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Quantum Algorithm for Modified Particle Size Problem by Central Limit Theorem

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Abstract

A quantum algorithm for a modified particle size problem by the central limit theorem and its example are reported. When a random variable Y_i [$1 \leq i \leq n$. i and n are integers.] becomes m_s [$1 \leq s \leq k$. s and k are integers. $m_1 < m_2 < \cdots < m_k$.] as each probability $1/k$, and a start size [M_0] becomes a final size [$M_n = M_0 Y_1 Y_2 \cdots Y_n$], one example in orders that reach at M_n is obtained. A computational complexity of a classical computation is k^n. The computational complexity becomes about $3(log_2 k)n$ by the quantum algorithm that uses quantum phase inversion gates, quantum inversion about mean gates, the log normal distribution and the standard normal distribution. Therefore, a polynomial time process becomes possible.

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1. Introduction

A quantum computation was started by Deutsch and Jozsa [1–3] who discovered the quantum algorithm of a high-speed process by a parallel computation that uses quantum entangled states. After that, Shor [2–4] found the method of solving the factoring in a polynomial time, and Grover [2,5,6] showed the algorithm for the databace search in a square root time. A quantum algorithm for the vertex coloring problem by the central limit theorem has recently been reported by Fujimura [7]. Its computational complexity becomes a polynomial time. In the particle size problem [8], its modified problem that is set up is examined this time. Therefore, its result is reported.

2. Modified Particle Size Problem

When a random variable Y_i [$1 \leq i \leq n$. i and n are integers.] becomes m_s [$1 \leq s \leq k$. s and k are integers. $m_1 < m_2 < \cdots < m_k$.] as each probability $1/k$, and a start size [M_0] becomes a final size [$M_n = M_0 Y_1 Y_2 \cdots Y_n$], one example in orders that reach at M_n is searched.

3. Quantum Algorithm

It is assumed that Y_i [$1 \leq i \leq n$. i and n are integers.] becomes m_s [$1 \leq s \leq k$. s and k are integers. $m_1 < m_2 < \cdots < m_k$.] as each probability $1/k$, a start size [M_0] becomes a final size [$M_n = M_0 Y_1 Y_2 \cdots Y_n$], the minimum value of m_s is m_{min}, the maximum value of m_s is m_{max}, and $Y_1 Y_2 \cdots Y_n$ follows the log normal distribution [8]. In $X_i = lnY_i[X = ln(M_n/M_0) = lnY_1 + lnY_2 + \cdots + lnY_n]$, a mean is $\mu_i = \sum_{s=1}^{k} (ln(m_s))/k$, and a dispersion is $\sigma_i^2 = \sum_{s=1}^{k} ((ln(m_s)) - \mu_i)^2/k$. Therefore, when a total mean is $\mu = \sum_{i=1}^{n} \mu_i = \mu_i n$ and a total dispersion $\sigma^2 = \sum_{i=1}^{n} \sigma_i^2 = \sigma_i^2 n$, $(\sum_{i=1}^{n} X_i - \mu)/\sigma$ follows the normal distribution from the central limit theorem. When the standard normal distribution $f(z)$ is $\int_0^z (e^{-z^2/2}/(2\pi)^{1/2})dz$, and values of $\int_{u_p}^{v_p} (e^{-z^2/2}/(2\pi)^{1/2})dz$ are $1/2^2, 1/2^4, 1/2^6, 1/2^8, \cdots$ and $1/2^{2p}$ [p is a positive integer], each value of z is assumed u_p and v_p that these range are contained a value $((ln(M_n/M_0)) - \mu)/\sigma$ that is searched. u_p and v_p are obtained from the table of $f(z)$. Each total number of the data between $u_p\sigma + \mu$ and $v_p\sigma + \mu$ is $k^n/2^2, k^n/2^4, k^n/2^6, k^n/2^8, \cdots$, respectively. A height at x is $k^n w e^{-((x-\mu)/\sigma)^2/2}/\ ((2\pi)^{1/2}\sigma)[= H(x)$. w is an effective unit width, for example, about $ln(m_2/m_1)$.].

Next, a quantum algorithm is shown as the following.

First of all, quantum registers $|a_1>, |a_2>, \cdots, |a_n>, |b>$ and $|c>$ are prepared. When α is the minimum integer that is $log_2 k$ or more, each of $|a_f>$ that f is an integer from 1 to n is consisted of α quantum bits [= qubits]. States of $|a_1>, |a_2>, \cdots, |a_n>, |b>$ and $|c>$ are $a_1, a_2, \cdots, a_n, b$ and c, respectively.

Step 1: Each qubit of $|a_1>, |a_2>, \cdots, |a_n>, |b>$ and $|c>$ is set $|0>$.

Step 2: The Hadamard gate $\boxed{H}$ [2, 3] acts on each qubit of $|a_1>, |a_2>, \cdots, |a_{n-1}>$ and $|a_n>$. It changes them for entangled states. The total states are $(2^\alpha)^n$.

Step 3: It is assumed that a quantum gate (A) doesn't change $|b>$ in $a_f < k$, or it changes $|b>$ for $|b+1>$ in the others of a_f. As a target state for $|b>$ is 0, quantum phase inversion gates (PI) and quantum inversion about mean gates

(IM) [2,5,6] act on $|b>$. When β is the minimum even integer that is $(2^{\alpha}/k)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is β, because they are a couple. Next, an observation gate (OB) observes $|b>$. These actions are repeated sequentially from $|a_1>$ to $|a_n>$. Therefore, each state of $|a_f>$ is $0, 1, 2, \cdots, k-2$ and $k-1$, and the total states become $k^n[=W_0]$.

Step 4: It is assumed that a quantum gate (B) changes $|b>$ for $|b + ln(m_s)>$ in $a_f = s-1$. This action repeats from 1 to n at f. Therefore, $|b>$ becomes from $|n(ln(m_{min}))>$ to $|n(ln(m_{max}))>$.

Step 5: It is assumed that a quantum gate (C_1) doesn't change $|c>$ in $u_1\sigma + \mu \leq b \leq v_1\sigma + \mu$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included in $u_1\sigma + \mu \leq b \leq v_1\sigma + \mu$ is $W_1 \approx k^n/2^2$. When γ_1 is the minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|c>$ is $\gamma_1 \approx 2$. Next, (OB) observes $|c>$, and the data of W_1 remain.

Similarly, (C_j) [$2 \leq j \leq g-1$. j is an integer. g that is an integer follows $W_0/H(ln(M_n/M_0)) = 1/(e^{-((ln(M_n/M_0)-\mu)/\sigma)^2/2}/((2\pi)^{1/2}\sigma)) \approx 2^{2g}$.] doesn't change $|c>$ in $u_j\sigma + \mu \leq b \leq v_j\sigma + \mu$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included in $u_j\sigma + \mu \leq b \leq v_j\sigma + \mu$ is $W_j \approx k^n/2^{2j}$. When γ_j is the minimum even integer that is $(W_{j-1}/W_j)^{1/2}$ or more, the total number that (PI) and (IM) act on $|c>$ is $\gamma_j \approx 2$. Next, (OB) observes $|c>$, and the data of W_j remain. These actions are repeated sequentially from 2 to $g-1$ at j.

(C_g) doesn't change $|c>$ at $b = ln(M_n/M_0)$ [$u_g\sigma + \mu \approx ln(M_n/M_0) \leq b \leq v_g\sigma + \mu \approx ln(M_n/M_0)$], or it changes $|c>$ for $|c+1>$ in $b \neq ln(M_n/M_0)$. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included at $b = ln(M_n/M_0)$ is $W_g \approx H(ln(M_n/M_0)) \approx k^n/2^{2g}$. When γ_g is the minimum even integer that is $(W_{g-1}/W_g)^{1/2}$ or more, the total number that (PI) and (IM) act on $|c>$ is $\gamma_g \approx 2$. Next, (OB) observes $|a_1>, |a_2>, \cdots, |a_n>, |b>$ and $|c>$, and one of the data of W_g remains. Therefore, one example of orders that reach at M_n is obtained.

4. Numerical Computation

It is assumed that there are $n = 5, m_1 = 1/2 = m_{min}, m_2 = 1, m_3 = 2 = m_{max}, ln(1/2) \approx -0.6931, ln1 = 0, ln2 \approx 0.6931, k = 3, M_0 = 1, M_5 = 16, \mu_i = \mu = 0, \sigma_i^2 \approx 0.3203, \sigma^2 = \sum_{i=1}^{5} \sigma_i^2 \approx 5\sigma_i^2 \approx 1.602, \sigma \approx 1.266, k^n = 3^5 = 243, w = ln(m_2/m_1) = ln(m_3/m_2) \approx 0.6931, H(ln(M_5/M_0)) = H(ln16) \approx 5, g = 3, u_1 \approx 0.6300, u_2 \approx 1.427, u_3 \approx 1.883$ and $v_1 = v_2 = v_3 \approx 2.190$.

First of all, $|a_1>, |a_2>, \cdots, |a_5>, |b>$ and $|c>$ are prepared. When α is the minimum integer that is $log_2k = log_23 \approx 1.585 \leq 2 = \alpha$, each of $|a_f>$ that f is the integer from 1 to 5 is consisted of $\alpha = 2$ qubits. States of $|a_1>, |a_2>, \cdots, |a_5>, |b>$ and $|c>$ are $a_1, a_2, \cdots, a_5, b$ and c, respectively.

Step 1: Each qubit of $|a_1>, |a_2>, \cdots, |a_5>, |b>$ and $|c>$ is set $|0>$.

Step 2: $\boxed{\text{H}}$ acts on each qubit of $|a_1>, |a_2>, |a_3>, |a_4>$ and $|a_5>$. It changes them for entangled states. The total states are $(2^\alpha)^n = (2^2)^5 = 4^5 = 1024$.

Step 3: (A) doesn't change $|b>$ in $a_f < k = 3$, or it changes $|b>$ for $|b+1>$ in the others of a_f. As the target state for $|b>$ is 0, (PI) and (IM) act on $|b>$. When β is the minimum even integer that is $(2^\alpha/k)^{1/2} = (2^2/3)^{1/2} = (4/3)^{1/2} \approx 1.155 \leq 2 = \beta$, the total number that (PI) and (IM) act on $|b>$ is $\beta \approx 2$. Next, (OB) observes $|b>$. These actions are repeated sequentially from $|a_1>$ to $|a_5>$. Therefore, each state of $|a_f>$ is 0, 1 and 2, and the total states become $k^n = 3^5 = 243[= W_0]$.

Step 4: (B) changes $|b>$ for $|b + ln(m_s)>$ in $a_f = s - 1$. This action repeats from 1 to 5 at f. Therefore, $|b>$ becomes $|-3.466>$ to $|3.466>$.

Step 5: (C_1) doesn't change $|c>$ in $u_1\sigma + \mu \approx 0.7976 \leq b \leq v_1\sigma + \mu \approx 2.772$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included in $0.7976 \leq b \leq 2.772$ is $W_1 \approx 3^5/2^2$. When γ_1 is the minimum even integer that is $(W_0/W_1)^{1/2} \approx (3^5/(3^5/2^2))^{1/2} = 2 \leq 2 = \gamma_1$, the total number that (PI) and (IM) act on $|c>$ is $\gamma_1 \approx 2$. Next, (OB) observes $|c>$, and the data of W_1 remain.

(C_2) doesn't change $|c>$ in $u_2\sigma + \mu \approx 1.807 \leq b \leq v_2\sigma + \mu \approx 2.772$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included in $1.807 \leq b \leq 2.772$ is $W_2 \approx 3^5/2^4$. When γ_2 is the minimum even integer that is $(W_1/W_2)^{1/2} \approx ((3^5/2^2)/(3^5/2^4))^{1/2} = 2 \leq 2 = \gamma_2$, the total number that (PI) and (IM) act on $|c>$ is $\gamma_2 \approx 2$. Next, (OB) observes $|c>$, and the data of W_2 remain.

(C_3) doesn't change $|c>$ at $b = ln(M_5/M_0) = 2.772$ $[u_3\sigma + \mu \approx 2.384 \leq b \leq v_3\sigma + \mu \approx 2.772]$, or it changes $|c>$ for $|c+1>$ in $b \neq 2.772$. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included at $b = 2.772$ is $W_3 \approx H(ln(M_5/M_0)) \approx H(2.772) \approx 5 \approx 3^5/2^6$. When γ_3 is the minimum even integer that is $(W_2/W_3)^{1/2} \approx ((3^5/2^4)/(3^5/2^6))^{1/2} = 2 \leq 2 = \gamma_3$, the total number that (PI) and (IM) act on $|c>$ is $\gamma_3 \approx 2$. Next, (OB) observes $|a_1>, |a_2>, \cdots, |a_5>, |b>$ and $|c>$, and one of the data of W_3 remains. For example, when $a_1, a_2, a_3, a_4, a_5, b$ and c are 2, 1, 2, 2, 2, 2.772 and 0, respectively, it is obtained that one example of orders is 2, 1, 2, 2 and 2.

5. Discussion and Summary

The computational complexity of this quantum algorithm [$= S$] becomes the following. In the order of the actions by the gates, the number of them is αn at $\boxed{\text{H}}$, n at (A), $\beta n \approx 2n$ at (PI) and (IM), n at (OB), n at (B), about g at (C_j) [$1 \leq j \leq g$. j is the integer.], about $2g$ at (PI) and (IM), and about g at (OB). Therefore, S becomes about $(\alpha + 5)n + 4g$. In the example of the section 4, S is 47. The computational complexity of the classical computation [$= Z$] is $k^n = 3^5 = 243$. After all, S/Z becomes $1/5$. When n is large enough, S becomes about $(\alpha + 5)n + 4g \approx 3(log_2 k)n$, where α is about $log_2 k$, and the maximum value of g is about $(n/2)log_2 k$, and S/Z is about $3(log_2 k)n/k^n$. For example, as for $k = 3$ and $n = 100$, S/Z is about $1/10^{45}$.

Therefore, a polynomial time process becomes possible.

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