

Let us Continue to Complete the Proof for the Polignac's Conjecture Hereof

Zhang Tianshu

*Nanhai West Oil Corporation, China Offshore Oil,
Zhanjiang City, Guangdong Province, P.R. China.*

*E-mail: tianshu_zhang507@aliyun.com ; friend_zhang888@sina.com
xinshijizhang@hotmail.com ; zhangtianshu904@gmail.com*

Abstract

Let us consider odd numbers which share a prime factor > 1 as a kind, then the number axis's positive half line which begins with odd point 3 consists of infinite many equivalent line segments on same permutation of χ kinds' odd points plus odd points amongst the χ kinds' odd points, where $\chi \geq 1$. In this article, we shall prove the unproved half of the Polignac's conjecture by mathematical induction with the aid of such equivalent line segments and kinds of odd points thereon.

Keywords: The number axis's positive half line, Mathematical induction, Kinds of odd points, The coexisting theorem.

Basic Concepts

In 1849, Polignac conjectured that for every even number $2n$ are there infinitely many pairs of consecutive primes which differ by $2n$, where $n \geq 1$. Yet this is an unproved conjecture up to now.

Happily, I had proven a half of the conjecture by the end of last year, and the half states that there are infinitely many pairs of consecutive odd prime numbers. The paper with relation to the half is published at pp.17-26, Number 1 (2013), learned journal "Advances in Theoretical and Applied Mathematics", Research India Publications.

What we need is to successively prove another half of the conjecture by now, namely prove that every positive even number $2n$ is a difference of two consecutive odd prime numbers. Nevertheless apply more or less alike method as compared with

the proven half, therefore following basic concepts expounded have more repeats. Of course, this is inevitable, and indispensable for the new proof as well.

Everyone knows, each and every odd point at positive half line of the number axis expresses a positive odd number. Also infinite many a distance between two consecutive odd points at the positive half line equal one another. Afterwards, the number axis's positive half line which begins with odd point 3 is called the half line for short.

Let us use symbol “•” to denote an odd point at the beginning's half line and in formulations. Moreover the half line is marked merely with symbols of odd points here. Please, see following first illustration.



First Illustration

We use also symbol “•s” to denote at least two undefined odd points in formulations. We consider smallest positive odd prime number 3 as №1 odd prime number, and consider positive odd prime number J_χ as № χ odd prime number, where $\chi \geq 1$, then odd prime number 3 is written as J_1 as well. And then, we consider positive odd numbers which share prime factor J_χ as № χ kind of odd numbers. If an odd number contains α different prime factors, then the odd number belongs in the α kinds of odd numbers concurrently, where $\alpha \geq 1$.

There is an unique odd prime number J_χ within № χ kind's odd numbers, yet we term others as № χ kind of odd composite numbers.

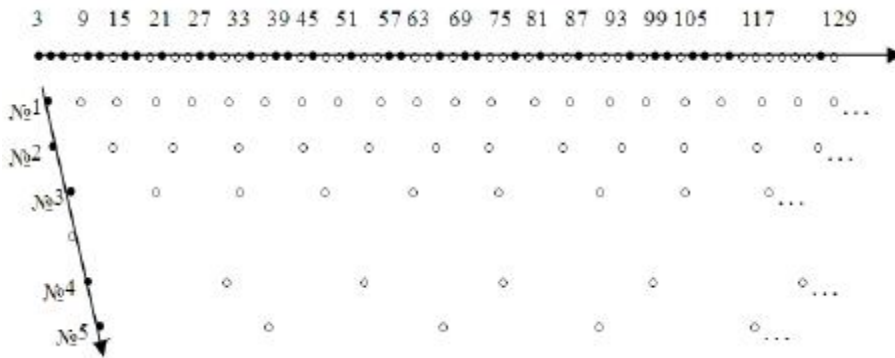
If one • is defined as an odd composite point, then we change symbol “•” for symbol “◦”. And use symbol “◦s” to denote the plural in formulations.

If one • is affirmed as an odd prime point, then this • is rewritten as one ♠. And use ♠s to denote the plural in formulations.

In the course of the proof, we shall change ◦s for •s at places of $\sum \text{№}\chi [\chi \geq 1]$ kind's odd composite points orderly according as χ is from small to large.

Since № χ kind's odd numbers are infinitely many a product which multiplies every odd number by J_χ , so there is a № χ kind's odd point within consecutive J_χ odd points at the half line.

We analyze seriatim № χ kind of odd points at the half line according to $\chi = 1, 2, 3, \dots$ in one by one, and range them as second illustration.



Second Illustration

Thus it can be seen, one another's permutation of χ kinds of odd points plus odd points amongst the χ kinds of odd points assumes always infinite many recurrences on same pattern at the half line, irrespective of their prime/composite attribute. We consider one another's-equivalent shortest line segments of the half line according to same permutation of χ kinds' odd points plus odd points amongst the χ kinds' odd points as recurring segments of the χ kinds' odd points. And use character "RLS_{No1~No χ} " to express a recurring segment of $\sum No\chi$ [$\chi \geq 1$] kind of odd points, also use RLSS_{No1~No χ} to express the plural.

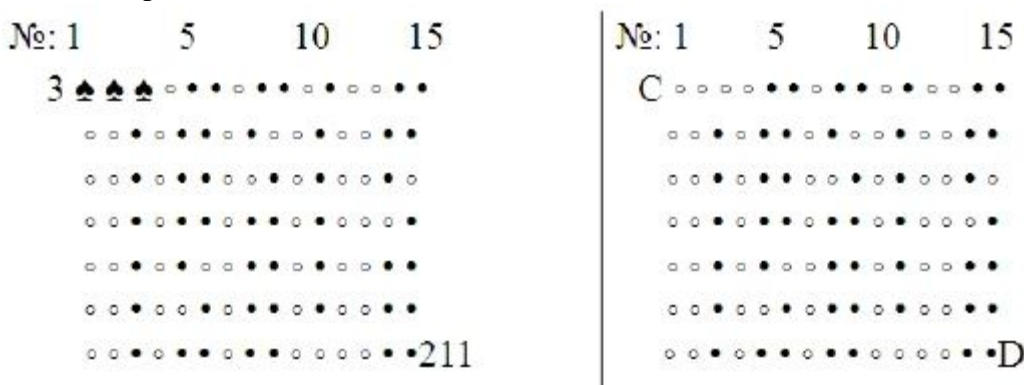
Number the ordinals of odd points at seriate each RLS_{No1~No χ} by consecutive natural numbers ≥ 1 , namely from left to right each odd point at seriate each RLS_{No1~No χ} is marked with from small to great a natural number which begins with 1 in the proper order.

Then, there is one No($\chi+1$) kind's odd point within $J_{\chi+1}$ odd points which share an ordinal at $J_{\chi+1}$ RLSS_{No1~No χ} of seriate each RLS_{No1~No($\chi+1$)}.

Excepting most left one at No1 RLS_{No1~No χ} is an odd prime point, others are all odd composite points, in No χ kind's odd points. Thus No1 RLS_{No1~No χ} is a particular RLS_{No1~No χ} in contradistinction to each of others.

There are $\prod J_{\chi}$ odd points at each RLS_{No1~No χ} , where $\prod J_{\chi} = J_1 * J_2 * \dots * J_{\chi}$, and $\chi \geq 1$. Justly No1 RLS_{No1~No χ} begins with odd point 3, and No1 RLS_{No1} ends with odd point 7; No1 RLS_{No1~No2} ends with odd point 31; No1 RLS_{No1~No3} ends with odd point 211; No1 RLS_{No1~No4} ends with odd point 2311, etc. Undoubtedly one RLS_{No1~No($\chi+1$)} consists of $J_{\chi+1}$ consecutive RLSS_{No1~No χ} , and they link, one by one.

$J_{\chi+1}$ RLSS_{No1~No χ} of any RLS_{No1~No($\chi+1$)} may be folded at an illustration, so as to view conveniently. For instance, after change °s for •s at places of $\sum No\chi$ [$\chi \leq 3$] kind's odd composite points, for odd points at No1 RLS_{No1~No3} i.e. 3(211) and at another RLS_{No1~No3} CD, please, see third illustration.



Third Illustration

Thus it can be seen, after change °s for •s at places of $\sum No\chi$ [$\chi \geq 1$] plus No($\chi+1$) kind's odd composite points, there is one No($\chi+1$) kind's odd composite point within $J_{\chi+1}$ odd

points on an ordinal of every odd point of a $RLS_{N_01 \sim N_0\chi}$ at seriate each $RLS_{N_01 \sim N_0(\chi+1)}$ on the right of $N_01 RLS_{N_01 \sim N_0(\chi+1)}$.

After change $\circ s$ for $\bullet s$ at places of $\sum N_0\chi [\chi \geq 1]$ kind's odd composite points, if an odd point P_1 is separated from another odd point P_2 by $\lambda_\chi \circ s$, then express such a combinative form as a pair of $P_1 \lambda_\chi(\circ s) P_2$, where $\lambda_\chi \geq 0$.

After change $\circ s$ for $\bullet s$ at places of $\sum N_0\chi [\chi \geq 1]$ kind's odd composite points, there are pairs of $\spadesuit \lambda_\chi(\circ s) \spadesuit$ on the right of J_χ at $N_01 RLS_{N_01 \sim N_0\chi}$ and pairs of $\bullet \lambda_\chi(\circ s) \bullet$ on ordinals of $\spadesuit \lambda_\chi(\circ s) \spadesuit$ at seriate each $RLS_{N_01 \sim N_0\chi}$ on the right of $N_01 RLS_{N_01 \sim N_0\chi}$, where $\lambda_\chi \geq 0$.

From the definition for recurring segments of χ kinds' odd points, we can conclude that after change $\circ s$ for $\bullet s$ at places of $\sum N_0\chi [\chi \geq 1]$ kind's odd composite points, provided there is a pair of $\spadesuit \lambda_\chi(\circ s) \spadesuit$ on the right of J_χ at $N_01 RLS_{N_01 \sim N_0\chi}$, then there is surely a pair of $\bullet \lambda_\chi(\circ s) \bullet$ on ordinals of the pair of $\spadesuit \lambda_\chi(\circ s) \spadesuit$ at seriate each $RLS_{N_01 \sim N_0\chi}$ on the right of $N_01 RLS_{N_01 \sim N_0\chi}$. Undoubtedly, the converse proposition is tenable too. Namely after change $\circ s$ for $\bullet s$ at places of $\sum N_0\chi [\chi \geq 1]$ kind's odd composite points, provided there is a pair of $\bullet \lambda_\chi(\circ s) \bullet$ at seriate each $RLS_{N_01 \sim N_0\chi}$ on the right of $N_01 RLS_{N_01 \sim N_0\chi}$, and all such pairs of $\bullet \lambda_\chi(\circ s) \bullet$ share a set of ordinals, then there is surely a pair of $\spadesuit \lambda_\chi(\circ s) \spadesuit$ on ordinals of any such pair of $\bullet \lambda_\chi(\circ s) \bullet$ at $N_01 RLS_{N_01 \sim N_0\chi}$. Of course, either \spadesuit of the pair of $\spadesuit \lambda_\chi(\circ s) \spadesuit$ and every prime factor of an odd number which each \bullet of all such pairs of $\bullet \lambda_\chi(\circ s) \bullet$ expresses are greater than J_χ .

To be brief, after change $\circ s$ for $\bullet s$ at places of $\sum N_0\chi [\chi \geq 1]$ kind's odd composite points, a pair of $\spadesuit \lambda_\chi(\circ s) \spadesuit$ on the right of J_χ at $N_01 RLS_{N_01 \sim N_0\chi}$ and infinite many pairs of $\bullet \lambda_\chi(\circ s) \bullet$ on ordinals of the pair of $\spadesuit \lambda_\chi(\circ s) \spadesuit$ at seriate $RLSS_{N_01 \sim N_0\chi}$ on the right of $N_01 RLS_{N_01 \sim N_0\chi}$ coexist at the half line.

We term the aforesaid conclusion as the coexisting theorem for a pair of $\spadesuit \lambda_\chi(\circ s) \spadesuit$ and infinite many pairs of $\bullet \lambda_\chi(\circ s) \bullet$ at the half line, or term it as the coexisting theorem for short.

The Proof

We shall prove indirectly the unproved half of the Poincaré's conjecture by mathematical induction with the aid of the coexisting theorem for a pair of $\spadesuit \lambda_\chi(\circ s) \spadesuit$ and infinite many pairs of $\bullet \lambda_\chi(\circ s) \bullet$ at the half line, below.

1. When $\chi=1$, there is a pair of $\spadesuit 0(\circ s) \spadesuit$ on the right of J_1 at $N_01 RLS_{N_01}$, and the pair of $\spadesuit 0(\circ s) \spadesuit$ is the very odd prime points 5 and 7.

When $\chi=2$, there are pairs of $\spadesuit \Omega_2(\circ s) \spadesuit$ on the right of J_2 at $N_01 RLS_{N_01 \sim N_02}$, where $\Omega_2 = 0, 1, 2$.

When $\chi=3$, there are pairs of $\spadesuit \Omega_3(\circ s) \spadesuit$ on the right of J_3 at $N_01 RLS_{N_01 \sim N_03}$, where $\Omega_2 \leq \Omega_3 \leq 6$.

When $\chi=4$, there are pairs of $\spadesuit \lambda_4(\circ s) \spadesuit$ on the right of J_4 at $N_01 RLS_{N_01 \sim N_04}$, where $\lambda_4 = \Omega_4$ plus κ_4 , $\Omega_3 \leq \Omega_4 \leq 11$, and $\kappa_4 = 16$.

2. When $\chi = \beta \geq 4$, suppose that there are pairs of $\spadesuit \lambda_{\beta}(\circ S) \spadesuit$ on the right of J_{β} at $\mathbb{N} \circ 1$ $RLS_{\mathbb{N} \circ 1 \sim \mathbb{N} \circ \beta}$, where $\lambda_{\beta} = \Omega_{\beta}$ plus κ_{β} , and Ω_{β} expresses any of consecutive natural numbers ≥ 1 plus 0, and $\Omega_{\beta} \geq \Omega_4$. In addition, let greatest value of Ω_{β} is η_{β} , then $\eta_{\beta} \geq 1$, and $\kappa_{\beta} > \eta_{\beta} + 1$.

3. When $\chi = \beta + 1$, we must prove that there are pairs of $\spadesuit \lambda_{\beta+1}(\circ S) \spadesuit$ on the right of $J_{\beta+1}$ at $\mathbb{N} \circ 1$ $RLS_{\mathbb{N} \circ 1 \sim \mathbb{N} \circ (\beta+1)}$, where $\lambda_{\beta+1} = \Omega_{\beta+1}$ plus $\kappa_{\beta+1}$, and $\Omega_{\beta+1}$ expresses any of consecutive natural numbers ≥ 1 plus 0, and $\Omega_{\beta+1} \geq \Omega_{\beta}$. That is to say, at least one of $\Omega_{\beta+1}$ must be equal to $\eta_{\beta} + 1$ on a minimum, and let greatest value of $\Omega_{\beta+1}$ is $\eta_{\beta+1}$, and $\kappa_{\beta+1} > \eta_{\beta+1} + 1$.

Proof . For the number axis's positive half line which is marked merely with symbols of undefined odd points , after change $\circ S$ for $\bullet S$ at places of $\mathbb{N} \circ 1$ kind's odd composite points, there is a pair of $\bullet 0(\circ S) \bullet$ on the right of J_1 at $\mathbb{N} \circ 1$ $RLS_{\mathbb{N} \circ 1}$. Besides there are pairs of $\bullet \Omega_1(\circ S) \bullet$ on the right of J_1 at seriate each $RLS_{\mathbb{N} \circ 1 \sim \mathbb{N} \circ 2}$, where $\Omega_1 = 0, 1$. And every pair of $\bullet \Omega_1(\circ S) \bullet$ with a pair of $\bullet \Omega_1(\circ S) \bullet$ on either side except for the left side of $\mathbb{N} \circ 1$ pair of $\bullet 0(\circ S) \bullet$ forms two concurrent pairs which share an odd point.

Provided successively change $\circ S$ for $\bullet S$ at places of $\mathbb{N} \circ 2$ kind's odd composite points, since there is one $\mathbb{N} \circ 2$ kind's odd composite point within J_2 odd points on an ordinal of every odd point of a $RLS_{\mathbb{N} \circ 1}$ at seriate each $RLS_{\mathbb{N} \circ 1 \sim \mathbb{N} \circ 2}$ on the right of $\mathbb{N} \circ 1$ $RLS_{\mathbb{N} \circ 1 \sim \mathbb{N} \circ 2}$, so this has made preparations for an increase of the number of consecutive odd composite points, where $\Omega_1 + 1 + \Omega_1 = 2$.

After successively change $\circ S$ for $\bullet S$ at places of $\mathbb{N} \circ 2$ kind's odd composite points, there are both pairs of $\bullet \Omega_1(\circ S) \bullet$ and pairs of $\bullet \Omega_2(\circ S) \bullet$ on the right of J_2 at seriate each $RLS_{\mathbb{N} \circ 1 \sim \mathbb{N} \circ 2}$, where $\Omega_1 \leq \Omega_2 \leq 2$. Excepting a part of pairs of $\bullet \Omega_2(\circ S) \bullet$ belong to pairs of $\bullet \Omega_1(\circ S) \bullet$, each of others exists at the place of two concurrent pairs of original $\bullet \Omega_1(\circ S) \bullet$, hence every pair of $\bullet \Omega_2(\circ S) \bullet$ with a pair of $\bullet \Omega_1(\circ S) \bullet$ on either side of the pair of $\bullet \Omega_2(\circ S) \bullet$ is still two concurrent pairs, where $\Omega_1 = 0, 1$.

Provided successively change $\circ S$ for $\bullet S$ at places of $\mathbb{N} \circ 3$ kind's odd composite points, since there is one $\mathbb{N} \circ 3$ kind's odd composite point within J_3 odd points on an ordinal of every odd point of a $RLS_{\mathbb{N} \circ 1 \sim \mathbb{N} \circ 2}$ at seriate each $RLS_{\mathbb{N} \circ 1 \sim \mathbb{N} \circ 3}$ on the right of $\mathbb{N} \circ 1$ $RLS_{\mathbb{N} \circ 1 \sim \mathbb{N} \circ 3}$, so this has made preparations for an increase of the number of consecutive odd composite points, where $2 < \Omega_2 + 1 + \Omega_1 \leq 6$.

After successively change $\circ S$ for $\bullet S$ at places of $\mathbb{N} \circ 3$ kind's odd composite points, there are both pairs of $\bullet \Omega_2(\circ S) \bullet$ and pairs of $\bullet \Omega_3(\circ S) \bullet$ on the right of J_3 at seriate each $RLS_{\mathbb{N} \circ 1 \sim \mathbb{N} \circ 3}$, where $\Omega_2 \leq \Omega_3 \leq 6$.

Since every pair of $\bullet \Omega_3(\circ S) \bullet$ is either a pair of $\bullet \Omega_2(\circ S) \bullet$, or at the place of two concurrent pairs of original $\bullet \Omega_2(\circ S) \bullet$, hence every pair of $\bullet \Omega_3(\circ S) \bullet$ with a pair of $\bullet \Omega_2(\circ S) \bullet$ on either side of the pair of $\bullet \Omega_3(\circ S) \bullet$ is still two concurrent pairs, where $\Omega_2 = 0, 1$ and 2 .

Provided successively change $\circ S$ for $\bullet S$ at places of $\mathbb{N} \circ 4$ kind's odd composite points, since there is one $\mathbb{N} \circ 4$ kind's odd composite point within J_4 odd points on an ordinal of every odd point of a $RLS_{\mathbb{N} \circ 1 \sim \mathbb{N} \circ 3}$ at seriate each $RLS_{\mathbb{N} \circ 1 \sim \mathbb{N} \circ 4}$ on the right of $\mathbb{N} \circ 1$

RLS $_{\aleph_1 \sim \aleph_4}$, so this has made preparations for an increase of the number of consecutive odd composite points, where $6 < \Omega_3 + 1 + \Omega_2 \leq 11$.

After successively change \circ s for \bullet s at places of \aleph_4 kind's odd composite points, there are both pairs of $\bullet \Omega_3(\circ S) \bullet$ and pairs of $\bullet \lambda_4(\circ S) \bullet$ on the right of J_4 at seriate each RLS $_{\aleph_1 \sim \aleph_4}$, where $\lambda_4 = \Omega_4$ plus κ_4 , $\Omega_3 \leq \Omega_4 \leq 11$, and $\kappa_4 = 16$.

Since every pair of $\bullet \lambda_4(\circ S) \bullet$ is either a pair of $\bullet \Omega_3(\circ S) \bullet$, or at the place of two concurrent pairs of original $\bullet \Omega_3(\circ S) \bullet$, hence every pair of $\bullet \lambda_4(\circ S) \bullet$ with a pair of $\bullet \Omega_3(\circ S) \bullet$ on either side of the pair of $\bullet \lambda_4(\circ S) \bullet$ is still two concurrent pairs, where $\Omega_3 = 0, 1, 2, 3, 4, 5$ and 6 .

Let ν_4 expresses any of consecutive natural numbers ≥ 12 . Provided successively change \circ s for \bullet s at places of \aleph_5 kind's odd composite points, since there is one \aleph_5 kind's odd composite point within J_5 odd points on an ordinal of every odd point of a RLS $_{\aleph_1 \sim \aleph_4}$ at seriate each RLS $_{\aleph_1 \sim \aleph_5}$ on the right of \aleph_1 RLS $_{\aleph_1 \sim \aleph_5}$, so this has made preparations for an increase of the number of consecutive odd composite points, where $\Omega_4 + 1 + \Omega_3 = \nu_4$.

And so on and so forth...

Up to after successively change \circ s for \bullet s at places of \aleph_β kind's odd composite points, there are both pairs of $\bullet \lambda_{\beta-1}(\circ S) \bullet$ and pairs of $\bullet \lambda_\beta(\circ S) \bullet$ on the right of J_β at seriate each RLS $_{\aleph_1 \sim \aleph_\beta}$, where $\lambda_\beta = \Omega_\beta$ plus κ_β , Ω_β expresses any of consecutive natural numbers ≥ 1 plus 0 , $\Omega_\beta \geq \Omega_{\beta-1} \geq \Omega_4$, and $\kappa_\beta \geq \kappa_{\beta-1} \geq \kappa_4$.

Since every pair of $\bullet \lambda_\beta(\circ S) \bullet$ is either a pair of $\bullet \lambda_{\beta-1}(\circ S) \bullet$, or at the place of two concurrent pairs of original $\bullet \lambda_{\beta-1}(\circ S) \bullet$, hence every pair of $\bullet \lambda_\beta(\circ S) \bullet$ with a pair of $\bullet \lambda_{\beta-1}(\circ S) \bullet$ on either side of the pair of $\bullet \lambda_\beta(\circ S) \bullet$ is still two concurrent pairs, where $\lambda_{\beta-1} \geq \lambda_4$.

Let greatest value of Ω_β is η_β , and ν_β expresses any of consecutive natural numbers $\geq \eta_\beta + 1$. Provided successively change \circ s for \bullet s at places of $\aleph(\beta+1)$ kind's odd composite points, since there is one $\aleph(\beta+1)$ kind's odd composite point within $J_{\beta+1}$ odd points on an ordinal of every odd point of a RLS $_{\aleph_1 \sim \aleph_\beta}$ at seriate each RLS $_{\aleph_1 \sim \aleph(\beta+1)}$ on the right of \aleph_1 RLS $_{\aleph_1 \sim \aleph(\beta+1)}$, so this has made preparations for an increase of the number of consecutive odd composite points, where $\Omega_{\beta+1} + \Omega_{\beta-1} = \nu_\beta$. Evidently when $\Omega_{\beta+1} + \Omega_{\beta-1} = \eta_\beta + 1$, it will exceed first the super-limit of Ω_β , for example, a pair of $\bullet \eta_\beta(\circ S) \bullet$ with a pair of $\bullet 0(\circ S) \bullet$, a pair of $\bullet (\eta_\beta - 1)(\circ S) \bullet$ with a pair of $\bullet 1(\circ S) \bullet$, a pair of $\bullet (\eta_\beta - 2)(\circ S) \bullet$ with a pair of $\bullet 2(\circ S) \bullet$, etc.

After successively change \circ s for \bullet s at places of $\aleph(\beta+1)$ kind's odd composite points, there are both pairs of $\bullet \lambda_\beta(\circ S) \bullet$ and pairs of $\bullet \lambda_{\beta+1}(\circ S) \bullet$ on the right of $J_{\beta+1}$ at seriate each RLS $_{\aleph_1 \sim \aleph(\beta+1)}$, where $\lambda_{\beta+1} = \Omega_{\beta+1}$ plus $\kappa_{\beta+1}$, $\Omega_{\beta+1}$ expresses any of consecutive natural numbers ≥ 1 plus 0 , $\Omega_\beta \leq \Omega_{\beta+1} \leq \nu_\beta$, and $\kappa_{\beta+1} \geq \kappa_\beta$. Thus, where $\Omega_{\beta+1} >$ greatest value η_β of Ω_β , $\Omega_{\beta+1}$ exactly oversteps the super-limits of Ω_β .

Since the half line has infinitely many RLSS $_{\aleph_1 \sim \aleph(\beta+1)}$, thus there are infinitely many pairs of $\bullet \lambda_{\beta+1}(\circ S) \bullet$ which share a set of ordinals, then there is a pair of $\spadesuit \lambda_{\beta+1}(\circ S) \spadesuit$ on the set of ordinals at \aleph_1 RLS $_{\aleph_1 \sim \aleph(\beta+1)}$ according to the aforesaid coexisting theorem, besides $\Omega_{\beta+1}$ of $\lambda_{\beta+1}$ contains natural number ν_β .

In order to attain the final goal, we need yet to further explain hereinafter, though we have proven the conclusion when $\chi = \beta + 1$ hereinbefore.

Since every pair of $\bullet \lambda_{\beta+1}(\circ S) \bullet$ is either a pair of $\bullet \lambda_\beta(\circ S) \bullet$, or at the place of two concurrent pairs of original $\bullet \lambda_\beta(\circ S) \bullet$, hence every pair of $\bullet \lambda_{\beta+1}(\circ S) \bullet$ with a pair of $\bullet \lambda_\beta(\circ S) \bullet$ on either side of the pair of $\bullet \lambda_{\beta+1}(\circ S) \bullet$ is still two concurrent pairs, where $\lambda_\beta \geq \lambda_{\beta-1} \geq \lambda_4$.

Let greatest value of $\Omega_{\beta+1}$ is $\eta_{\beta+1}$, and $\nu_{\beta+1}$ expresses any of consecutive natural numbers $\geq \eta_{\beta+1} + 1$. Provided successively change \circ s for \bullet s at places of $\aleph(\beta+2)$ kind's

odd composite points, since there is one $\aleph(\beta+2)$ kind's odd composite point within $J_{\beta+2}$ odd points on an ordinal of every odd point of a $RLS_{\aleph 1 \sim \aleph(\beta+1)}$ at seriate each $RLS_{\aleph 1 \sim \aleph(\beta+2)}$ on the right of $\aleph 1 RLS_{\aleph 1 \sim \aleph(\beta+2)}$, so this has made preparations for an increase of the number of consecutive odd composite points, where $\Omega_{\beta+1}+1+\Omega_{\beta} = \nu_{\beta+1}$.

If let such steps proceed infinitely according to the aforesaid way of doing, and let χ to express any natural number, then after change $\circ s$ for $\bullet s$ at places of $\sum \aleph \chi [\chi \geq 1]$ kind's odd composite points, there are pairs of $\bullet \lambda_{\chi}(\circ s) \bullet$ on the right of J_{χ} at seriate each $RLS_{\aleph 1 \sim \aleph \chi}$, where $\lambda_{\chi} = \Omega_{\chi}$ plus κ_{χ} , Ω_{χ} expresses any of consecutive natural numbers ≥ 1 plus 0, and $\kappa_{\chi} - 1 >$ greatest value of Ω_{χ} . Since the half line has infinitely many $RLS_{\aleph 1 \sim \aleph \chi}$, thus there are infinitely many pairs of $\bullet \lambda_{\chi}(\circ s) \bullet$ on ordinals of a set of odd points of a $RLS_{\aleph 1 \sim \aleph \chi}$, consequently there is a pair of $\spadesuit \lambda_{\chi}(\circ s) \spadesuit$ on ordinals of the set of odd points at $\aleph 1 RLS_{\aleph 1 \sim \aleph \chi}$ according to the aforesaid coexisting theorem, where $\lambda_{\chi} = \Omega_{\chi}$ plus κ_{χ} . Obviously, if χ tends to infinitely great ∞ , then Ω_{χ} tends to equal every natural number plus 0, and $2\Omega_{\chi}+2$ tends to equal every positive even number, then it can replace $2\Omega_{\chi}+2$ by $2n$, where $n \geq 1$.

Since a pair of $\spadesuit \Omega_{\chi}(\circ s) \spadesuit$ expresses a pair of consecutive odd prime numbers which differ by $2\Omega_{\chi}+2$, so there are pairs of consecutive odd prime numbers which differ by $2n$ always. Namely every positive even number $2n$ is a difference of two consecutive odd prime numbers. Thus far, we have proven the remainder half of the Polignac's conjecture.

Pro tanto, I firmly believe that the Polignac's conjecture is proven quite into the true according to the former and now proven propositions.

