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Profit Optimization in Insurance Business Facing Customer Impatience

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Abstract

Insurance sector is crowded with a number of insurance players. Due to increasing awareness the customers' expectations are very high. Based on the quality of service, and expected returns the customers may be attracted towards other service providers. As a result customers get impatient and may discontinue their policies with a particular firm. This phenomenon is quite harmful to the growth and sustainability of any insurance firm. Customer impatience has become a key issue amongst insurance companies. Firms employ a number of customer retention strategies to sustain their business. In this paper, a stochastic queuing model is proposed to study the retention of impatient customers when any customer retention mechanism is applied. It is envisaged that if the firms employ some customer retention mechanism, then there are chances that a certain proportion of impatient customers may be retained. The cost profit analysis of the model is performed. The optimization of the model is also carried out to obtain the optimum service rate, optimum total cost, the optimum total revenue and the optimum total profit of the system.

The results of the model may be highly useful for any insurance firm to design its service system for optimizing its profit and to tackle the problem of customer impatience. The analysis carried out in this paper is very useful to any insurance firm, as it helps to choose the best customer retention strategy.

Introduction

Insurance sector is crowded with a number of insurance players. Due to increasing awareness the customers' expectations are very high. Based on the quality of service, and expected returns the customers may be attracted towards other service providers. As a result customers get impatient and may discontinue their policies with a particular firm. This phenomenon is quiet harmful to the growth and sustainability of any insurance firm. Customer impatience has become a key issue amongst insurance companies. Firms employ a number of customer retention strategies to sustain their business. [Vestrov et al., 2003] present a model for stochastic modeling of insurance business with dynamical control of investments. Many other authors have applied mathematical models for managing the risk and other factors in insurance business [Shapiro, 1998] and [Kutub et. al., 2011].

Cost analysis of queuing theory occupies a prominent place in the research of queuing theory. Optimization techniques are widely used in the areas of production, manufacturing and planning including the communication systems to effectively assess the performance of systems. Nowadays, a trend has been redirected and shifted to investigate more realistic performance measures of the system as compared to general theoretical approach that embodies hardly a bit of application. [Taha, 1997] discusses the queuing decision models namely a cost model and an aspiration level model recognizing that higher service levels reduce the waiting time in the system. [Ke and Wang, 1999] analyze the cost of analysis of the M/M/R machine repair problem with balking, reneging and server breakdowns. Cost analysis of finite M/M/R queuing system with balking reneging and server breakdown is discussed as in [Wang and Chang, 2002]. [Mishra and Mishra, 2004] perform the cost analysis of machine interference model with balking reneging and spares.

The notion of customer impatience appears in queuing theory in the work of [Haight, 1957]. He studies M/M/1 queue with balking in which there is a greatest queue length at which an arrival will not balk. [Haight, 1959] studies queuing with reneging. [Ancker and Gafarian, 1963a] studies M/M/1/N queuing system with balking and reneging and derive its steady state solution. [Ancker and Gafarian, 1963b] obtain results for a pure balking system (no reneging) by setting the reneging parameter equal to zero. The phenomenon of customer impatience in single-server queues is discussed in the works of [Gavish and Schweitzer 1977], [Robert, 1979], [Baccelli et al. 1984], and [Wu et al., 2005]. [Choudhury and Medhi, 2010] study customer impatience in multi-server queues. They consider both balking and reneging as functions of system state by taking into consideration the situations where the customer is aware of its position in the system. [Kapodistria, 2011] studies a single server Markovian queue with impatient customers and considers the situations where customers abandon the system simultaneously. He considers two abandonment scenarios. In the first one, all present customers become impatient and perform synchronized abandonments, while in the second scenario; the customer in service is excluded from the abandonment procedure. He extends this analysis to the M/M/c queue under the second abandonment scenario also. [Kumar, 2012] investigates a correlated queuing problem with catastrophic and restorative effects with impatient customers which have special applications in agile broadband communication networks. Recently [Kumar and Sharma, 2012] study M/M/I/N queuing system with retention of reneged customers. They obtain the steady state solution of the model and study the effect of probability of customer retention on the expected system size. In this paper we extend the work of [Kumar and Sharma, 2012] by performing the optimization of the model. The application of this model in insurance sector is described and the optimal values of total expected profit are discussed with respect to the different customer retention strategies.

Rest of the paper is arranged as follows. Section 2, the model is described is done. Section 3 describes the optimization of the model. In section 4, the conclusions are presented.

Description of the model:

In this study, we propose a single server finite capacity Markovian queuing model with retention of impatient (reneged) customers for any insurance firm facing the problem of customer impatience and implementing different customer retention strategies. The customers arrive to the system in a Poisson fashion with mean rate λ. An arrival to the system represents the sale of one insurance policy. There is a single server and the customers are served in order of their arrival. The service time distribution is exponential with parameter μ. Here the service of a customer represents the claim processing at the maturity of the policy. The insured customers at any stage before maturity may get impatient due to various reasons like dissatisfaction of service, immediate requirement of money, better opportunities with the other insurers etc. and withdraw them. This phenomenon is analogous to reneging in case of queuing theory. The customers get impatient (reneged) following exponential distribution with parameter ξ. As the customer impatience has highly negative impact on the business the firms, they employ different customer retention strategies to retain their customers. It is envisaged that if the firms employ certain customer retention strategies, then there are chances that a certain proportion of impatient customers may be retained. Thus, an impatient customer may be retained in the system with some probability q (say) and he may not be convinced to stay in the system for his complete service with probability 1 – q (= p). The customer retention strategies in insurance business can be the convincing of customers by phone, better and reliable service, providing better returns, discounts on premium etc.

Here, we consider a single server, finite capacity Markovian queuing model as studied by [Kumar and Sharma, 2012] whose steady state probabilities are given by:

$$P_n = \prod_{k=1}^{n} \frac{\lambda}{\mu+(k-1)\xi p} P_0 \;; 1 \leq n \leq N-1 \qquad (1)$$

Also for $n = N$ we get

$$P_N = \prod_{k=1}^{N} \frac{\lambda}{\mu+(k-1)\xi p} P_0 \tag{2}$$

Using the normalization condition, $\sum_{n=0}^{N} P_n = 1$, we get

$$P_0 = \frac{1}{\left(1+\sum_{n=1}^{N} \prod_{k=1}^{n} \frac{\lambda}{\mu+(k-1)\xi p}\right)} \tag{3}$$

and the expected number of customers in the system is:

$$L_s = \sum_{n=1}^{N} n \left(\prod_{k=1}^{n} \frac{\lambda}{\mu+(k-1)\xi p}\right) P_0 \tag{4}$$

Optimization of the Model:

In this section the optimization of the model is performed. We obtain the optimum value of the service rate at which the total expected cost of the system is minimum. We study the variation in total optimum profit w.r.t. the probability of customer retention associated with a particular customer retention strategy. The total optimum cost and total optimum revenue are also computed. The numerical results are presented for cost – profit analysis of the model. Following notations are used to perform the cost – profit analysis.

Notations:

$\frac{1}{\lambda}$ = mean inter arrival time.

$\frac{1}{\mu}$ = mean service time.

P_N = probability that the system is full.

L_s = expected number of customers in the system.

R_r = average rate of reneging.

R_R = average rate of Retention.

TEC = total expected cost.

C_s = cost per service per unit time.

C_h = holding cost per unit per unit time.

C_L = cost associated to each lost unit per unit time.

C_r = cost associated to each reneged unit per unit time.

C_R = cost of retaining a customer per unit time.

R = earned revenue by providing service to each customer per unit time.

TER = total expected revenue of the system.

TEP = total expected profit of the system.

Now, we present the cost – profit analysis and perform the optimization of the model under investigation.

We define the total expected cost (TEC) of the system as

$$TEC = C_s\mu + C_h L_s + C_L\lambda P_N + C_r R_r + C_R R_R$$

$$TEC = C_s\mu + C_h \sum_{n=1}^{N} n\left(\prod_{k=1}^{n} \frac{\lambda}{\mu + (k-1)\xi p}\right) P_0 + C_L\lambda \prod_{k=1}^{N} \frac{\lambda}{\mu + (k-1)\xi p} P_0$$
$$+ C_r \left[\sum_{n=1}^{N} (n-1)\xi p \prod_{k=1}^{n} \frac{\lambda}{\mu + (k-1)\xi p}\right] P_0$$
$$+ C_R \left[\sum_{n=1}^{N} (n-1)\xi q \prod_{k=1}^{n} \frac{\lambda}{\mu + (k-1)\xi p}\right] P_0 \quad (5)$$

Where the average reneging rate R_r and the average retention rate R_R are given by:

$R_r = \sum_{n=1}^{N}(n-1)\xi p P_n$ and $R_R = \sum_{n=1}^{N}(n-1)\xi q P_n$

Let R be the earned revenue for providing service to each customer per unit time, then RL_s is the total earned revenue for providing service to average number of customers in the system. $R\lambda P_N$ and RR_r are the losses in the revenue of the system due to number of lost customers per unit time and due to reneging of customers. Hence, total expected revenue (TER) of the system is given by

$$TER = RL_s - R\lambda P_N - RR_r$$

$$TER = R \sum_{n=1}^{N} n\left(\prod_{k=1}^{n} \frac{\lambda}{\mu + (k-1)\xi p}\right) P_0 - R\lambda \prod_{k=1}^{N} \frac{\lambda}{\mu + (k-1)\xi p} P_0$$
$$- R \left[\sum_{n=1}^{N} (n-1)\xi q \prod_{k=1}^{n} \frac{\lambda}{\mu + (k-1)\xi p}\right] P_0 \quad \ldots (6)$$

Now, total expected profit (TEP) of the system is defined as:

$$TEP = TER - TEC$$

$$TEP = (R - C_h)\sum_{n=1}^{N} n\left(\prod_{k=1}^{n}\frac{\lambda}{\mu + (k-1)\xi p}\right)P_0$$
$$- \ (R + C_L)\lambda \ \prod_{k=1}^{N}\frac{\lambda}{\mu + (k-1)\xi p}P_0$$
$$- \ (R + C_r)\left[\sum_{n=1}^{N}(n-1)\xi q\prod_{k=1}^{n}\frac{\lambda}{\mu + (k-1)\xi p}\right]P_0 - \ C_s\mu$$
$$- C_R\left[\sum_{n=1}^{N}(n-1)\xi q\prod_{k=1}^{n}\frac{\lambda}{\mu + (k-1)\xi p}\right]P_0 \qquad \dots (7)$$

Thus, we have the TEC, TER and TEP functions in terms of various parameters involved. The cost – profit analysis of the model is performed numerically by using these functions and the results are discussed accordingly. The optimization of the model is also carried out in order to obtain the optimal service rate and to obtain the optimum TEC, TER and TEP. The impact of various customer retention strategies on the total optimal profit is also analyzed. It becomes quiet intractable to proceed analytically for optimum value of μ at which the TEC is minimum. Therefore, we have used MATLAB software to get the optimum values using a computational algorithm.

Computational Algorithm:

Step 1: Define variables
Step 2: Write the formula of function TEC in terms of μ
Step 3: Obtain critical values for TEC
Step 4: Find the value of μ at which TEC is minimum (let it be μ*)
Step 5: Compute the values of TEC, TER and TEP at μ*

Table -1 Variation in TEC, TER and TEP with the change in probability of customer retention, q We take $\lambda = 3$, $\mu = 4$, $\xi = 0.15$, $N = 4$, $C_s = 4$, $C_h=3$, $C_L=12$, $C_r=8$, and R=100.

(q)	(C_R)	Total Expected Return (TER)	Total Expected Cost (TEC)	Total Expected Profit (TEP)
0	0	88.9785	24.9273	64.0512
0.1	6	91.3261	24.9676	66.3585
0.2	8	93.6923	25.0948	68.5975
0.3	12	96.0775	25.439	70.6385
0.4	14	98.4818	25.7871	72.6947
0.5	20	100.9053	26.6592	74.2461

0.6	25	103.3482	27.6702	75.678
0.7	32	105.8105	29.2151	76.5954
0.8	36	108.2926	30.5546	77.738
0.9	40	110.7944	32.0827	78.7117
1	45	113.3163	34.023	79.2933

Table-1 presents the cost-profit analysis of the model and the effect of customer retention strategies on the total expected profit. It is obvious that a better customer retention strategy involves higher retention cost than the others. Therefore, in the third column of the table we have associated different customer retention costs to different strategies. It is observed that as the probability of customer retention associated with a particular strategy increases the total expected profit of the system also increases. When the probability of customer retention is zero, the total expected profit is minimum. This is the case of no customer retention. When q = 1 the total expected profit is maximum. In this case all the impatient customers are retained. Thus, one may have a snapshot of the long run performance of different customer retention strategies on the business.

Table – 2 Variation in TOC, TOR and TOP with the change in probability of customer retention, q We take $\lambda = 3$, $\xi = 0.15$, $N = 4$, $C_s = 4$, $C_h=3$, $C_L=12$, $C_r=8$, and R=100.

(q)	(C_R)	(μ*)	Total Optimum Revenue (TOR)	Total Optimum Cost (TOC)	Total Optimum Profit (TOP)
0	0	3.771	95.2737	24.7395	70.5342
0.1	6	3.795	96.9132	24.8070	72.1062
0.2	8	3.8276	98.3449	24.9631	73.3818
0.3	12	3.8809	99.2513	25.3453	73.9060
0.4	14	3.9337	100.2267	25.7335	74.4932
0.5	20	3.974	101.5823	26.6365	74.9458
0.6	25	4.1428	99.7291	27.8199	71.9092
0.7	32	4.2921	98.5837	29.5772	69.0065
0.8	36	4.4176	98.1704	31.1291	67.0413
0.9	40	4.5518	97.6984	32.9186	64.7798
1	45	4.7089	96.8802	35.2097	61.6705

We have performed the optimization of the model under investigation to determine the optimum service rate and the optimal values of total expected revenue, total expected cost and total expected profit. A computational algorithm translated in MATLAB is used to obtain the optimum values. In table – 1 simple cost profit analysis of the model is presented, while in table – 2 the optimum results are described. As the probability of customer retention increases, the optimum service rate also increases this may be attributed to the fact that the increase in probability of customer retention leads to the increase in the number of customers in queue and therefore higher service rate is needed to serve more and more customers for revenue generation.

The total optimum cost of the system increases with increase in probability of customer retention. For $q = 0$, i.e. in the case of no customer retention the total optimum cost is higher than the total optimum cost for $q = 0.1$ to $q = 1.0$. The reason for this is that for $q = 0$, no customer retention is used. The total optimum profit of the system increases as the probability of customer retention increases up to $q = 0.5$, but the total optimum profit is maximum for $q = 0.5$ and then it decreases slowly from $q = 0.5$ to $q = 1$. From table – 2 one can conclude that the strategy corresponding to $q = 0.5$ is the best one as the corresponding total optimum profit is maximum in this case.

The results presented in table – 2 may be of great importance to insurance firms facing customer impatience and implementing various customer retention strategies. As the results are obtained in steady state, the inferences can be drawn for long run performance of the system.

Conclusions:

A queuing theory approach is used to model insurance business problems. A single server finite capacity Markovian queuing model with retention of impatient (reneged) customers is proposed for any insurance firm facing the problem of customer impatience and implementing different customer retention strategies.

The cost – profit analysis of the model is performed and the impact of various customer retention strategies on the total expected profit of the system is studied.

The optimization of the model is performed to minimize the total expected cost of the system by obtaining the optimum value of service rate. It is found that the total optimum profit of the system increases with increase in probability of customer retention. The analysis carried out in this paper is very useful to any insurance firm, as it helps to choose the best customer retention strategy.

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