining the solution of equation (3.26), the solution of equation (3.28) after imposing the boundary conditions of (3.19) is

$$V_{11}(Z) = (C_1 + U)e^{m_3 Z} + Ue^{m_4 Z} + C_1 e^{m_2 Z} \qquad (3.29)$$

Following the same method in determining the complete solution of equation (3.13), equation (3.11) can be written as

$$\theta_{01}(Z) = e^{m_1 Z} \qquad (3.30)$$

We substitute equation (3.30) into equation (3.10) and after simplification, we get

$$V_{01}''(Z) - AV_{01}(Z) = A_4 - A_2 e^{m_1 Z} \qquad (3.31)$$

Following the method adopted in determining the complementary function of equation (3.12), and a similar method in solving for the particular integral of the same equation, the solution of equation (3.31), is given by

$$V_{01}(Z) = (U - C_3)e^{m_5 Z} + (C_2 + U)e^{-m_5 Z} - C_2 - C_3 e^{m_1 Z} \qquad (3.32)$$

Substituting equations (3.23) and (3.32) into equations (3.9a), gives

$$V_0(Z) = (U - C_3)e^{m_5 Z} + (C_2 + U)e^{-m_5 Z} - C_2 - C_3 e^{m_1 Z} + Ec\left(A_1 e^{m_6 Z} + Ue^{-m_6 Z} - A_2 e^{m_1 Z}\right) \qquad (3.33)$$

Also, substituting equations (3.21) and (3.30) into equation (3.9b) results,

$$\theta_0(Z) = e^{m_1 Z} + Ece^{m_1 Z} \qquad (3.34)$$

Again, substituting equations (3.26) and (3.29) into equation (3.9c) results,

$$V_1(Z) = (C_1 + U)e^{m_3 Z} + Ue^{m_4 Z} + C_1 e^{m_2 Z} + Ec(C_1 + U)e^{m_3 Z} + Ue^{m_4 Z} + C_1 e^{m_2 Z} \qquad (3.35)$$

Finally, putting equations (3.24) and (3.27) into equation (3.9d), we get

$$\theta_1(Z) = e^{m_2 Z} + Ece^{m_2 Z} \qquad (3.36)$$

Similarly, if we put equations (3.33) and (3.35) into equation (3.1) and equations (3.34) and (3.36) into equation (3.2), we obtain the velocity and temperature profiles of the flow respectively as:

$$V(Z,t)=(U-C_3)e^{m_5 Z}+(C_2+U)e^{-m_5 Z}-C_2-C_3 e^{m_1 Z}$$
$$+Ec\left(A_1 e^{m_6 Z}+Ue^{-m_6 Z}-A_2 e^{m_1 Z}\right) \quad (3.37)$$
$$+\mathrm{Re}\left[(C_1+U)e^{m_3 Z}+Ue^{m_4 Z}+C_1 e^{m_2 Z}+Ec(C_1+U)e^{m_3 Z}+Ue^{m_4 Z}+C_1 e^{m_2 Z}\right]$$

$$\theta(Z,t')=(1+Ec)e^{m_2 Z}+\left(\mathrm{Re}\,e^{m_1 Z}+\mathrm{Re}\,Ece^{m_1 Z}\right) \quad (3.38)$$

We have formulated and solved the problem of the influence of viscous dissipation and radiation on the onset of instability of magneto hydrodynamic plane Couette-Poiseuille flow in a porous medium based on fairly realistic assumptions and approximations. The solutions and the field variables, show that the parameters entering the problem are Reynolds number (Re), free convection parameter or Grashof number (Gr), Prandtl number (Pr), viscous dissipation parameter or Eckert number (Ec), dimensionless radiation parameter (α), dimensionless magnetic parameter (M) and dimensionless permeability term (k^a). Others are constant dimensionless frequency of oscillation (ω), constant thermal diffusivity (β),and dimensionless constant time (t).In order to get physical insight and numerical validation of the problem, a typical value of the Prandtl number corresponding to an astrophysical body (Air) at $25^0 c$ is chosen as 0.71. Air is chosen because it is weakly electrically conducting under assumed circumstances and the problem under study in particular. The values of the other parameters made use of are

$$\omega=2.0; t=2.0; \beta=0.8; k_p=2.5; U=5, \mathrm{Re}=10,20,30,40.50., Ec=0.01,0.5,1.0,1.5,2.0$$
$$M^2=2,4,6,8,10, K^a=o.4,0.8,1.2,1.6,2.0, \alpha^2=2,4,6,8,10, Gr=2,4,6,8,10$$

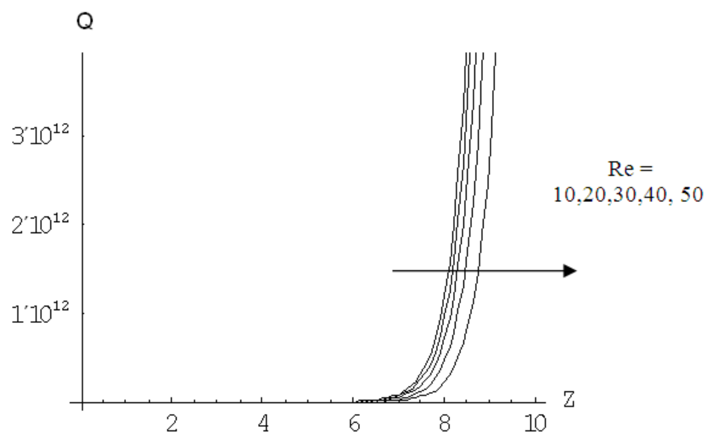

Figure 5.1: Variation of Reynolds number on the plot of temperature profile against the boundary layer.

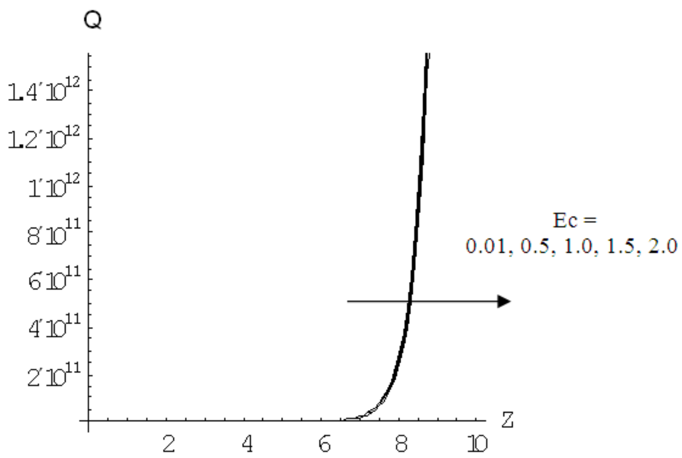

Figure 5.2: Variation of Eckert number on the plot of temperature profile against the .boundary layer.

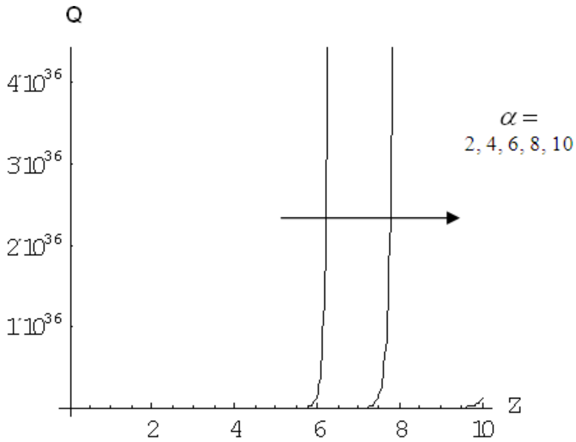

Figure 5.3: Variation of radiation parameter on the plot of temperature profile against the boundary layer.

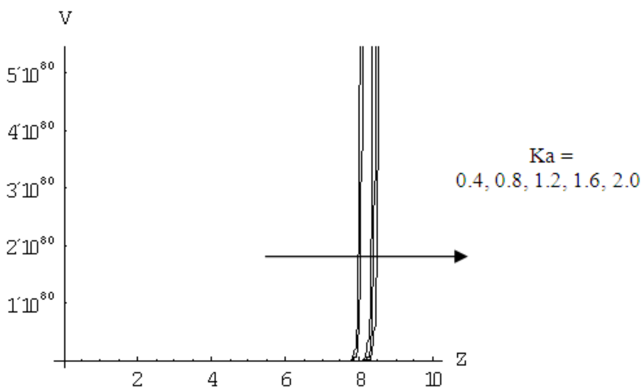

Figure 5.4: The dependence of velocity profiles on the boundary layer with permeability term varying.

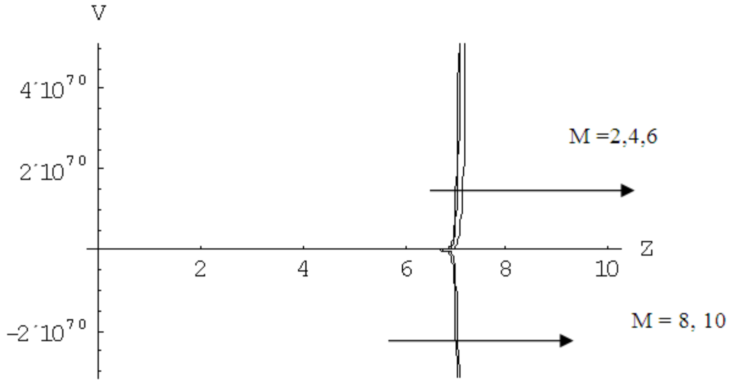

Figure 5.5: The dependence of velocity profiles on the boundary layer with magnetic field parameter varying.

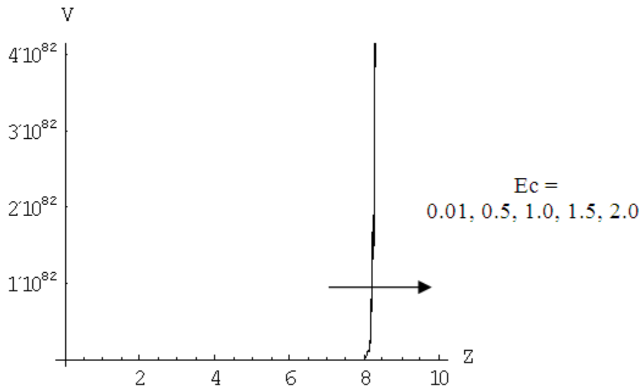

Figure 5.6: The dependence of velocity profiles on the boundary layer with Eckert number varying.

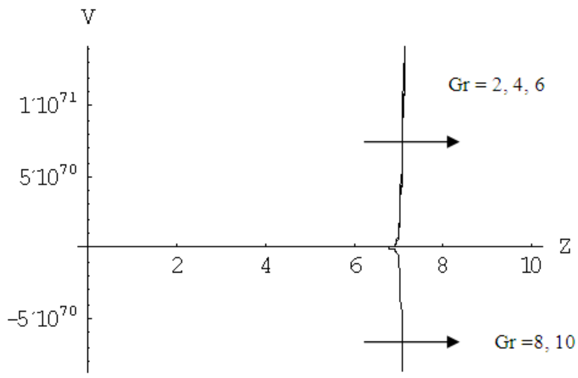

Figure 5.7: The dependence of the velocity profiles on the boundary layer with Grashof number varying.

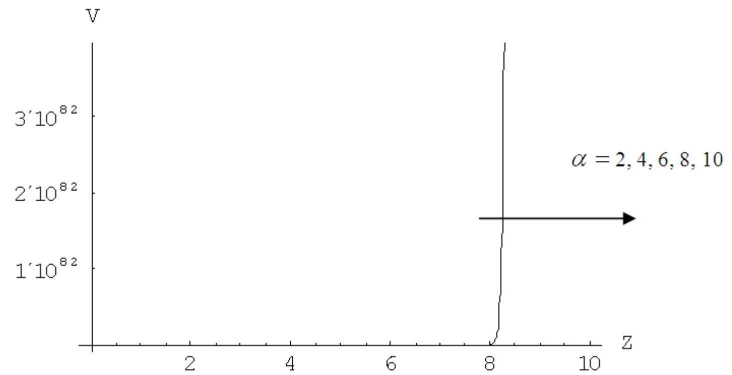

Figure 5.8: The dependence of the velocity profiles on the boundary layer with radiation parameter varying.

Discussion

In the analysis, we start with the temperature profile due to its primary importance in astrophysics environments; the next is velocity profile and finally the stability of the fluid flow.

Temperature Profile

The effect of increase in Reynolds number (Re) is shown in Figure 5.1. It is evident that an increase in Reynolds number (Re), shows an increase in the temperature distribution. Figure 5.2 shows the effect of viscous dissipation parameter or Eckert number (Ec) on temperature and the result shows that further increase in Eckert number leads to a slight increase in temperature and later did not increase further. This observation is in agreement with the findings of Israel-Cookey *et al* (2003), that a more heating by viscous dissipation caused a rise in temperature. Figure 5.3 displays the variations of temperature profile with various values of the radiation parameter (α). It is evident that an increase in radiation parameter leads to a decrease in the temperature. This result is consistent with the findings of Mebine (2007) that increase in radiation function brings about a decrease in temperature.

Velocity Profile

In the analysis of the velocity profile for different material parameters, we examine first the effect of permeability as depicted in Figure 5.4. It shows that increase in permeability is associated with a decrease in the velocity profile and this result agrees with the earlier results of Israel-Cookey *et al* (2003). Figure 5.5 shows the effect of magnetic field on the velocity profile. Analysis of Figure 5.5 reveals that an increase in the magnetic field parameter caused a pronounced decrease in the velocity profile. This observation, when compared with the results of Israel-Cookey *et al* (2003) and Gbadeyan and Idowu (2006), agrees quantitatively. In the absence of viscous dissipation parameter and magnetic field parameter, the results in part are in agreement with the earlier findings of Ngiangia and Wonu (2007) and Ngiangia (2007). In the absence of viscous dissipation alone, the findings are consistent with the results of Ngiangia *et al* (2011). In the presence of viscous dissipation as shown in Figure 5.6, increase in the Eckert number results in a slight increase in thermal internal energy of the fluid which in turn increases the velocity profiles caused by viscous dissipation. This observation laid credence to the earlier works of Israel-Cookey *et al* (2003) and Oladele *et al* (2006). Figure 5.7 displays the increase of the free convection parameter on the velocity profile. The findings as depicted in Figure 5.7 show that heating of the plate brings about increase in the velocity profile rapidly. From the physical point of view the value of the Grashof number (Gr) indicates the state of the lower plate. This observation was shared by Mebine (2007) who described the free convection currents flow towards the lower plate as externally heated since the Grashof number (Gr) depends upon the temperature of the plates. Also, cooling of the plates $Gr > 0$ brings about decrease in the velocity distribution and this observation is consistent with the earlier findings of Israel-Cookey *et al* (2003). Figure 5.8, shows the effect of radiation on the velocity profiles. Increase in the radiation parameter increase minimally the velocity profile and later stopped increasing. This observation agrees with the findings of Adesanya and Ayeni (2006) and Mebine (2007). In the absence of Eckert number, the results are consistent with the report of Ngiangia *et al* (2011).

Conclusion

In this study, we have provided an approximate solution to the governing hydrodynamic equations. Generally, difficulty in closed-form solutions owing to non linearity and sometimes difficult geometries is well known but realistic assumptions and approximations employed in analyzing the problem revealed that the results are in qualitative and quantitative agreement with results of earlier works and also shed light on the applicability of problems of this nature.

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