Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 8, Number 5 (2012), pp. 541–545
© Research India Publications
http://www.ripublication.com/gjpam.htm

Quantum Algorithm for Modified Roulette Problem by Central Limit Theorem

Toru Fujimura
Chemical Department, Industrial Property Cooperation Center,
1-2-15, Kiba, Koto-ku, Tokyo 135-0042, Japan
E-mail: tfujimura8@gmail.com

Abstract

A quantum algorithm for a modified roulette problem by the central limit theorem and its example are reported. When a random variable V_i [$1 \le i \le n$. i and n are integers.] becomes m_s [$1 \le s \le t$. m_s, s and t are integers.] as each probability k_s/K [$\sum_{s=1}^{t} k_s = K$. k_s and K are positive integers. It is contained which we bet.], one unit at each time is betted, and start units [M_0] become final units [M_n], one example in orders that reach at M_n is obtained. A computational complexity of a classical computation is K^n. The computational complexity becomes about $3(log_2 K)n$ by the quantum algorithm that uses quantum phase inversion gates, quantum inversion about mean gates, and the standard normal distribution. Therefore, a polynomial time process becomes possible.

AMS subject classification: Primary 81-08; Secondary 68R05, 68W40.
Keywords: Quantum algorithm, modified roulette problem, central limit theorem, computational complexity, standard normal distribution, polynomial time.

1. Introduction

About 20 years passed after Deutsch and Jozsa [1–3] discovered the quantum algorithm of a high-speed process by a parallel computation that uses quantum entangled states. In the period, Shor [2–4] found the method of solving the factoring in a polynomial time, and Grover [2,5,6] showed the algorithm for the databace search in a square root time. A quantum algorithm for the vertex coloring problem by the central limit theorem has recently been reported by Fujimura [7]. Its computational complexity becomes a polynomial time. In the roulette problem [8], its modified problem that is set up is examined this time. Therefore, its result is reported.

2. Modified Roulette Problem

When a random variable V_i [$1 \leq i \leq n$. i and n are integers.] becomes m_s [$1 \leq s \leq t$. m_s, s and t are integers.] as each probability k_s/K [$\sum_{s=1}^{t} k_s = K$. k_s and K are positive integers. It is contained which we bet.], one unit at each time is betted, and start units [M_0] become final units [M_n], one example in orders that reach at M_n is searched.

3. Quantum Algorithm

It is assumed that V_i [$1 \leq i \leq n$. i and n are integers.] becomes m_s [$1 \leq s \leq t$. m_s, s and t are integers.] as each probability k_s/K [$\sum_{s=1}^{t} k_s = K$. k_s and K are positive integers. It is contained which we bet.], one unit at each time is betted, start units [M_0] become final units [M_n], the minimum value of m_s is m_{min}, and the maximum value of m_s is m_{max}. In V_i, a mean is $\mu_i = \sum_{s=1}^{t} m_s k_s/K$, and a dispersion is $\sigma_i^2 = \sum_{s=1}^{t} (m_s - \mu_i)^2 k_s/K$. Therefore, when a total mean is $\mu = \sum_{i=1}^{n} \mu_i = \mu_i n$ and a total dispersion $\sigma^2 = \sum_{i=1}^{n} \sigma_i^2 = \sigma_i^2 n$, $(\sum_{i=1}^{n} V_i - \mu)/\sigma$ follows the normal distribution from the central limit theorem. When the standard normal distribution $f(z)$ is $\int_0^z (e^{-z^2/2}/(2\pi)^{1/2})dz$, and values of $\int_{u_p}^{v_p} (e^{-z^2/2}/(2\pi)^{1/2})dz$ are $1/2^2, 1/2^4, 1/2^6, 1/2^8, \cdots$ and $1/2^{2p}$ [p is a positive integer], each value of z is assumed u_p and v_p that these range are contained a value $(M_n - \mu)/\sigma$ that is searched. u_p and v_p are obtained from the table of $f(z)$. Each total number of the data between $u_p\sigma + \mu$ and $v_p\sigma + \mu$ is $K^n/2^2, K^n/2^4, K^n/2^6, K^n/2^8, \cdots$, respectively. A height at m is $K^n e^{-((m-\mu)/\sigma)^2/2}/ ((2\pi)^{1/2}\sigma)[= H(m)]$.

Next, a quantum algorithm is shown as the following.

First of all, quantum registers $|a_1>, |a_2>, \cdots, |a_n>, |b>$ and $|c>$ are prepared. When α is the minimum integer that is $log_2 K$ or more, each of $|a_f>$ that f is an integer from 1 to n is consisted of α quantum bits [= qubits]. States of $|a_1>, |a_2>, \cdots, |a_n>$, $|b>$ and $|c>$ are $a_1, a_2, \cdots, a_n, b$ and c, respectively.

Step 1: Each qubit of $|a_1>, |a_2>, \cdots, |a_n>, |b>$ and $|c>$ is set $|0>$.

Step 2: The Hadamard gate $\boxed{\mathrm{H}}$ [2, 3] acts on each qubit of $|a_1>, |a_2>, \cdots, |a_{n-1}>$ and $|a_n>$. It changes them for entangled states. The total states are $(2^\alpha)^n$.

Step 3: It is assumed that a quantum gate (A) doesn't change $|b>$ in $a_f < K$, or it changes $|b>$ for $|b+1>$ in the others of a_f. As a target state for $|b>$ is 0, quantum phase inversion gates (PI) and quantum inversion about mean gates (IM) [2,5,6] act on $|b>$. When β is the minimum even integer that is $(2^{\alpha}/K)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is β, because they are a couple. Next, an observation gate (OB) observes $|b>$. These actions are repeated sequentially from $|a_1>$ to $|a_n>$. Therefore, each state of $|a_f>$ is $0, 1, 2, \cdots, K-2$ and $K-1$, and the total states become $K^n[=W_0]$.

Step 4: It is assumed that a quantum gate (B) changes $|b>$ for $|b+m_s>$ in $\sum_{d=1}^{s} k_{d-1} \leq a_f \leq \sum_{d=1}^{s} k_d - 1$ [d is an integer. k_0 is 0.]. This action repeats from 1 to n at f. Therefore, $|b>$ becomes from $|m_{min}n>$ to $|m_{max}n>$.

Step 5: It is assumed that a quantum gate (C_1) doesn't change $|c>$ in $u_1\sigma + \mu \leq b \leq v_1\sigma + \mu$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included in $u_1\sigma + \mu \leq b \leq v_1\sigma + \mu$ is $W_1 \approx K^n/2^2$. When γ_1 is the minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|c>$ is $\gamma_1 \approx 2$. Next, (OB) observes $|c>$, and the data of W_1 remain.

Similarly, (C_j) [$2 \leq j \leq g-1$. j is an integer. g that is an integer follows $W_0/H(M_n - M_0) = 1/(e^{-((M_n-M_0-\mu)/\sigma)^2/2}/((2\pi)^{1/2}\sigma)) \approx 2^{2g}$.] doesn't change $|c>$ in $u_j\sigma + \mu \leq b \leq v_j\sigma + \mu$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included in $u_j\sigma + \mu \leq b \leq v_j\sigma + \mu$ is $W_j \approx K^n/2^{2j}$. When γ_j is the minimum even integer that is $(W_{j-1}/W_j)^{1/2}$ or more, the total number that (PI) and (IM) act on $|c>$ is $\gamma_j \approx 2$. Next, (OB) observes $|c>$, and the data of W_j remain. These actions are repeated sequentially from 2 to $g-1$ at j.

(C_g) doesn't change $|c>$ at $b = M_n - M_0$ [$u_g\sigma + \mu \approx M_n - M_0 \leq b \leq v_g\sigma + \mu \approx M_n - M_0$], or it changes $|c>$ for $|c+1>$ in $b \neq M_n - M_0$. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included at $b = M_n - M_0$ is $W_g \approx H(M_n - M_0) \approx K^n/2^{2g}$. When γ_g is the minimum even integer that is $(W_{g-1}/W_g)^{1/2}$ or more, the total number that (PI) and (IM) act on $|c>$ is $\gamma_g \approx 2$. Next, (OB) observes $|a_1>, |a_2>, \cdots, |a_n>, |b>$ and $|c>$, and one of the data of W_g remains. Therefore, one example of orders that reach at M_n is obtained.

4. Numerical Computation

It is assumed that there are $n = 3, m_1 = -1 = m_{min}, m_2 = 1, m_3 = 2 = m_{max}, k_1 = 3, k_2 = 2, k_3 = 1, K = 6, M_0 = 10, M_n = 15, \mu_i = 1/6, \mu = \sum_{i=1}^{3} \mu_i = 3\mu_i =$

$1/2, \sigma_i^2 \approx 1.472, \sigma^2 = \sum_{i=1}^{3} \sigma_i^2 \approx 3\sigma_i^2 \approx 4.416, \sigma \approx 2.101, K^n = 6^3 = 216, H(M_n - M_0) = H(15 - 10) = H(5) \approx 4, g = 3, u_1 \approx 0.6245, u_2 \approx 1.415, u_3 \approx 1.856$ and $v_1 = v_2 = v_3 \approx 2.142$.

First of all, $|a_1>, |a_2>, |a_3>, |b>$ and $|c>$ are prepared. When α is the minimum integer that is $log_2 K = log_2 6 \approx 2.585 \leq 3 = \alpha$, each of $|a_f>$ that f is the integer from 1 to 3 is consisted of $\alpha = 3$ qubits. States of $|a_1>, |a_2>, |a_3>, |b>$ and $|c>$ are a_1, a_2, a_3, b and c, respectively.

Step 1: Each qubit of $|a_1>, |a_2>, |a_3>, |b>$ and $|c>$ is set $|0>$.

Step 2: $\boxed{\text{H}}$ acts on each qubit of $|a_1>, |a_2>$ and $|a_3>$. It changes them for entangled states. The total states are $(2^\alpha)^n = (2^3)^3 = 8^3 = 512$.

Step 3: (A) doesn't change $|b>$ in $a_f < K = 6$, or it changes $|b>$ for $|b+1>$ in the others of a_f. As the target state for $|b>$ is 0, (PI) and (IM) act on $|b>$. When β is the minimum even integer that is $(2^\alpha/K)^{1/2} = (2^3/6)^{1/2} = (8/6)^{1/2} \approx 1.155 \leq 2 = \beta$, the total number that (PI) and (IM) act on $|b>$ is $\beta \approx 2$. Next, (OB) observes $|b>$. These actions are repeated sequentially from $|a_1>$ to $|a_3>$. Therefore, each state of $|a_f>$ is 0, 1, 2, 3, 4 and 5, and the total states become $K^n = 6^3 = 216[= W_0]$.

Step 4: (B) changes $|b>$ for $|b+m_s>$ in $\sum_{d=1}^{s} k_{d-1} \leq a_f \leq \sum_{d=1}^{s} k_d - 1$ [d is the integer. k_0 is 0.]. This action repeats from 1 to 3 at f. Therefore, $|b>$ becomes $|-3>$ to $|6>$.

Step 5: (C_1) doesn't change $|c>$ in $u_1\sigma + \mu \approx 2 \leq b \leq v_1\sigma + \mu \approx 5$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included in $2 \leq b \leq 5$ is $W_1 \approx 6^3/2^2$. When γ_1 is the minimum even integer that is $(W_0/W_1)^{1/2} \approx (6^3/(6^3/2^2))^{1/2} = 2 \leq 2 = \gamma_1$, the total number that (PI) and (IM) act on $|c>$ is $\gamma_1 \approx 2$. Next, (OB) observes $|c>$, and the data of W_1 remain.

(C_2) doesn't change $|c>$ in $u_2\sigma + \mu \approx 4 \leq b \leq v_2\sigma + \mu \approx 5$, or it changes $|c>$ for $|c+1>$ in the others of b. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included in $4 \leq b \leq 5$ is $W_2 \approx 6^3/2^4$. When γ_2 is the minimum even integer that is $(W_1/W_2)^{1/2} \approx ((6^3/2^2)/(6^3/2^4))^{1/2} = 2 \leq 2 = \gamma_2$, the total number that (PI) and (IM) act on $|c>$ is $\gamma_2 \approx 2$. Next, (OB) observes $|c>$, and the data of W_2 remain.

(C_3) doesn't change $|c>$ at $b = M_n - M_0 = 15 - 10 = 5$ [$u_3\sigma + \mu \approx 5 \leq b \leq v_3\sigma + \mu \approx 5$], or it changes $|c>$ for $|c+1>$ in $b \neq 5$. As the target state for $|c>$ is 0, (PI) and (IM) act on $|c>$. The number of the data that is included at $b = 5$ is $W_3 \approx H(5) \approx 4 \approx 6^3/2^6$. When γ_3 is the minimum even integer that is

$(W_2/W_3)^{1/2} \approx ((6^3/2^4)/(6^3/2^6))^{1/2} = 2 \leq 2 = \gamma_3$, the total number that (PI) and (IM) act on $|c>$ is $\gamma_3 \approx 2$. Next, (OB) observes $|a_1>, |a_2>, |a_3>, |b>$ and $|c>$, and one of the data of W_3 remains. For example, when a_1, a_2, a_3, b and c are 5, 5, 4, 5 and 0, respectively, it is obtained that one example of orders is 2, 2 and 1.

5. Discussion and Summary

The computational complexity of this quantum algorithm $[= S]$ becomes the following. In the order of the actions by the gates, the number of them is αn at $\boxed{\mathrm{H}}$, n at (A), $\beta n \approx 2n$ at (PI) and (IM), n at (OB), n at (B), about g at (C_j) $[1 \leq j \leq g.$ j is the integer.], about $2g$ at (PI) and (IM), and about g at (OB). Therefore, S becomes about $(\alpha + 5)n + 4g$. In the example of the section 4, S is 36. The computational complexity of the classical computation $[= Z]$ is $K^n = 6^3 = 216$. After all, S/Z becomes $1/6$. When n is large enough, S becomes about $(\alpha + 5)n + 4g \approx 3(log_2 K)n$, where α is about $log_2 K$, and the maximum value of g is about $(n/2)log_2 K$, and S/Z is about $3(log_2 K)n/K^n$. For example, as for $K = 7$ and $n = 100$, S/Z is about $1/10^{82}$.

Therefore, a polynomial time process becomes possible.

References

[1] Deutsch D., and Jozsa R., Rapid solution of problems by quantum computation, *Proc. Roy. Soc. Lond. A*, 439:553–558, 1992.

[2] Takeuchi S., Ryoshi Konpyuta (Quantum Computer), Kodansha, Tokyo, Japan, 2005 [in Japanese].

[3] Miyano K., and Furusawa A., Ryoshi Konpyuta Nyumon (An Introduction to Quantum Computation), Nihonhyoronsha, Tokyo, Japan, 2008 [in Japanese].

[4] Shor P.W., Algorithms for quantum computation: discrete logarithms and factoring, *Proc. 35th Annu. Symp. Foundations of Computer Science*, IEEE, pp. 124–134, 1994.

[5] Grover L.K., A fast quantum mechanical algorithm for database search, *Proc. 28th Annu. ACM Symp. Theory of Computing*, pp. 212–219, 1996.

[6] Grover L.K., A framework for fast quantum mechanical algorithms, *Proc. 30th Annu. ACM Symp. Theory of Computing*, pp. 53–62, 1998.

[7] Fujimura T., Quantum algorithm for vertex coloring problem by central limit theorem, *Glob. J. Pure Appl. Math.*, 7:401–405, 2011.

[8] Barboianu C., Roulette Odds and Profits: The Mathematics of Complex Bets, Infarom, Craiova, Romania, 2007.