Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 9, Number 2 (2013), pp. 151-168

http://www.ripublication.com/gjpam.htm

Logic about Isolated System—Mathematical Foundation for Science Unification

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Abstract

This paper provides a model to explain the Universe as an ideal isolated system with freedom, mutual mapping and growth properties. They are common properties for all pursuit systems, including action pursuit in the Universe between momentum and coordinate, knowledge pursuit in research between concepts and facts, happiness pursuit in society between desires and commodities. Therefore, it is a mathematical foundation for science unification. It also provides an understanding for $SU(2)XU(1)$ and $SU(3)$ symmetries in the Universe. When elements have motion freedom, an ordered set gets some main properties of the Universe, including momentum set, four interactions, photons and intermediate Bosons , three generations of Fermions. The set and its momentum set is symmetrical. Therefore, in the Universe, space-time distance ought to be small and conserved in momentum space, just like momentum-energy in coordinate space. Individuals in the two spaces have different wave functions. One wave function only describes distribution in one space. Because all momentum-energy in coordinate space came from momentum space, it seemed that they came from a starting time and a small area of the coordinate space, like a Big Bang. However, the bang will last forever.

Keywords: isolated system; geometry with coordinate freedom; mapping rule; largest freedom principle; energy-time symmetry; free set theory

1. Introduction

According to free spatial transformation theory, [1], freedom ought to be the kernel of mathematics. If every point is movable and decomposable, there will be two-space geometry. One is coordinate space. The other is momentum space.

Movable coordinates function as particles. In each space, movement freedom means that any two points A and B can be both symmetrical and anti-symmetrical. A can move to B when they are symmetrical. If they are static at the beginning, there will be two points at B and zero point at A, which corresponds to electron-positron pair production in the Universe. So it is necessary to differentiate coordinate site from coordinate. Free spatial translation, $\{\boldsymbol{l}\}_A$, represents that there are $\boldsymbol{l}$ coordinates at site A. In three-dimensional space, it corresponds to fermions with charge $(3-\boldsymbol{l})/3$, $0 \le \boldsymbol{l} \le 6$. If point is decomposable, there will be coordinate distribution $\{3\boldsymbol{n}+1_i\}$ and $\{3\boldsymbol{n}-1_i\}$, corresponding to quarks and anti-quarks respectively. Weak interaction keeps point and point site one-one mapping, and its general form is:

$$\begin{aligned} &\{\boldsymbol{l}\}+\{3\}^0 \to \{\boldsymbol{l}-3\boldsymbol{n}+3\}+\{3\boldsymbol{n}\leftrightarrow 3\}, \\ &\boldsymbol{n}=0,1,2. \end{aligned} \tag{1}$$

$\{3\boldsymbol{n}\leftrightarrow 3\}$ represents that 3n coordinates exchanges 3 coordinates, corresponding to intermediate bosons. If coupling constant is unrelated with $\boldsymbol{n}$, there is $SU(2)$ symmetry for the interaction. Strong interaction keeps three coordinates together, and its general form is:

$$\begin{aligned} &\{3\boldsymbol{n}+1_i\}+\{3\}^0 \to \{3\boldsymbol{n}+1_j\}+\{1_i \leftrightarrow 1_j\}, \\ &\{3\boldsymbol{n}-1_i\}+\{3\}^0 \to \{3\boldsymbol{n}-1_j\}+\{1_j \leftrightarrow 1_i\}, \\ &\boldsymbol{i},\boldsymbol{j}=1,2,3. \end{aligned} \tag{2}$$

$\{1_j \leftrightarrow 1_i\}$ represents that one coordinate at site x_j exchanges one at x_i. Since $\{3\leftrightarrow 3\}$ is weak interaction, there are eight independent $\{1_j \leftrightarrow 1_i\}$, corresponding to gluons. $\{3\boldsymbol{n}\leftrightarrow 3\}$ and $\{1_j \leftrightarrow 1_i\}$ are both free spatial reflections. Therefore, geometry with coordinate freedom could become a system similar to the Universe.

On the other hand, the Universe ought to be an isolated system, [2], and freedom is a natural result of isolation postulate. An isolated system does not exchange resources and laws with environment. Its evolution ought to be natural, without any unnecessary limitations. Therefore, if there is no reason to confine, there will be freedom. In fact, freedom ought to be the largest, see Sec. 3. Pursuing largest freedom ought to be the basic natural law for an isolated system.

If there are reasons to confine freedom, they must exist in the system, called inside causality, which is a result of principle of universal causality. For an isolated system, outside reason is unacceptable, including objective existence, first push, etc. In geometry, there is no reason to forbid a point from moving or decomposing, so there is free spatial transformation. Besides, there is no reason to differentiate these coordinates, no way to discriminate without losing freedom. From inside causality, they should be identical and can be both symmetrical and anti-symmetrical.

An isolated system is a system of all freedoms or all possibilities, but there will be laws. If some freedom can grow exponentially and infinitely, sacrificing some other static freedom would be worthy. The sacrificed freedom forms methodology, the laws for growth. Advantage of methodology is irrelevant with the dimension of the growing quantity, like happiness in society, action in the Universe, knowledge in research, etc.

An isolated system pursues largest freedom automatically. Ration is unnecessary in an isolated system. The best pursuit methodology will emerge, just like a society of creatures with perfect intellectual. Basic laws in the Universe form the best pursuit methodology. It is the system openness and wrong notions that prevent society from being an isolated system.

Mutual mapping is the way to define items in an isolated system. Coordinate space and momentum space define each other by Fourier expansion. However, “A is a” is not good mutual definition. Good mutual definition is 1-N, explaining better when N is larger.

According to Truth Evolutionism Project, [3], it is possible to discover knowledge about the Universe purely by ration or mathematics, not experimental. Experiment is not the only source of truth. However, the foundation must be true and cannot be experimental, just like principles of isolation, largest freedom and inside causality. Knowledge from experiment or induction is never safe. For some basic knowledge about the Universe, experiment ought to be secondary. For example, even if money and technology were not problems, experiment could not prove the isolation of the Universe, could not observe the inside structure of black holes.

According to the project, the Universe is self-realized, [4], growing the fastest unconditionally. For a large isolated system, its scale ought to be the result of growth. Several quantities can form an independent perturbation or isolated system. If every quantity can grow with the help of the other quantities, they will form a self-growing isolated system. Then, a world with nothing cannot exist. In the world, the Universe is a necessity, not an accident.

The system with the fastest growth will be the largest system, which ought to be the Universe. Hence, the knowledge about the largest system becomes truth by evolution.

Mathematical truth is not only applicable inside the Universe, but also outside. This is an advantage to study the Universe by pure ration. If a self-growing isolated system can be established mathematically, such a system is constructible. For systems like economy and research, after adopting the methodology of the Universe, they will be man-made mini-Universes. This is called “imitating the Universe”, [5].

2. Energy-time symmetry

If momentum space is symmetrical with coordinate space, [1, 6], energy should replace time in momentum space. It also helps to satisfy relativity invariance.

Energy-time symmetry principle:

Time and energy mutually replace each other when changing space.

Here are some results.

First, in momentum space, space-time length becomes a conservative quantity. At every point, energy is always growing in momentum space, just like time does in coordinate space. For particles in momentum space, rest time replaces rest energy. Energy symmetry replaces time symmetry. Least time principle and least energy principle replace each other when crossing spaces.

For any microscopic event, there is one and only one clock running. When a particle jumps from coordinate space to momentum space, its energy clock will start to move and time clock will stop.

Second, relative to coordinate space, space-time is scarce and momentum-energy is abundant in momentum space. Therefore, in the standard of space-time length in coordinate space, momentum space is small and early, and looks like the coordinate space at the beginning time, but with much more momentum-energy. The difference ought to be symmetrical. For example, time level in coordinate space is about 10^{15} hours, if average time level in momentum space is 1 hour, average energy level in momentum space will be 10^{15} higher than in coordinate space. This forms postulate C1 in Sec. 9.

In each space, when matters fall into black holes, at the first stage, they increase the scarce quantity to catch up with the time in object space; at the second stage, they release most of the abundant quantity by scattering and radiating and go back to very early time in the source space. For example, in coordinate space, matters get energy in strong gravity when falling into black hole; then, they will cool down by releasing space-time in momentum space.

Third, momentum-energy conservation is valid in coordinate space only. Momentum and energy grows in momentum space, just like space expansion and time running in coordinate space. Energy from momentum space causes the forever expansion of coordinate space; time from coordinate space causes the forever expansion of momentum space.

Fourth, moving in one space is reversible, but moving between the two spaces is irreversible. A particle cannot exit a space by reducing the abundant quantity. It has only one choice: increasing the scarce quantity. This forms postulate C2 in Sec. 9.

If the two spaces follow C2, their difference between energy or time will enlarge. Supposing that there are two identical spaces having equal energy and time at the beginning, when there is an undulation, there will be two different spaces, each of which with a relatively abundant quantity and a relatively scarce quantity. In each space, the relatively abundant quantity becomes extensive quantity, scarce quantity becomes intensive quantity. By pursuing largest freedom, the abundant quantity increases the abundance by expansion. Therefore, uniform distribution is unstable.

Symmetry between momentum space and coordinate space is a kind of large-small symmetry. The Universe does not set which space is larger or more important. A

relatively small area in coordinate space can be a relatively large area in momentum space, and vice versa.

Fifth, in momentum space, $\frac{dp}{dE}$ cannot surpass light velocity, and in the standard of coordinate space, $\frac{dr}{dt}$ can surpass light velocity, so particle can appear in a large area of the object space when falling into black holes. This provides a mechanism for cosmic inflation. In momentum space, when a particle enters black hole, it has large space-time distance, so it can appear in a very large area in coordinate and time.

Therefore, the reverse process of black hole is not white hole. In each space, flowing into a point is the opposite process of releasing uniformly in whole space, not bursting out from a point. At least part of high-energy cosmic ray ought to come from the black holes of momentum space.

Sixth, if the Universe has two independent spaces, one wave function cannot describe the distribution in the two spaces. There must be two complex wave functions.

In classical Fourier expansion, there is only one complex function, because there are only two variables, coordinate and momentum. When point is movable in both spaces, there are four variables in a state: coordinate, momentum and their sites. Therefore, there must be two complex wave functions. Wave function in coordinate space describes the relation between coordinate and momentum sites; wave function in momentum space describes the relation between momentum and coordinate sites. Wave function in momentum representation is not the wave function in momentum space.

If the two spaces are symmetrical, the Big Bang ought to be a matter creating process lasting forever, generate background radiation all the time. It is not necessary to happen during a very short time, in coordinate space standard; or during a short energy, in momentum space standard. For the coordinate space, if all matters had little space-time and large momentum-energy when they came from momentum space, it looks like a Big Bang.

Symmetry group does not enlarge when energy is enough high, two sets of symmetries just replace each other. Symmetry between momentum-energy and space-time might be the only symmetry that is strictly true in the Universe. However, the symmetrical event always happens in the other space, which is unobservable or difficult to observe.

3. Largest freedom principle

In free spatial transformation, in order to decide movement of a point, it is necessary to give a payback function. If it is negative, it corresponds to action of the Universe and has minimum. If it is positive, it corresponds to negative action and has maximum.

In [1], largest freedom and largest negative action are two keys to understand free spatial transformation. However, a pursuit system cannot pursue two independent

purposes the best simultaneously, [7]. Therefore, negative action must be freedom. Largest freedom principle:

Total freedom is the largest in an isolated system.

This forms A12. In an isolated system, there is no limit for freedom, so largest freedom is a natural law. If another quantity is the largest, it will be hard to explain.

In an isolated system, freedom is a procedure and measurable. In a small area, there is always a space providing the intensity of freedom, and the other space providing extensity of freedom. Without largest freedom postulate, the system will be neutral for freedom growth: growth can be either positive or negative.

For each particle, it is a hypothesis that the freedom to move between spaces is a constant when the size of the object space is fixed. In coordinate space, it is rest mass m_0; in momentum space, it is rest space-time distance s_0. They provide the unit to measure the intensive of freedom. If m_0 is treated as a constant and serves as the unit for intensive quantity, space-time distance *ds* provide the unit for fundamental extensive quantity, there will be a measurable coordinate space. Therefore, if there are areas observable in both spaces, choice of unit decides which space it is.

Photon and gluon do not move between the two spaces, so their m_0 and s_0 are zero.

Freedom is calculable. In D-dimensional space, motion freedom in coordinate space is

$$S_x = \left(Edt - \sum_{i=1}^{D} p_i dx_i \right) / \hbar \quad ; \tag{3}$$

in momentum space, it is, [1],

$$S_p = \left(tdE - \sum_{i=1}^{D} x_i dp_i \right) / \hbar \, . \tag{4}$$

They are dimensionless. When there is interaction, corrections are necessary.

Therefore, when wave functions in the two spaces are independent, the state is

$$\begin{Bmatrix} \Psi_p \\ \Psi_x \end{Bmatrix} = \begin{Bmatrix} \sum_x A_x \exp\left(iS_p\right) \\ \sum_p A_p \exp\left(iS_x\right) \end{Bmatrix} . \tag{5}$$

It is sum because the space can be discrete, like an ordered set. Details of wave function depend on largest freedom principle.

$\begin{Bmatrix} \Psi_p \\ \Psi_x \end{Bmatrix}$ can be treated as a combination of two states, $\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$ and $\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$. The former is a wave function in coordinate space; the latter is a wave function in momentum space. For simplicity, they can be viewed as a point and a momentum, as in two-space

geometry. However, (3) and (4) are valid when coordinate and momentum are independent, and in most situations, the two waves are not independent.

Freedom mainly comes from symmetry between the two spaces. In the Universe, Edt and tdE terms are the main positive parts of freedom. Freedom of moving between the two spaces is positive. Freedom of moving in each space is negative. Motion between the two spaces is the pursued purpose, motion in space just helps to pursue.

4. Mapping Rule

Classical Fourier expansion is treated as the equivalence or symmetry between momentum space and coordinate space. Therefore, free motion must have the form of $\begin{Bmatrix}1\\1\end{Bmatrix}$, (or $\begin{Bmatrix}0\\0\end{Bmatrix}$), representing that there is one point and one momentum.

In free spatial translation, Fourier expansion in coordinate space is treated as the equivalence between coordinates and momentum sites, and hence there must be another Fourier expansion to represent the equivalence between momentum and coordinate sites. Then, free motion in coordinate space ought to be $\begin{Bmatrix}0\\1\end{Bmatrix}$, representing that there is one point and zero momentum. Therefore, coordinate space is not a regroup of momentum space. In coordinate space, momentum in Fourier expansion is momentum site, not momentum. A basic state in one space includes a point in the space and a vacancy in the other space. This forms A8.

When observing by $\begin{Bmatrix}0\\1\end{Bmatrix}$, as in coordinate space, $\begin{Bmatrix}1\\0\end{Bmatrix}$ is unobservable, so momentum space forms dark matter. When observing the Universe from the coordinate space, momentum space stretches from black hole to dark matter. When observing in momentum space, it is black hole when dark matter is enough dilute. Therefore, the gauges are very different in the two spaces.

It is impossible to measure coordinate space when there is too little or even no $\begin{Bmatrix}0\\1\end{Bmatrix}$. If general relativity is valid in an area, at least there must be basic states in coordinate space. Therefore, general relativity cannot describe the inside structure of black holes. In order to observe momentum space, there must be enough $\begin{Bmatrix}1\\0\end{Bmatrix}$ in the area.

The simplest rule will be that site is invariant, as in [1], but that violates many principles. First, it violates general relativity from the stand of experiment. Second, it violates self-realization principle. Third, it requires sites be a priori, so violating inside causality. From the viewpoint of experiment, observable momentum-energy in coordinate space can not provide enough stability for coordinate site. Therefore, site

not only maps with observable momentum in coordinate space, but also with momentum in $\begin{Bmatrix}1\\0\end{Bmatrix}$, including black holes and dark matters. Mapping rule:

A coordinate site is a unique mapping with all momentum; a momentum site is a unique mapping with all coordinates.

This is A4 in Sec. 9. A point is a unity of whole and individual. Each site is a unique representation of a whole space. Each coordinate is a free individual with the right to choose site. Site provides freedom. This principle corresponds to A10. Mapping rule ought to be the foundation for electromagnetic interaction and gravitational interaction.

5. Principle of gravity

When momentum and momentum site increases in balance, coordinates and coordinate sites will increase in balance instantly, providing more freedom for momentum and momentum sites.

In Euclidean space, when new coordinate and sites are created in a region, the space becomes non-Euclidean. From A12, more freedom always means attraction; less freedom always means repulsion. Therefore, there will be attraction between momentum-energy in coordinate space, and between space-time in momentum space.

Principle of electromagnetic interaction:

Coordinate and vacancy reduce freedom of the same kind, increase freedom for the opposite kind.

In a region, when there are less coordinates or more coordinate vacancies, there will be more freedom for coordinates and less freedom for coordinate vacancies, because two coordinates or vacancies cannot occupy the same state. From A12, it will lead to the outflow of coordinate vacancies and inflow of coordinates. This principle corresponds to A11.

These two principles decide the change of freedom, but attraction or repulsion depends on A12. For all four interactions, detailed distribution of field depends on largest freedom principle. For example, if there were least freedom principle, there would be attraction between coordinates.

6. Symmetry between points

In $\begin{Bmatrix}\Psi_p\\\Psi_x\end{Bmatrix}$, the two wave functions are not necessary to be independent. If treating the two wave functions as two fermions, there will be two possibilities. First, when the two wave functions are anti-symmetrical, there will be a singlet, $\{\ \}^A$. In coordinate space, it is $\begin{Bmatrix}0\\1\end{Bmatrix}^A$; in momentum space, it is $\begin{Bmatrix}1\\0\end{Bmatrix}^A$. W hen they are symmetrical, there

will be a triplet $\{\ \}^1$, $\{\ \}^2$ and $\{\ \}^3$, or simply $\{\ \}^S$. They form A7 and A13 in Sec. 9. Particles in $\{\ \}^A$ series correspond to free spatial reflection, or bosons; those in $\{\ \}^S$ series correspond to free spatial translation, or fermions.

Comparing $\begin{Bmatrix}\Psi_p\\ \Psi_x\end{Bmatrix}$ with $\begin{Bmatrix}0\\1\end{Bmatrix}^A$ and $\begin{Bmatrix}1\\0\end{Bmatrix}^A$, there are four possibilities for symmetry: $\begin{Bmatrix}A\\A\end{Bmatrix}^A$, $\begin{Bmatrix}S\\S\end{Bmatrix}^A$, $\begin{Bmatrix}A\\S\end{Bmatrix}^S$ and $\begin{Bmatrix}S\\A\end{Bmatrix}^S$. For $\begin{Bmatrix}A\\A\end{Bmatrix}^A$, its wave function in momentum space is anti-symmetrical with $\begin{Bmatrix}1\\0\end{Bmatrix}^A$, its wave function in coordinate space is anti-symmetrical with $\begin{Bmatrix}0\\1\end{Bmatrix}^A$. Two basic states for $\begin{Bmatrix}\Psi_p\\ \Psi_x\end{Bmatrix}$ can be noted as $\begin{Bmatrix}0^A\\1^A\end{Bmatrix}^A$ and $\begin{Bmatrix}1^A\\0^A\end{Bmatrix}^A$. All coordinates must be anti-symmetrical with each other, see A9. Therefore, two coordinates cannot occupy the same wave function.

Photons have two independent kinds. One is photon in momentum space, the other is photon in coordinate space. Since simultaneity relates with photon, energy simultaneity must replace time simultaneity in momentum space. This proves energy-time symmetry in Sec. 2.

Different symmetry leads to different particle. $\begin{Bmatrix}A\\S\end{Bmatrix}^S$ and $\begin{Bmatrix}S\\A\end{Bmatrix}^S$ corresponds to Fermions, $\begin{Bmatrix}A\\A\end{Bmatrix}^A$ to photons with $U(1)$ symmetry, $\begin{Bmatrix}S\\S\end{Bmatrix}^A$ to intermediate Bosons with $SU(2)$ symmetry.

$\begin{Bmatrix}S\\S\end{Bmatrix}^A$ is a singlet, and freedom is exchanged between $\begin{Bmatrix}\Psi_p\\ \Psi_x\end{Bmatrix}$ and a basic state. The two wave functions in momentum space are symmetrical, and so do the two wave functions in coordinate space. Coordinate distribution can be $\{0\leftrightarrow 1\}^A$, $\{1\leftrightarrow 1\}^A$ and $\{2\leftrightarrow 1\}^A$. Symmetry always brings a triplet. This time, it is a charge triplet.

For two points with $\begin{Bmatrix}A\\A\end{Bmatrix}^A$ symmetry, there are only two freedoms of phase shift (freedom shift), which follows $U(1)$ symmetry, see (5). In coordinate space, photons propagate in space-time and deliver momentum-energy. Similarly, in momentum space, photons propagate in momentum-energy and deliver space-time distance.

A basic state can be excited without changing number of coordinates. For example, In coordinate space, a neutrino $\{1\}^S$ has the same number of coordinate as $\begin{Bmatrix} 0^A \\ 1^A \end{Bmatrix}^A$.

Symmetry between $\{\ \}^S$ and $\begin{Bmatrix} A \\ A \end{Bmatrix}^A$ is a mixture of $\begin{Bmatrix} A \\ S \end{Bmatrix}^S$ and $\begin{Bmatrix} S \\ A \end{Bmatrix}^S$. So $\{\ \}^S$ sometimes moves in coordinate space, sometimes rests in coordinate space and moves in momentum space, and its velocity cannot surpass light velocity in either space. $\begin{Bmatrix} A \\ S \end{Bmatrix}^S$ and $\begin{Bmatrix} S \\ A \end{Bmatrix}^S$ are anti-symmetrical. When state $\begin{Bmatrix} A \\ S \end{Bmatrix}^S_{\mathrm{G}}$ moves to $\begin{Bmatrix} A \\ S \end{Bmatrix}^S_{\mathrm{H}}$ or when $\begin{Bmatrix} S \\ A \end{Bmatrix}^S_{\mathrm{G}}$ moves to $\begin{Bmatrix} S \\ A \end{Bmatrix}^S_{\mathrm{H}}$, $\{\ \}^S$ moves with two $U(1)$ symmetries and interacts with $\begin{Bmatrix} 0^A \\ 1^A \end{Bmatrix}^A$ and $\begin{Bmatrix} 1^A \\ 0^A \end{Bmatrix}^A$. When there is cross-flow between $\begin{Bmatrix} A \\ S \end{Bmatrix}^S$ part and $\begin{Bmatrix} S \\ A \end{Bmatrix}^S$ part, it moves with $SU(2)$ symmetry and interacts with $\begin{Bmatrix} S \\ S \end{Bmatrix}^A$.

A state moves and interacts with the same symmetry. This is A14 in Sec. 9, and leads to $SU(2)\times U(1)$ symmetry for $\{\ \}^S$. With still unknown reasons, the $SU(2)$ part of $\{\ \}^S$ is left-handed in the Universe, the $U(1)$ part of $\{\ \}^S$ is right-handed.

7. Symmetry between dimensions

A point has one site in each dimension. This forms B1. Therefore, if a point is decomposable, coordinate in each dimension ought to have a wave function. In a D-dimensional space, wave functions of all coordinate components form a D-dimensional complex space. ${}_D\{\ \}^A_{\mathrm{G}}$ means that it is a D-dimensional space, the state locates at G, momentum part is anti-symmetrical with coordinate part.

Maybe it will be easier to understand the similarities between $SU(2)$ and $SU(3)$ by writing coordinate components as ${}_3\begin{Bmatrix} \Psi_x \\ \Psi_y \\ \Psi_z \end{Bmatrix}^A$, similar to three spaces. However, there is no ${}_3\begin{Bmatrix} \Psi_x \\ \Psi_y \\ \Psi_z \end{Bmatrix}^S$, because coordinate components in different dimensions are always independent or anti-symmetrical. This forms B3.

In a point, all components have the same symmetries between the two wave functions, so there is no distribution like ${}_2\left\{(\ )^S,(\ )^A\right\}$. This is B2.

When $D\geq 2$, in coordinate space, $\left\{0^A\right\}^A$ becomes ${}_D\left\{\left(0^A\right)_i\right\}^A$. ${}_D\left\{(n)...(n+1)_i...(n)_D\right\}^S$ is quark with i color. ${}_D\left\{(n)...(n-1)_i...(n)_D\right\}^S$ is quark with anti-i color. $\left\{1_i\leftrightarrow 1_j\right\}^A$ is gluon.

In a point, when number of coordinates is given, number of coordinates on each dimension has the smallest difference. This is B4. Therefore, in two-dimensional space, there will be two kind of quarks, ${}_2\left\{(1),(2)\right\}^S$ and ${}_2\left\{(1),(0)\right\}^S$, but no ${}_2\left\{(0),(2)\right\}^S$. They will annihilate and cannot form stable nucleon, which should not annihilate itself or with electron and positron, a necessary condition for growth. A nucleon has the same number of coordinates in each dimension, corresponding to colorless in strong interaction. In three-dimensional space, there will be four quarks, ${}_3\left\{(1),(2),(2)\right\}^S$, ${}_3\left\{(1),(1),(2)\right\}^S$, ${}_3\left\{(1),(1),(0)\right\}^S$ and ${}_3\left\{(1),(0),(0)\right\}^S$.

Three is the smallest dimension to form stable nucleons, However, it is unnecessary to be three-dimensional in whole space. On the border between the two spaces, it is possible to be two-dimensional or even one-dimensional space, which might be the reason to break symmetry between particle and antiparticle and produce particles for both spaces.

When symmetry between ${}_D\{\ \}^A_{\mathrm{G}}$ and ${}_D\{\ \}^A_{\mathrm{H}}$ is $\left\{\begin{matrix}A\\A\\A\end{matrix}\right\}^A$, phase-shift of different components can be different, so there can be phase-shift between different components, corresponding to spin of a particle. If the symmetry is $\left\{\begin{matrix}S\\S\\S\end{matrix}\right\}^A$, the three components form a 3-dimensional complex space, and their variations follows $SU(3)$ symmetry. $SU(2)$ symmetry represents the cross-flow between momentum and point; $SU(3)$ represents the cross-flow among different dimensions.

8. Free set theory

Mystery of set not only exists in large sets and paradoxes, but also in each element. Why element cannot move or exchange? What is the definition of each element? They

need an answer if the set is isolated. These two basic questions are related. Motion is possible when definition is variable.

Element ought to have several kinds of freedom. First, it is movable. Classical transformation group keeps site and element one-one mapping, but it is unnecessary. Second, if an element has more than one dimension, it has the freedom to decompose. Third, an element can move to momentum set. Fourth, site has freedom to change.

Space itself need to be constructed, and hence it is not a good start to construct the Universe. However, space is close to ordered set, postulates in free set theory are the same as those in free spatial transformation. For example, a basic state in a set includes an element in the set and a vacancy in the other set. In one-dimensional set, basic states are $\begin{Bmatrix} 0_P^A \\ 1_E^A \end{Bmatrix}^A$ in element set, $\begin{Bmatrix} 0_E^A \\ 1_P^A \end{Bmatrix}^A$ in momentum set.

In order to construct two mutually mapped sets, motion of element is a good start.

With motion freedom, every set has its momentum set. For example, element set $\{E_1, E_2\}$ ought to allow E_1 to move to E_2 or exchange site with E_2. So it will be better to treat the set as two element sites 1_E, 2_E and two identical elements. The two elements can be both symmetrical and anti-symmetrical. When symmetrical, there will be state of motion $E_1 \to E_2$ and $E_2 \to E_1$, called P_1 and P_3 respectively. $E_1 \to E_1$ and $E_2 \to E_2$ forms P_2. The new set is called momentum set, noted as $\{P_i\}$. There are three momentum sites 1_p, 2_p, 3_p and three identical momentum. Therefore, motion freedom causes the separation between element and element site, momentum and momentum site.

For an isolated system of two mutually 1-N mapped sets, there will be uncertainty principle. When a state distributes in a smaller area in one set, it distributes in a larger area in the other set. Similar to Fourier expansion, 1_E can be expanded as the sum of all momentum including 1_E, which is $P_1(1_E)+P_2(1_E)+P_3(1_E)$. Without $P_3(1_E)$, 2_E term must join in, like $P_1(2_E)$. States like $P_1(1_E)+P_2(1_E)$ are forbidden.

There will be interaction. For example, if $P_1(1_E)+P_2(1_E)+P_3(1_E)$ releases $P_3(1_E) \leftrightarrow P_1(2_E)$, which is absorbed by $P_1(2_E)+P_2(2_E)+P_3(2_E)$ later, then the two static elements will become two moving elements, $P_1(1_E)+P_2(1_E)+P_1(2_E)$ and $P_3(2_E)+P_2(2_E)+P_3(1_E)$. Therefore, freedom leads to uncertainty principle, then interaction and dynamic evolution are inevitable.

No matter element set is ordered or not, momentum is internally ordered. For an disordered set, when covering three element sites, 123、312 and 231 means different momentum, but 321 and 123 are opposite momentum. For $P(j)$ covering j sites, there are $j!/2$ independent permutations. Therefore, if there is no order in element set, momentum set will be much larger than element set. When number of elements is not

equal in the two sets, the two sets and their elements are not equal. And elements have different freedom in the two sets.

It is a hypothesis that set equality between the two sets is a necessary condition for best growth. With the equality, all states have the same freedom, but some have larger freedom in element set, some in momentum set. If there is order in element set, there will be order in momentum set automatically, and there is only one $P(j)$ for each j, therefore, the two sets will be equal automatically. Then, distribution on sites will be two complex wave functions. In D-dimensional ordered set, distribution in each set will be a D-component complex wave function.

9. Some Fact Evidences

In [5], there are detailed discussions about similarities and differences among these pursuit systems. For example, gravitational interaction causes star in action pursuit, subjects in knowledge pursuit, cities in happiness pursuit and large companies in profit pursuit. Repulsion between like charges is similar to competition in other pursuit systems. System growth causes space expansion, social development and knowledge progress.

However, some similarities are more fundamental for pursuit. Freedom, mutual mapping and fastest growth ought to be the three pillars to study all pursuit systems. They have close relationships with isolation postulate. Here are some facts to prove their universality.

Isolation postulate is universally valid. Society is the isolated system of happiness; the Universe is the isolated system of action; research is the isolated system of knowledge; economy is the isolated system of profit. Among them, the Universe ought to be the only strictly isolated system. However, when a system grows larger, its openness usually decreases, more and more exchanges are between individual pursuers, not between pursuer and environment. For example, more and more freedom comes from exchanges between persons, not between person and nature. Present economy, society and thinking are more isolated than one thousand years ago. Therefore, isolated system is a better model for a more developed pursuit system. It is possible that the Universe is not a completely isolated system, but the degree of openness must be very small.

An isolated system includes two mutually mapped sets. The mappings between desires and commodities form happiness pursuit. The mappings between concepts and facts form knowledge pursuit. The mappings between momentum and coordinate form action pursuit.

An isolated system always pursues largest freedom in the two sets. Economy, society and thinking pursue largest freedom in profit, happiness and knowledge respectively.

Mutual mapping must be 1-more, not one-one. In the Universe, it is a 1-all mapping. In economy, a commodity satisfies many desires, and vice versa. A concept explains many facts, and vice versa.

Mapping has freedom to change. Site and element ought to be separable. A site is a possible state, including all properties of an element; elements are identical pursuers. For example, in economy, a job is a site, a worker ought to have freedom to choose job or job distribution, [8]. In knowledge pursuit, a concept can change site or meaning when necessary.

Elements in one set decide sites in the other set. When there are new commodities, there are more sites and freedom for desires, but desires are not sure to flow into these sites. When there are new desires, there are more sites and freedom for commodities, but new commodities are not sure to be invented or produced. When there are new facts, there are new sites for concepts, but not sure to be realized or established. When there are new concepts, there are new possibilities for facts, but not sure to be true.

There are basic states and activated states. In set of commodities, unconsumed commodities are at their basic states, corresponding to $\begin{Bmatrix} 0^A \\ 1^A \end{Bmatrix}^A$. Desires and commodities are symmetrical when consuming, corresponding to $\{\ \}^S$. In set of concepts, there are many possible concepts but only a few have corresponding fact foundations. For realized concepts, facts and concepts are symmetrical. They explain each other.

When a commodity is occupied by desire, there are three possibilities, corresponding to three generations. It can be dominated by commodity, by desire or by a mixture independent with the former two. In coordinate space, a first generation fermion mainly exists in the space, hence has the smallest mass; a third generation fermion mainly exists in momentum space, and has the largest mass; a second generation fermion exists in the middle.

When a set grows larger, for elements in the set, freedom almost keeps the same because they must move continuously. For element in the other set, their freedom increases linearly with the growth. The main beneficiaries are elements in the other set, because they map with whole set. It helps to explain that freedom of moving between sets is always larger than freedom of moving inside set.

In society, positive happiness mainly comes from the mapping between desires and commodities. People prefer not to change their mapping when there is no benefit. In thinking, mapping between concept and fact forms positive knowledge. A concept or fact cannot get positive knowledge by changing its position relative to other concepts or facts. In fact, it means negative knowledge, so people prefer not to change. Similarly, a desire cannot get positive satisfaction by simply adjusting itself. In the Universe, mapping between energy and time forms the main positive part of negative action.

Although the Universe provides little clues for growth, other pursuit areas give some hints for possible mechanisms. In the unified science, [5], they are some sort of proofs.

In each pursuit system, there are two areas. Each with one abundant quantity and one scarce quantity, abundant in one area is scarce in the other. Each area needs its scarce quantity to be the power of growth but not easy to input, frontiers between the two areas are the sources of growth. In the Universe, two spaces are the two areas, and

black holes are the frontiers that should provide the detailed mechanism about growth. In society, there are not only market areas with abundant commodities and scarce desires, but also that with scarce commodities and abundant desires, like living longer, knowing more truth, travelling in space, etc.. In thinking, there are not only mature areas with many stable concepts and few new tests from facts, but also areas with many unexplained facts and few stable concepts.

Although time jump is not easy to understand in the Universe, it is ordinary in other pursuit areas. In economy and thinking, new products and thoughts are power of growth. They grow from nothing to ripe very rapidly, no matter how developed the economy or knowledge system is. There is no upper limit for the catch-up velocity.

However, unlike the Universe, these pursuit systems are imperfect. One of the main differences is that there is not a unique order in the set of commodities, desires, concepts, facts and so on. Nevertheless, they are not totally disorder. Some orders are more popular. It is possible that completely disordered set cannot grow. The best growth only permits one order or its reverse order. Therefore, a pursuit system needs to be partially ordered at least, whereas strictly ordered sets ought to be the best.

In the Universe, each coordinate site should map with all momentum. But a desire does not map with all commodities, and a concept does not map with all facts. Therefore, a pursuit system requires two 1-more mapping sets at least, but 1-all mapping sets ought to be the best.

10. Postulate group

In [1] and this paper, there are some postulates about an ideal isolated system. They are organized into four groups:

Postulate group of mutual mapping:

A1: An isolated system includes two sets.

A2: The two sets are both ordered.

A3: Each set includes elements and element sites.

A4: A site in one set is determined by a unique mapping with all elements in the other set.

A5: All elements and vacancies are equal.

A6: A state is the distribution of an element in the two sets.

A7: For a basic state in a set, its distributions in the two sets are anti-symmetrical.

A8: A basic state in a set includes an element in the set and a vacancy in the other set.

A9: In a set, two basic states are always anti-symmetrical with each other.

A10: Site provides freedom for both element and vacancy.

A11: Element provides freedom for vacancy, and vice versa.

A12: Total freedom in the isolated system is the largest.

A13: Between two states, there is a triplet for each kind of symmetry, and a singlet for each kind of anti-symmetry.

A14: A state moves and interacts with the same symmetry.

Postulate group of dimension:

B1: In a D-dimensional set, an element site has one coordinate site in each dimension.

B2: If $D \geq 2$, in a state, different components have the same symmetry.

B3: In an element site, distributions in different dimensions are always anti-symmetrical.

B4: If $D \geq 2$, in a site, number of elements have the smallest difference among different dimensions.

Postulate group of energy-time:

C1: There is an abundant quantity and a scarce quantity in each set. Scarce quantity in one set is abundant quantity in the other set.

C2: The abundant quantity decides the threshold value for element to enter a set.

C3: Freedom of moving between the two sets is positive. Freedom of moving in each set is negative.

Postulate group of coupling constant:

D1: Coupling constants of interactions are invariant with site.

D2: When interacting with $\{mD \leftrightarrow D\}$, $m = 0,1,2$, coupling constant is invariant with *m*.

D3: If $D \geq 2$, when interacting with $\{1_i \leftrightarrow 1_j\}$, coupling constant is invariant with i and j.

Conclusion

Classical logic does not provide much useful knowledge about the matter world. Lack of freedom is the main reason. Mathematics had been a tool for various sciences for a long time, but it ought to be ultimate science itself. Freedom is the key. Without freedom, mathematics has no life, so it needs life or dynamic mechanism from different sciences, like person, particle, corporation, etc. With freedom, it no longer needs them. Without freedom, mathematics might be more beautiful. However, there are some items more important than beauty, like causality, isolation, life, pursuit and growth. In addition, science unification is a result with far more beauty.

In the phenomenon world, there is set equality or set symmetry, and it is broken in each specific system. Before symmetry breaking, various sets are equal. After symmetry breaking, there is a free set or an isolated system. Then, it starts the forever journey of freedom pursuit. By now, people have observed three such systems.

In an isolated system, the freedom with best growth will make all static freedom be of no importance. Element has the freedom to be static; set has the freedom to be disordered. However, such freedom is infinitesimal relative to the infinitely increasing freedom.

This paper tried to discover the sacrificed freedom, or methodology, logically. The ultimate goal for Truth Evolutionism is to reveal all truth from largest freedom principle, make all laws natural and abolish all unnecessary bondage to the freedom of human beings. However, there is still a long way from that. Many postulates in Sec. 9

have no direct relationship with freedom pursuit, but there are some clues. For example, if equality is the best pursuit method, A2, A5 and group D will be natural laws. If private ownership is the best method for an element to protect its freedom, there will be anti-symmetry, because anti-symmetry is the precise form of private ownership. It is suggested that symmetry be the best way to pursue new freedom, to search new chances, and to change ownership.

Isolated system is a general mathematical foundation for many different evolutions and pursuits. There are three ideal conditions: the two sets are ordered; a site maps with all elements in the other set; the system is completely isolated. It was highly suggested that the Universe meet all these conditions.

Isolated system is not a model to explain a particular system. It is the nature of all dynamic systems, including life, cognition and the Universe. A special dynamic system should adjust itself to accommodate truth, not vice versa. In consequence there will be one science for all systems, not many sciences.

If knowledge of the Universe is purely mathematical, it is constructive. As shown in this paper, freedom and mutual mapping decide uncertainty and interaction; number of sets decides symmetry of weak interaction; dimension decides symmetry of strong interaction; gravitational and electromagnetic interactions come from mapping rule between the two sets. If freedom is optimal, kinds of particles, interactions and symmetries will be the same as the Universe. They are not unique properties of the Universe. The Universe has no unique property. Its knowledge is universally valid.

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