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Peristaltic Transport of Three Layered Viscous Incompressible Fluid

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Abstract

Peristaltic transport of three layered viscous incompressible fluid (Herschel-Bulkley) in reference to axi-symmetric case (sinusoidal wave) with low Reynolds number at moderate amplitude ratio. Flow is considered as three-layered model consisting of three layers (incompressible fluids) of different viscosities μ_1,μ_2 and μ_3 occupying core, intermediate and peripheral layers in a circular tube. Theoretical analysis with the use of Herschel-Bulkley fluid gives the effective insight to show the positive pumping action considered under contraction and expansion of waves. Theoretical analysis is focused on obtaining the velocity, flux, stream function, the limitation of flux and the friction force. These computations provide better accuracy in numerical results. Comparisons have been made between theoretical and experimental observations.

Keywords: Peristaltic transport, Herschel-Bulkley fluid, core, intermediate and peripheral.

Introduction

The peristaltic transport is traveling contraction wave a long tube like structure and it results physiologically from neuron - muscular properties of any tubular smooth muscle. The fluid is transported by regular coordinated waves of muscular contractions along the wall of the vessel. The vagus nerve functions as a fine tuner to stimulate the peristaltic moments. The chemical and mechanical stimulations of peristaltic wave are well supported for its movement by the gel (mucus).

Over the past few years, analytical and experimental studies have been carried out to analyze the flow parameters under the peristaltic transport. Burns et al. [1967] studied the peristaltic motion through pipe and channel flow under the assumptions of small Reynolds number. Fung et al. [1968] analyzed the flow of urine associated with peristaltic action in a two-dimensional channel. Shapiro et al. [1969] described the peristaltic pumping using long wave length at low Reynolds number for dissipation and mechanical efficiency in relevance to ureter function. Weinberg et al. [1971] made the experimental investigations on two-dimensional peristaltic pumping for the measurements of flow parameters at fixed locations of the tube. Michle [1973] investigated inertial and stream line curvature effects on peristaltic pumping. Thomas et al. [1977] presented the computational and experimental investigations of two-dimensional non linear peristaltic flows on the assumptions of highest stress and energy exchange. Kaimal [1978] studied the peristaltic pumping of a Newtonian fluid with particles suspension at low Reynolds number under long wavelength approximations. Shukla et al. [1980] discussed the peristaltic transport of a power-law fluid with variable consistency taking into account the existence of a peripheral layer. Shukla et al. [1982] analyzed the effects of peripheral layer viscosity on peristaltic transport of bio-fluid with varying viscosity across the duct. Srivastava et al. [1984] investigated the peristaltic transport of blood using Casson model under zero Reynolds number and long wave length approximation. Liepsch [1998] discussed a detailed discussion on blood circulatory system by which the human heart operates as a double working pump for the flow of blood similar to a piston in the tube network. Mizuno et al. [2001] studied the sucking pressure expressive, frequency and duration for measured parameters. Basavarajappa et al. [2002] discussed the peristaltic transport of two-layered viscous incompressible fluid to approximate the stream functions and the interface. Mezneb et al. [2006] studied the effect of wall compliance on peristaltic transport of Newtonian fluid in an axiymmetric channel and investigated the effect of amplitude ratio. Vajravelu et al. [2006] presented the peristaltic transport of a Herschel-Bulkley fluid in contact with Newtonian fluid. Hakeem et al. [2007] discussed the slip effects on peristaltic transport of power law fluid through an inclined tube. Agman Mahamoud [2008] studied as a model for the blood flow in living creatures using long wave length approximation Numerical computation were carried out to investigate the effect of couple stress parameter α and kundson number kn, Prasad K M et al. [2009] presented the effect of peripheral layer peristaltic transport of a micropolar fluid in the core region and Newtonian fluid in the peripheral layer under the assumption of long wave length approximation with low Reynolds number. Eldabe and Abou zeid et al. [2010] analyzed three wall properties effect on peristaltic transport of Micropolar non Newtonian fluid with heat is main transfer the flow include the viscoelastic wall properties and micropolar fluid parameters using the equations of the fluid as well as deformatic boundaries Pandey S K et al. [2012] made theoretical study of two-dimensional peristaltic flow of power-law fluids in three layers with different viscosities and the analysis was carried out under low Reynolds number and long wavelength approximations, observed that a medicinal intervention that creates a more viscous intermediate layer and reduces pseudo plasticity may reduce constipation.

In view of the above factors an attempt has been made in the present study to investigate the effective peristaltic pumping, limitations of flux at various amplitude ratios and the resistance of flow using Herschel-Bulkley fluid.Core layer filled with shear thickening and peripheral layer filled with shear-thinning fluid are employed in the analysis. Positive mean flow with decrease of friction force is determined by solving the equation of interface by numerical method. The volume of the fluid in peripheral layer increases with viscosity ratio. Flow rates for core layer and peripheral layer with varying viscosity are determined.

Formulation

Consider the peristaltic transport of axisymmetric flow of Herschel-Bulkley fluid through the tube of radius 'a' of which the core layer with radius h_1 is filled with 90 % of the Herschel-Bulkley fluid, intermediate layer with radius h_2 and another immiscible fluid as plasma in peripheral layer with radius as h. The wall is subjected to periodic peristaltic movement with wave length 'λ'. The flow becomes steady in the reference frame moving in the direction of the wave propagation with speed 'c'. Geometry of the fluid flow is sinusoidal, traveling with amplitude b_1 and b in two layers respectively.

Under the peristaltic action R = H(z) be the instantaneous radius when the fluid is surrounded co axially (R , Z) be the fixed frame and (R' , Z') be the reference frame, then $z' = Z + ct'$, $r' = R$, $w_i' = w_i + c$

Taking Herschel-Bulkley fluid modeled as

$$\tau_i = \mu_i e^{n_i} + \tau_0, \qquad \tau_i \ge \tau_0$$

$$e_i = 0, \qquad \tau_i \le \tau_0 \tag{1}$$

i =1 core layer, i = 2 intermediate layer and i = 3 peripheral layer

μ_1 - Viscosity of core layer,

μ_2 - Viscosity of intermediate layer and

μ_3 - Viscosity of peripheral layer

$$m_r = \mu_r = \mu = \frac{\mu_1}{\mu_2} \text{ in } h_1 \le r \le h \text{ and } m_r = \mu_r = \mu = 1 \text{ in } 0 \le r \le h$$

the corresponding viscosities is given by

$$\mu = \begin{cases} 1 & \text{in } 0 \le r \le h_1 \\ \mu_1 & \text{in } h_1 \le r \le h_2 \\ \mu_2 & \text{in } h_2 \le r \le h \end{cases}$$

Geometry of the tube wall is taken for cylindrical polar coordinates (r, θ, z) to study the problem,

$$H(z) = 1 + \varepsilon \sin\left(\frac{2\pi z}{\lambda}\right), \qquad \varepsilon = \frac{b}{a} \text{, is the amplitude ratio} \tag{2}$$

Using long wave length approximation, neglecting wall slope and inertia forces for steady flow under lubrication theory, the equations of motion is

$$\frac{\partial p'}{\partial z'} = \frac{1}{r'}\frac{\partial}{\partial r'}\left\{ r' \, m_r \frac{\partial w_i'}{\partial r'} \left| \frac{\partial w_i'}{\partial r'} \right|^{n_i - 1} \right\} \quad \textit{and} \quad \frac{\partial p'}{\partial z'} = 0 \tag{3}$$

Non-dimensional quantities are,

$$r = \frac{r'}{a}, \; z = \frac{z'}{\lambda}, \; h_1 = \frac{H_1}{a} \; w_i = \frac{w_i'}{a}, \; m_r = \frac{\mu_2}{\mu_1}, \; p = \frac{p' a^{n_i + 1}}{m_1 \lambda c^{n_i}}$$

Then equation (3) becomes

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left\{ r \, m_r \frac{\partial w_i}{\partial r} \left| \frac{\partial w_i}{\partial r} \right|^{n_i - 1} \right\} \textit{ with } p = \frac{1}{r}\frac{\partial}{\partial r}\left\{ r \, m_r \frac{\partial w_i}{\partial r} \left[\frac{\partial w_i}{\partial r} \right]^{n_i - 1} \right\} \tag{4}$$

Solving equation (4) with initial conditions

$$\frac{\partial w_i}{\partial r} = 0 \text{ at } r = 0 \text{ (at the wall) , } w_2 = -1 \text{ at } r = h \text{ (at the axis)}$$

The conditions at the fluid interface are the continuity of the velocity and the stress across it then we have

$$\left| \frac{\partial w_i}{\partial r} \right| = \left| \frac{Pr}{2m_r} \right|^{\frac{1}{n_i}} \tag{5}$$

Analysis

The peristaltic transport of bio-fluid consisting of three layers (incompressible fluids) of different viscosities occupying core, intermediate and peripheral layers in a circular tube. With the basic assumptions the peristaltic transport of three layered (Herschel-Bulkley fluid in circular tube) in an axisymmetric flow for three different layers (core, intermediate and peripherals)

$0 \le r \le h_1$, $h_1 \le r \le h_2$ and $h_2 \le r \le h_3$

From (4) and (5) and using above conditions we get

$$w_i = -1 - \left| \frac{pr}{2m} \right|^{k_2} \frac{h^{k_2 + 1}}{k_2 + 1} + \left| \frac{pr}{2m} \right|^{k_i} \frac{r^{k_i + 1}}{k_i + 1} \tag{6}$$

For three layers varies from r - h_1 , $h_1 - h_2$ and $h_2 - h$
$mk_i < 1$ corresponding to shear thinning, $mk_i > 1$ corresponding to Newtonian and $mk_i = 1$ corresponding to shear thickening fluids.
For i = 1, the velocity for core layer is given by

$$w_1 = -1 + \left|\frac{p}{2m}\right|^{k_2}\left[\frac{r^{k_1+1} - h_1^{k_1+1}}{k_1+1}\right] + \left|\frac{p}{2m}\right|^{k_2}\left[\frac{h_1^{k_2+1} - h^{k_2+1}}{k_2+1}\right] \tag{6a}$$

For i = 2, the velocity for intermediate layer is given by

$$w_2 = -1 + \left|\frac{pr}{2m}\right|^{k_2}\left[\frac{r^{k_2+1} - h_1^{k_2+1}}{k_2+1}\right] \tag{6b}$$

For i =3 the velocity for the peripheral layer

$$w_3 = -1 + \left|\frac{p}{2m}\right|^{k_3}\left[\frac{r^{k_3+1} - h_1^{k_3+1}}{k_3+1}\right] + \left|\frac{p}{2m}\right|^{k_2}\left[\frac{h_1^{k_2+1} - h^{k_2+1}}{k_2+1}\right] \tag{6c}$$

The instantaneous volume flow rate for three layers is given by

$$q = q_1 + q_2 + q_3$$

$$q = 2\int_0^{h_1} r\, w_1\, dr + 2\int_{h_1}^{h_2} r\, w_2\, dr + 2\int_{h_2}^{h_3} r\, w_3\, dr$$

$$q = -\left|\frac{p}{2}\right|^{k_1}\frac{h_1^{k_1+3}}{k_1+3} + \left|\frac{p}{2m}\right|^{k_3}\left[\frac{(k_3+3)h_1^{k_3+1}\left(h_2^{\,2} - h^2\right) + 2\left(h^{k_3+3} - h_2^{\,k_3+3}\right)}{(k_3+1)(k_3+3)}\right]$$

$$+ \left|\frac{p}{2m}\right|^{k_2}\left[\begin{array}{c}\dfrac{h_1^{k_2+3} - h_1^{\,2}h^{k_2+1}}{k_2+1} + \dfrac{h_1^{k_2+1} - h^{k_2+1}}{(k_2+1)}\left(h^2 - h_2^{\,2}\right)\\ \dfrac{(k_3+3)h^{k_2+1}\left(h_1^{\,2} - h_2^{\,2}\right) + 2\left(h_2^{\,k_3+3} - h^{k_2+3}\right)}{(k_3+1)(k_2+3)}\end{array}\right] - h^2 \tag{7}$$

Dimensionless average time flux in terms of flow rate is obtained as,

$$Q = q + 1 + \frac{\varepsilon^2}{2}$$

Where ε - the amplitude ratio.

The prescription of these values $\overline{Q}$ serves as boundary conditions at the ends of the tube

$$\overline{Q} = -\left|\frac{pr}{2m}\right|^{k_1}\frac{h_1^{mk_1+3}}{(mk_2+3)} - \left|\frac{pr}{2m}\right|^{k_2}\left[\frac{h^{mk_2+3} - h_1^{mk_2+3}}{mk_2+1}\right] \tag{8}$$

The conservation of mass across the interface at the every axial station is modeled for computing two stream functions ψ_1, ψ_2, and ψ_3 respectively can be obtained using the conditions $\Psi_1 = 0$ at r = 0 ,$\Psi_2 = q$ at r = h and $\psi_3 = q$, r = 0 and $w_i = \dfrac{-1}{r}\dfrac{\partial \psi_i}{\partial r}$, $i = 1, 2, 3$

$$\psi_1 = \frac{r^2}{2}\left[1- \left|\frac{pr}{2m}\right|^{k_1}\left\{\frac{2r^{k_1+1}-(k_1+3)h_1^{k_1+1}}{(k_1+3)(k_1+1)}\right\}-\left|\frac{pr}{2m}\right|^{k_2}\left\{\frac{h_1^{k_2+1}-h^{k_2+1}}{k_2+1}\right\}\right] \tag{9a}$$

$$\psi_2=\left(\frac{2q+r^2-h^2}{2}\right)-\left|\frac{pr}{2m}\right|^{k_2}\left\{\frac{2r^{k_2+3}-(k_2+3)h^{k_2+1}+(k_2+1)h^{k_2+3}}{2(k_2+3)(k_2+1)}\right\} \tag{9b}$$

$$\psi_3 = \frac{r^2}{2}+\left|\frac{pr}{2m}\right|^{k_3}\left[\frac{2r^{k_3+3}-(k_3+3)h_1^{k_3+1}r^2}{(k_3+3)(k_3+1)}\right]+\left|\frac{pr}{2m}\right|^{k_2}\left[\frac{h_1^{k_2+1}-h^{k_2+1}}{k_2+1}\right]\frac{r^2}{2} \tag{9c}$$

The equation for interface in terms of stream function for a single layer fluid model (m = 1)

$$Q = q + h^2 \Rightarrow \overline{Q_1} = q_1 + h^2, \frac{Q_1}{2} = \frac{q_1+h^2}{2}, \ \psi i = \frac{q_i}{2}, i = 1,2,3$$

At $r = h_1$

$$\left(\frac{2q+r^2-h^2}{2}\right)-\left|\frac{pr}{2m}\right|^{k_2}\left\{\frac{2r^{k_2+3}-(k_2+3)h^{k_2+1}r^2+(k_2+1)h^{k_2+3}}{2(k_2+3)(k_2+1)}\right\}$$

$$= \frac{-h_1^2}{2}-\left|\frac{pr}{2m}\right|^{k_1}\frac{h_1^{k_1+3}(k_2+1)}{2(k_2+3)(k_1+1)}+\left|\frac{pr}{2m}\right|^{k_2}\left\{\frac{h_1^{k_2+3}-h^{k_2+1}h_1^2}{2(k_2+1)}\right\}$$

$$Q = q + h^2 \Rightarrow \overline{Q_1} = q_1 + h_1^2 \Rightarrow \frac{q+h^2}{2} = \frac{Q}{2}$$

$$\frac{Q_1}{2} = \frac{Q}{2} - \left|\frac{pr}{2m}\right|^{k_2}\left\{\frac{2h_1^{k_2+3}-(k_2+3)h^{k_2+1}h_1^2+(k_2+1)h^{k_2+3}}{2(k_2+3)(k_2+1)}\right\} \tag{10}$$

For, $\psi_2 = \frac{q_1}{2}+\frac{h^2{}_1}{2}=\frac{Q_1}{2}$ at $r = h_1$ We obtain the equation governing the interface as,

$$\frac{Q_1}{2} = \frac{\left|\frac{pr}{2m}\right|^{k_2} h_1^{k_1+3}}{2(k_2+3)}+\left|\frac{pr}{2m}\right|^{k_2}\left\{\frac{h_1^{k_2+3}-h^{k_2+1}h_1^2}{2(k_2+1)}\right\}$$

To eliminate $\frac{pr}{2m}$ from the previous equations $\frac{Q_1}{2}$ and $\frac{q_1}{2}$, we take $\frac{pr}{2m}$ as common as

$$\left|\frac{pr}{2m}\right|^{k_1} = \left|\frac{pr}{2m}\right|^{k_2} = \left|\frac{pr}{2m}\right|^{k_2}$$

On simplifying and extract $\frac{pr}{2m}$ as

$$\frac{pr}{2m} = \frac{(Q_1 - Q_2)^{1/k_2}[(k_2+3)(k_2+1)]^{1/k_2}}{[2h_1^{mk_2+3} - (k_2+3)h^{k_2+1}h_1^2 + (k_2+1)h^{k_2+3}]^{1/k_2}} \tag{11}$$

Substituting the value of $\frac{pr}{2m}$ in equation (7)

we get

$$(Q - Q_1)\left[\frac{-(Q_1 - Q_2)^{\frac{k1}{k_2}-1}[(k_2+3)(k_2+1)]^{\frac{k1}{k_2}} h_1^{k_1+3}}{[2h_1^{k_2+3} - (k_2+3)h^{k_2+1}h_1^2 + (k_2+1)h^{k_2+3}]^{\frac{k1}{k_2}}(k_2+1)} + \frac{(k_2+3)[h_1^{k_2+3} - h^{k_2+1}h^2]}{2h_1^{k_2+3} - (k_2+3)h^{k_2+1}h_1^2 + (k_2+1)h^{k_2+3}}\right] - Q_1 = 0 \tag{12}$$

The axial pressure gradient $\frac{\partial p}{\partial z} = p$, Put $k_1 = k_2 = k$ in the equation (10) we get

$$\frac{pr}{2m} = \frac{(q + h^2)^{1/k}(mk+1)^{1/k}}{(h^{mk+3})^{1/k}} \tag{13}$$

For stream functions at $k_1 = k_2 = k$ we get,

$$\psi_1 = \frac{r^2}{2} + \frac{q + h^2}{h^{k+3}(k+1)}\left[r^{k+3} - (k+3)h_1^{k+1}r^2 + (k+3)h^{k+1}\frac{r^2}{2}\right] \tag{14a}$$

$$\psi_2 = \left(\frac{2q + r^2 - h^2}{2}\right) + \frac{q + h^2}{2h^{k+3}(k+1)}\left[2r^{k+3} - (k+3)h^{k+1}r^2 + (k+1)h^{k+3}\right] \tag{14b}$$

$$\psi_3 = \frac{r^2}{2} + \left|\frac{pr}{2m}\right|^k\left[\frac{2r^{k+3} - (k+3)h_1^{k+1}r^2}{(k+3)(k+1)}\right] + \left|\frac{pr}{2m}\right|^k\left[\frac{h_1^{k+1} - h^{k+1}}{k+1}\right]\frac{r^2}{2} \tag{14c}$$

at $h = h_1$, $(\psi_1)_{r=h_1} = (\psi_2)_{r=h_1} = (\psi_3)_{r=h_1}$ Equation (12) gives

$$\frac{Q_1}{2} = \frac{Q}{2} - |C_2|^{k_2}\left\{\frac{2h_1^{mk_2+3} - (mk_2+3)h^{mk_2+1}h_1^2 + (mk_2+1)h^{mk_2+3}}{2(mk_2+3)(mk_2+1)}\right\}$$

$$\text{where } C_2 = \frac{pr}{2m} = \frac{(q + h^2)^{1/k}(mk+1)^{1/k}}{(h^{mk+3})^{1/k}}$$

Taking $k_2 = k$, the above equation reduces to $q_1 + h_1^2 = 0 = L(h_1)$

$$\Rightarrow \quad \left(q+h^2\right)\left\{1-\frac{2h_1^{mk+3}-\left(mk+3\right)h^{mk+1}\,h_1^{\,2}+\left(mk+1\right)h^{mk+3}}{h^{mk+3}\left(mk+1\right)}\right\}$$

$$\left[q_1\left\{h^{mk+3}(mk+1)\right\}-(q+h^2)\left\{h^{mk+3}(mk+1)+2h_1^{\,mk+3}-h^{mk+1}(mk+3)h_1^{\,2}+h^{mk+3}(mk+1)\right\}\right]=0 \quad (15)$$

For $h_1 = 0$

$$L(0) = h^{mk+3}(mk+1)[q_1 - q - h^2] - (q+h^2)(mk+1)h^{mk+3} \quad (16)$$

Equation of the interface is obtained for a particular case at $h_1 = \alpha$ and $\frac{1}{n_1} = \frac{1}{n_2} = \frac{1}{n_3}$ with the condition that the flow rate in core layer is twice the stream function ($r = h_1$) substituting equation (12) in Q_1 and further $Q_1 = q_1 + h^2$ gives

$$Q_1 = 2\left\{\frac{\alpha^5 - \alpha^3\left(h^2 + 7\overline{Q}\right) - \alpha(h^4 + h^2) - 16\overline{Q}\,\alpha^2}{-13\alpha^4 - 3\alpha^2 h^2 - 23\alpha^2}\right\} \quad (17)$$

For a given flow rate, equation (15) is reduced to 5th degree polynomial in ‘α’. By applying Newton-Raphson method (with ten iterations in each step to obtain the set of values for α). Series of values of α represent interface. The initialization is made with α = 0.3 chosen as sufficiently close to the root in comparison with H(z). The pressure gradient under the assumptions of (20) with $\frac{1}{n_1} = \frac{1}{n_2} = \frac{1}{n_3} = n$ is obtained by separating P from equation (11) and for single layer model.$m_r = 1$ and n =1.

The expressions for pressure difference Δp studied between the extreme locations of each wavelength as,

$$\Delta p = \frac{-8q[2+3\varepsilon^2]}{[1-\varepsilon^2]^{7/2}} - \frac{8}{[1-\varepsilon^2]^{3/2}} \quad (18)$$

Friction force at the wall is given by,

$$F = \frac{8q}{[1-\varepsilon^2]^{3/8}} + 8$$

$$F = \int_0^\lambda h^2\left(\frac{\partial p}{\partial z}\right) dz$$

$$F = \frac{8}{[1-\varepsilon^2]^{3/8}} - h^2 - \left[\frac{p}{2m_r}\right]^{1/n_1}\left[\frac{h_1^{\frac{1}{n_1}+3}}{\frac{1}{n_1}+3}\right] - \left[\frac{p}{2m_r}\right]^{1/n_2}\left[\frac{h_1^2 h^{\frac{1}{n_2}+3} - h_1^{\frac{1}{n_2}+3}}{\frac{1}{n_2}+1}\right]$$
$$+\left[\frac{p}{2m_r}\right]^{1/n_2}\left[\frac{-h^{\frac{1}{n_2}+3}}{\frac{1}{n_2}+3} + \frac{\left(\frac{1}{n_2}+3\right)h_1^2 h^{\frac{1}{n_1}+1} - 2h_1^{\frac{1}{n_2}+3}}{\left(\frac{1}{n_2}+1\right)\left(\frac{1}{n_2}+3\right)}\right] \tag{19}$$

Analyzing the pumping characteristics for the two layered model, it can be seen that, for $\varepsilon = 0$ from equations (17) and (18) there is no peristalsis comparing with equation H (z). Peak peristalsis appears at $\varepsilon = 0.55$ and onwards and full occlusion occur for $\varepsilon = 0.85$ and onwards.

From equation (19) when friction force starts decreasing at the wall against $\Delta P = 0, 0.005, 0.05$ the enhanced positive mean flow is noticed with the model fluid.

From finite range of flux $\overline{Q_L}$ the limitation of the flux $\overline{Q_L}$ is calculated as

$$\overline{Q_L} = \frac{\varepsilon^2}{2} + \left[\frac{\alpha^{k+3} - (k+1) + 2(k+3)\alpha^{k+2} - (k+3)\alpha^{k+2} - (k+3)\alpha^{k+3}}{4\alpha^{k+1} - 2(k+3)\alpha^k + (k+3)\alpha^{k+1} - (\alpha+3)}\right] \tag{20}$$

Best fit for $\overline{Q_L}$ is approximated by computing α above the surface of the core layer $\overline{Q_L}$ increases as ε varies from 0.3 to 0.8. Use of special non-Newtonian (shear thickening) fluid enhances the positive pumping, so that the transport of the fluid for the range of ε : 0.2 – 0.8 is found to be more effective. Absolute resistance of flow R_f is observed in the direction of the sinusoidal wave. The ratio of average pressure rise and the time averaged flux is determined as R_f.

$$R_f = \left[\frac{-8q[2+3\varepsilon^2]}{[1-\varepsilon^2]^{7/2}\left(q+1+\frac{\varepsilon^2}{2}\right)} - \frac{8}{[1-\varepsilon^2]^{3/2}\left(q+1+\frac{\varepsilon^2}{2}\right)}\right] \tag{21}$$

Various values of R_f are obtained at ε : 0. 28 to 0.75

Results and Discussions

Consistency of the peristaltic flow is obtained with $m_r = 0.02$, $\varepsilon = 0.2, 0.4, 0.6, 0.8$.Flow consistency changes when the amplitude b exceeds the range of ε. Then the present model is to be further modified which approximates the limitations of the flux within the range $\varepsilon = 0.3$ to 0.8. Therefore the flow is motivated in the specified range of amplitude ratio. Streamline movements corresponding to particle lines can be visualized from the experimental work compared with experimental values. The bolus of the fluid forms around the axis above and below the axis as small fluid blocks separated from streamline location. Bolus so formed at this point constitutes the trapping zone for sinusoidal wall profile. Numbers of trials have been observed, the stagnation points are located at this trapping zone. Beyond the trapping range $\varepsilon = 0$ to 0.05, dilation occurs. This checks the axial velocity expected to be zero or very minimum near the axis. But on the axis, velocity is zero. Also up to certain height, where the trapped bolus occurs, there is no split of the streamline, but after the bolus ends; we can visualize the split in the streamline so that the flow dilates in that region. Reflux (backflow) starts from the trapped bolus moves to some distance in the negative axial direction, but due to the successive pumping, the net-trapped bolus moves towards the positive axial direction. This phenomenon is verified by equation the (17) at $\frac{\partial p}{\partial z} = p$ as positive pressure and $\frac{\partial p}{\partial z} = p$ as adverse pressure. Model proposed confirms there is no back flow as the figure indicates the occurrence of positive pumping with shear thickening and shear thinning nature of the fluid. For further study, we propose that the time dependent flow by considering the flow of blood like fluids for flow visualization.

It has been shown that the flow flux increases and friction force decreases as the thickness of the peripheral layer increases or its fluid viscosity decreases. Even for a small value of pressure drop, the increase in the flow or decrease in the friction force is considerable for smaller peripheral layer viscosity in comparison to their respective values when the peripheral layer viscosity is equal to the central layer viscosity.

Conclusion

The three layered model for viscous incompressible fluid considering the core, intermediate and peripheral layer, stream line movements corresponding to practical lines can be visualized from the experimental values and the three layered model proposes that there is no back flow as it indicates the positive pumping exists with shear thinning and shear thickening nature of the fluid.

The viscosity of core layer may be of the same order of magnitude as that of faces which varies $10^3 \sim 10^6$ cP. For zero pressure drop, it has been noted that the flux does not depend upon the peripheral layer viscosity but the force of friction decreases considerably with its decrease.

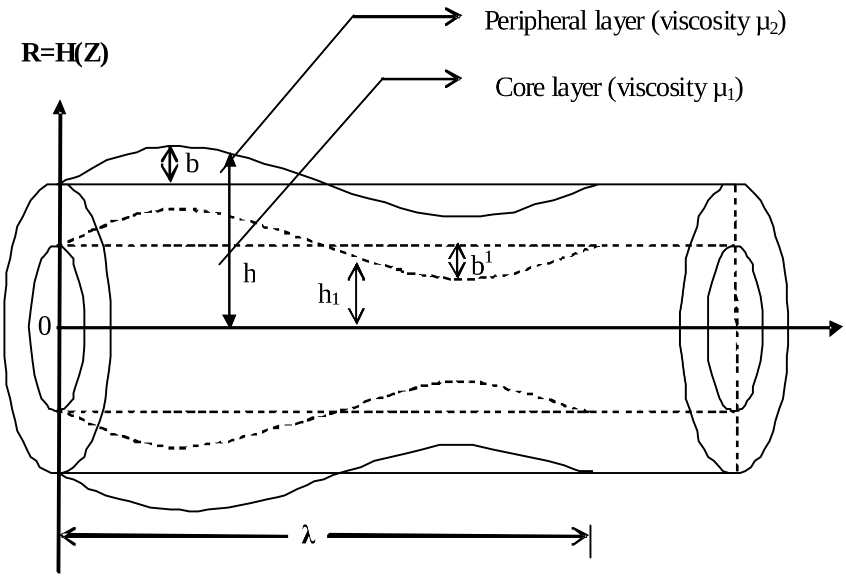

Figure 1: Geometry of two layered Peristaltic Transport

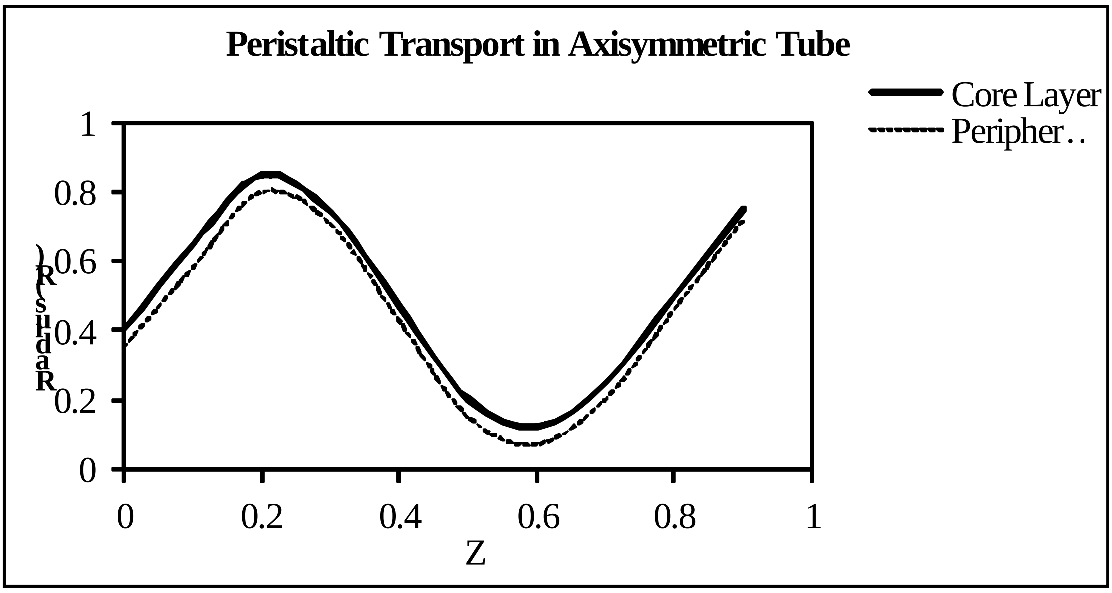

Figure 2: Peristaltic transport in axi-symmetric tube

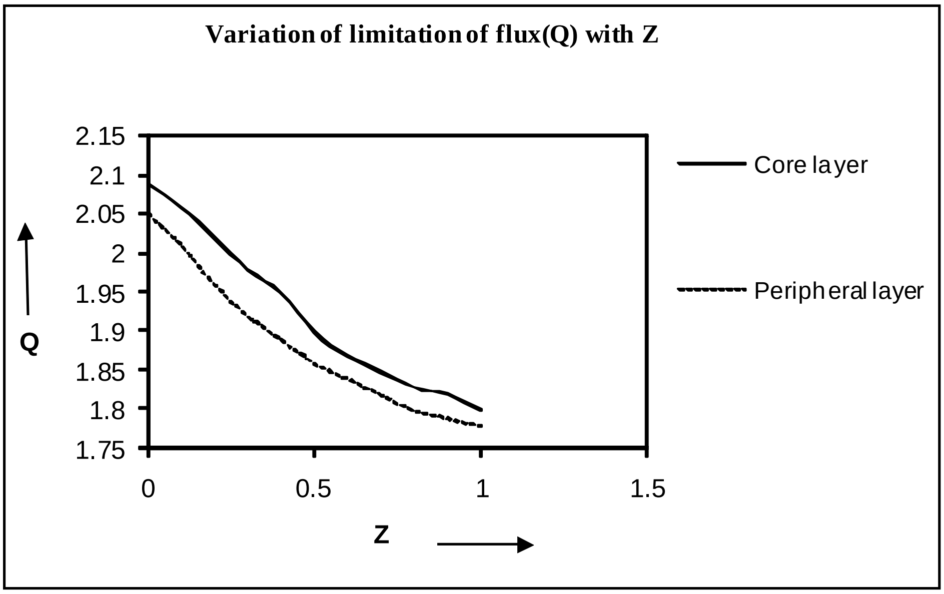

Figure 3: Variation of flux Q with Z

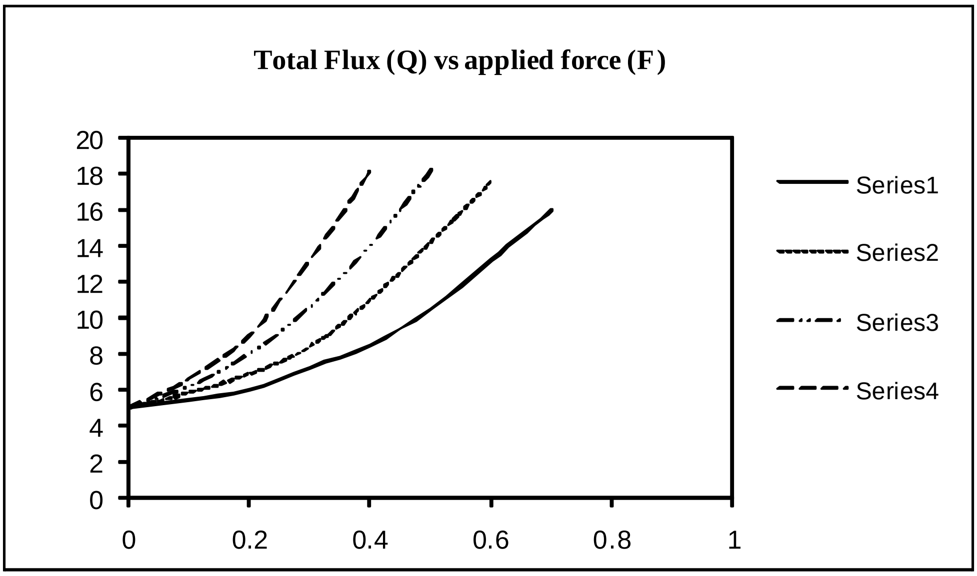

Figure 4: Total flux Q Vs applied force F

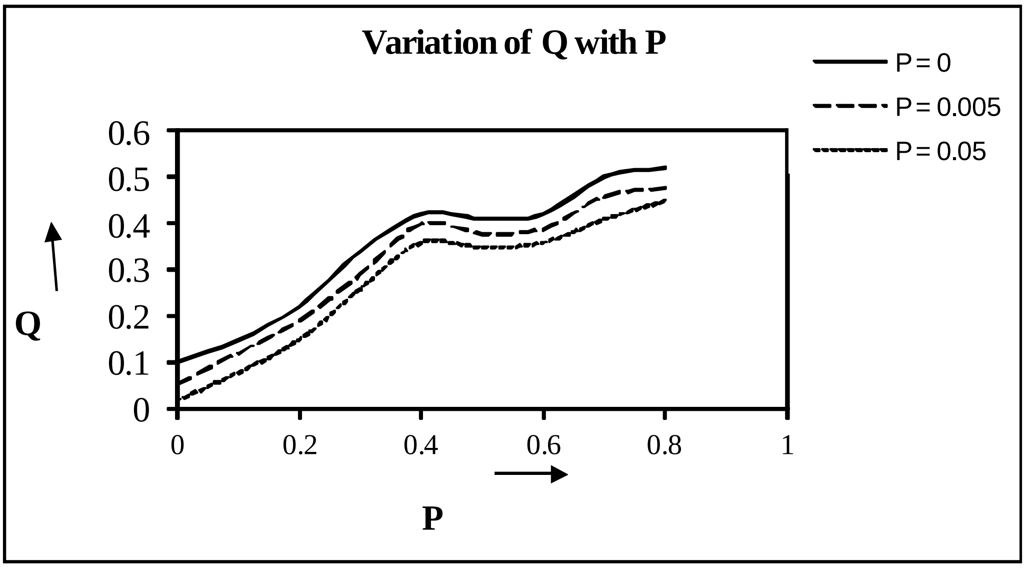

Figure 5: Variation of Q with P

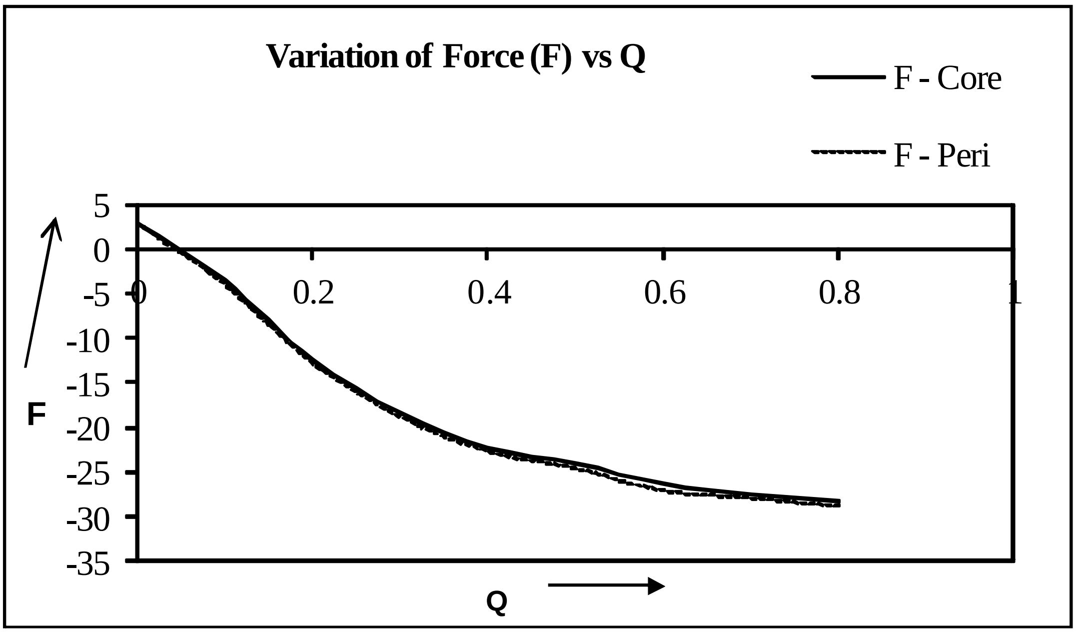

Figure 6: Variation of forces F Vs Q

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