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Hermite-Hadamard type inequalities for m-convex functions[1]

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Abstract

The aim of this work is to establish some new inequalities for functions whose first derivative in absolute value are m-convex. Some estimates on the right hand side of Hermite-Hadamard type inequality for m-convex functions are given. Some applications to special means of positive real numbers are considered.

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1. Introduction

The following inequality is well known as the Hermite-Hadamard inequality:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x \leq \frac{f(a)+f(b)}{2}$$

where $f : I \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ is a convex function on the interval I of real numbers and $a, b \in I$ with $a < b$.

In the paper [7], G. Toader defined the concept of m-convexity as the following:

Definition 1.1. The function $f : [0, b] \longrightarrow \mathbb{R}$ is said to be m-convex, where $m \in [0, 1]$, if for every $x, y \in [0, b]$ and $t \in [0, 1]$ we have

$$f(tx + m(1-t)y) \leq tf(x) + m(1-t)f(y).$$

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Some interesting and important inequalities for m-convex functions can be found in the papers [1, 2, 6]. In [4, 5], M. Özdemir et al. used the following lemma in order to establish some inequalities for the mappings whose absolute value of second derivative are m-convex.

Lemma 1.2. Let $f : I \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ be a twice differentiable mapping on I°. If $f'' \in L^1[a,b]$, where $a, b \in I$ with $a < b$ and $m \in (0,1]$, then the following equalities hold:

$$\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x = \frac{(b-a)^2}{2}\int_0^1 t(1-t)f''(ta+(1-t)b)\mathrm{d}t,$$

and

$$\frac{f(a)+f(mb)}{2} - \frac{1}{mb-a}\int_a^{mb} f(x)\mathrm{d}x = \frac{(mb-a)^2}{2}\int_0^1 t(1-t)f''(ta+m(1-t)b)\mathrm{d}t.$$

The aim of this work is to establish some estimates to the right hand side of Hadamard type inequality for m-convex functions. We obtain different lemma, which use first derivative of m-convex functions. Along this paper we consider a real interval $I \subseteq \mathbb{R}$ and we denote I° is the interior of I.

2. Inequalities for m-convex functions

In order to prove main theorems, we need the following lemma.

Lemma 2.1. Let $f : I \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ be a differentiable mapping on I° and assume that $a, b \in I$ with $a < b$ and $m \in (0,1]$. If $f' \in L^1[a,b]$, then the following equality holds:

$$\frac{f(a)+f(mb)}{2} - \frac{1}{mb-a}\int_a^{mb} f(x)\mathrm{d}x = \frac{(mb-a)}{2}\int_0^1 (1-2t)f'(ta+m(1-t)b)\mathrm{d}t.$$

A simple proof of the equality can be done by performing an integration by parts in the integral on the right side and changing the variable. The details are left to the interested reader.

Theorem 2.2. Let $f : I \subseteq [0,\infty) \longrightarrow \mathbb{R}$ be a differentiable mapping on I° such that $f' \in L^1[a,b]$, where $a, b \in I$ with $a < b$. If $|f'|^q$ is m-convex on $[a,b]$ for some fixed $m \in (0,1]$ and $q \geq 1$, then the following inequalities hold:

$$\left|\frac{f(a)+f(mb)}{2} - \frac{1}{mb-a}\int_a^{mb} f(x)\mathrm{d}x\right| \leq \frac{|mb-a|}{4}2^{\frac{1}{q}}\left[\frac{|f'(a)|^q + m|f'(b)|^q}{4}\right]^{\frac{1}{q}}$$
$$\leq \frac{|mb-a|}{4}\left(\frac{1}{2}\right)^{\frac{1}{q}}\left[|f'(a)| + m^{\frac{1}{q}}|f'(b)|\right]. \tag{1}$$

Proof. Suppose that $q = 1$. From Lemma 2.1 and using the m-convexity of $|f'|$, we have

$$\begin{aligned}
&\left|\frac{f(a)+f(mb)}{2}-\frac{1}{mb-a}\int_a^{mb} f(x)\mathrm{d}x\right| \\
&\leq \frac{|mb-a|}{2}\int_0^1 |1-2t||f'(ta+m(1-t)b)|\mathrm{d}t \\
&\leq \frac{|mb-a|}{2}\int_0^1 |1-2t|[t|f'(a)|+m(1-t)|f'(b)|]\mathrm{d}t \\
&= \frac{|mb-a|}{2}\left\{\int_0^{\frac{1}{2}} \left[t(1-2t)|f'(a)|+m(1-2t)(1-t)|f'(b)|\right]\mathrm{d}t\right. \\
&\quad \left. +\int_{\frac{1}{2}}^1 \left[t(2t-1)|f'(a)|+m(2t-1)(1-t)|f'(b)|\right]\mathrm{d}t\right\} \\
&= \frac{|mb-a|}{8}\left[|f'(a)|+m|f'(b)|\right],
\end{aligned}$$

which completes the proof for $q = 1$. Now, suppose that $q > 1$. Using well known Hölder inequality for $q > 1$ and $p = \frac{q}{q-1}$, we have

$$\begin{aligned}
&\int_0^1 |1-2t||f'(ta+m(1-t)b)|\mathrm{d}t = \int_0^1 |1-2t|^{1-\frac{1}{q}}|1-2t|^{\frac{1}{q}}|f'(ta+m(1-t)b)|\mathrm{d}t \\
&\leq \left(\int_0^1 |1-2t|\mathrm{d}t\right)^{\frac{q-1}{q}}\left(\int_0^1 |1-2t||f'(ta+m(1-t)b)|^q\mathrm{d}t\right)^{\frac{1}{q}} \qquad (2)
\end{aligned}$$

Since $|f'|^q$ is m-convex on $[a,b]$, we know that for every $t \in [0,1]$

$$|f'(ta+m(1-t)b)|^q \leq t|f'(a)|^q + m(1-t)|f'(b)|^q \qquad (3)$$

from the inequalities (2), (3) and Lemma 2.1, we have

$$\begin{aligned}
&\left|\frac{f(a)+f(mb)}{2}-\frac{1}{mb-a}\int_a^{mb} f(x)\mathrm{d}x\right| \\
&\le \frac{|mb-a|}{2}\int_0^1 |1-2t||f'(ta+m(1-t)b)|\mathrm{d}t \\
&\le \frac{|mb-a|}{2}\left(\int_0^1 |1-2t|\mathrm{d}t\right)^{\frac{q-1}{q}}\left(\int_0^1 |1-2t|[t|f'(a)|^q+m(1-t)|f'(b)|^q]\mathrm{d}t\right)^{\frac{1}{q}} \\
&= \frac{|mb-a|}{2}\left(\frac{1}{2}\right)^{\frac{q-1}{q}}\left\{\int_0^{\frac{1}{2}}\left[t(1-2t)|f'(a)|^q+m(1-2t)(1-t)|f'(b)|^q\right]\mathrm{d}t\right. \\
&\quad \left.+\int_{\frac{1}{2}}^1\left[t(2t-1)|f'(a)|^q+m(2t-1)(1-t)|f'(b)|^q\right]\mathrm{d}t\right\}^{\frac{1}{q}} \\
&= \frac{|mb-a|}{2}\left(\frac{1}{2}\right)^{\frac{q-1}{q}}\left[\frac{|f'(a)|^q+m|f'(b)|^q}{4}\right]^{\frac{1}{q}}.
\end{aligned}$$

Now, using the fact that

$$\sum_{i=1}^{n}(a_i+b_i)^r \le \sum_{i=1}^{n} a_i^r+\sum_{i=1}^{n} b_i^r \tag{4}$$

for $0 < r < 1$, $a_1, a_2, \ldots, a_n \ge 0$ and $b_1, b_2, \ldots, b_n \ge 0$, we obtain the inequality (1). ■

Remark 2.3. If in Theorem 2.2 we choose $m = q = 1$, then we have

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \le \left(\frac{b-a}{8}\right)[|f'(a)|+|f'(b)|]$$

which is as in the paper [3].

Theorem 2.4. Let $f : I \subseteq [0,\infty) \longrightarrow \mathbb{R}$ be a differentiable mapping on I° such that $f' \in L^1[a,b]$, where $a, b \in I$ with $a < b$. If $|f'|^q$ is m-convex on $[a,b]$ for some fixed

$m \in (0,1]$ and $q > 1$, then the following inequalities hold:

$$\begin{aligned}
&\left|\frac{f(a)+f(mb)}{2} - \frac{1}{mb-a}\int_a^{mb} f(x)\mathrm{d}x\right| \\
&\le \frac{|mb-a|}{2}\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left[\frac{|f'(a)|^q + m|f'(b)|^q}{2}\right]^{\frac{1}{q}} \\
&\le \frac{|mb-a|}{2}\left(\frac{1}{2}\right)^{\frac{1}{q}}\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left[|f'(a)| + m^{\frac{1}{q}}|f'(b)|\right].
\end{aligned}$$

Proof. From Lemma 2.1 and using the Hölder's inequality for $p = \dfrac{q}{q-1}$, $q > 1$ and inequality (4), we obtain

$$\begin{aligned}
&\left|\frac{f(a)+f(mb)}{2} - \frac{1}{mb-a}\int_a^{mb} f(x)\mathrm{d}x\right| \\
&\le \frac{|mb-a|}{2}\int_0^1 |1-2t||f'(ta+m(1-t)b)|\mathrm{d}t \\
&\le \frac{|mb-a|}{2}\left(\int_0^1 |1-2t|^p\mathrm{d}t\right)^{\frac{1}{p}}\left(\int_0^1 |f'(ta+m(1-t)b)|^q\mathrm{d}t\right)^{\frac{1}{q}} \\
&\le \frac{|mb-a|}{2}\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left(\int_0^1 [t|f'(a)|^q + m(1-t)|f'(b)|^q]\mathrm{d}t\right)^{\frac{1}{q}} \\
&= \frac{|mb-a|}{2}\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left[\frac{|f'(a)|^q + m|f'(b)|^q}{2}\right]^{\frac{1}{q}} \\
&\le \frac{|mb-a|}{2}\left(\frac{1}{2}\right)^{\frac{1}{q}}\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left[|f'(a)| + m^{\frac{1}{q}}|f'(b)|\right],
\end{aligned}$$

which completes the proof. ■

Remark 2.5. Since

$$\lim_{q\to\infty}\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}} = \frac{1}{2}, \qquad \lim_{q\to 1^+}\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}} = 1,$$

we have

$$\frac{1}{2} < \left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}} < 1, \quad q \in (1,\infty),$$

so for $q \in (1, \infty)$

$$\begin{aligned}
&\frac{|mb-a|}{4}\left(\frac{1}{2}\right)^{\frac{1}{q}}\left[|f'(a)|+m^{\frac{1}{q}}|f'(b)|\right] \\
&\leq \frac{|mb-a|}{2}\left(\frac{1}{2}\right)^{\frac{1}{q}}\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left[|f'(a)|+m^{\frac{1}{q}}|f'(b)|\right] \\
&\leq \frac{|mb-a|}{2}\left(\frac{1}{2}\right)^{\frac{1}{q}}\left[|f'(a)|+m^{\frac{1}{q}}|f'(b)|\right],
\end{aligned}$$

This means that the estimation given in Theorem 2.2 is better than one given in Theorem 2.4. Also, the one given in Theorem 2.4 becomes better as q increases.

3. Hermite-Hadamard type inequalities

Lemma 3.1. Let $f : I \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ be a differentiable mapping on I° and assume that $a, b \in I$ with $a < b$. If $f' \in L^1[a, b]$, then the following equality holds:

$$\begin{aligned}
\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x = \frac{b-a}{2}\int_0^1 (-t)f'(ta+(1-t)b)\mathrm{d}t \\
+\frac{b-a}{2}\int_0^1 tf'(tb+(1-t)a)\mathrm{d}t.
\end{aligned}$$

Proof. We get a simple proof of the equality by performing an integration by parts in the integrals from the right side and changing the variable. ■

Theorem 3.2. With the assumptions of Theorem 2.2, we have the following inequalities:

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \\
&\leq \left(\frac{b-a}{4}\right)\left(\frac{2}{3}\right)^{\frac{1}{q}}\left\{\left[|f'(a)|^q+\frac{m}{2}\left|f'\left(\frac{b}{m}\right)\right|^q\right]^{\frac{1}{q}}+\left[|f'(b)|^q+\frac{m}{2}\left|f'\left(\frac{a}{m}\right)\right|^q\right]^{\frac{1}{q}}\right\} \\
&\leq \left(\frac{b-a}{4}\right)\left(\frac{2}{3}\right)^{\frac{1}{q}}\left[|f'(a)|+|f'(b)|+\left(\frac{m}{2}\right)^{\frac{1}{q}}\left(\left|f'\left(\frac{a}{m}\right)\right|+\left|f'\left(\frac{b}{m}\right)\right|\right)\right].
\end{aligned}$$

Proof. Suppose that $q = 1$, by Lemma 3.1 and using the m-convexity of $|f'|$, we get

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \\
&\leq \frac{b-a}{2}\left[\int_0^1 t|f'(ta+(1-t)b)|\mathrm{d}t + \int_0^1 t|f'(tb+(1-t)a)|\mathrm{d}t\right] \\
&\leq \frac{b-a}{2}\int_0^1 \left[t^2(|f'(a)|+|f'(b)|) + mt(1-t)\left(\left|f'\left(\frac{a}{m}\right)\right| + \left|f'\left(\frac{b}{m}\right)\right|\right)\right]\mathrm{d}t \\
&= \frac{b-a}{6}\left[|f'(a)|+|f'(b)| + \frac{m}{2}\left(\left|f'\left(\frac{a}{m}\right)\right| + \left|f'\left(\frac{b}{m}\right)\right|\right)\right].
\end{aligned}$$

which completes the proof for $q = 1$.

Now, suppose that $q > 1$. Using well known Hölder's inequality for $q > 1$ and $p = \frac{q}{q-1}$, we obtain

$$\begin{aligned}
\int_0^1 t|f'(ta+(1-t)b)|\mathrm{d}t &= \int_0^1 t^{1-\frac{1}{q}}\, t^{\frac{1}{q}}|f'(ta+(1-t)b)|\mathrm{d}t \\
&\leq \left(\int_0^1 t\ \mathrm{d}t\right)^{\frac{1}{p}}\left(\int_0^1 t|f'(ta+(1-t)b)|^q\mathrm{d}t\right)^{\frac{1}{q}}. \qquad (5)
\end{aligned}$$

Since $|f'|^q$ is m-convex on $[a,b]$, we know that for every $t \in [0,1]$

$$|f'(ta+(1-t)b)|^q \leq t|f'(a)|^q + m(1-t)\left|f'\left(\frac{b}{m}\right)\right|^q \qquad (6)$$

From inequalities (4), (5), (6) and Lemma 3.1, we get

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \\
&\leq \frac{b-a}{2}\left(\frac{1}{2}\right)^{\frac{q-1}{q}}\left\{\left[\int_0^1\left(t^2|f'(a)|^q + mt(1-t)\left|f'\left(\frac{b}{m}\right)\right|^q\right)\mathrm{d}t\right]^{\frac{1}{q}}\right. \\
&\quad \left. + \left[\int_0^1\left(t^2|f'(b)|^q + mt(1-t)\left|f'\left(\frac{a}{m}\right)\right|^q\right)\mathrm{d}t\right]^{\frac{1}{q}}\right\} \\
&= \left(\frac{b-a}{4}\right)\left(\frac{2}{3}\right)^{\frac{1}{q}}\left\{\left[|f'(a)|^q + \frac{m}{2}\left|f'\left(\frac{b}{m}\right)\right|^q\right]^{\frac{1}{q}} + \left[|f'(b)|^q + \frac{m}{2}\left|f'\left(\frac{a}{m}\right)\right|^q\right]^{\frac{1}{q}}\right\} \\
&\leq \left(\frac{b-a}{4}\right)\left(\frac{2}{3}\right)^{\frac{1}{q}}\left[|f'(a)|+|f'(b)| + \left(\frac{m}{2}\right)^{\frac{1}{q}}\left(\left|f'\left(\frac{a}{m}\right)\right| + \left|f'\left(\frac{b}{m}\right)\right|\right)\right],
\end{aligned}$$

which is required. ■

Theorem 3.3. With the assumptions of Theorem 2.4, we have the following inequalities:

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \\
&\leq \left(\frac{b-a}{4}\right)\left(\frac{2q-2}{2q-1}\right)^{\frac{q-1}{q}}\left\{\left[|f'(a)|^q + m\left|f'\left(\frac{b}{m}\right)\right|^q\right]^{\frac{1}{q}}\right. \\
&\quad \left. + \left[|f'(b)|^q + m\left|f'\left(\frac{a}{m}\right)\right|^q\right]^{\frac{1}{q}}\right\} \\
&\leq \left(\frac{b-a}{4}\right)\left(\frac{2q-2}{2q-1}\right)^{\frac{q-1}{q}}\left[|f'(a)|+|f'(b)|+m^{\frac{1}{q}}\left(\left|f'\left(\frac{a}{m}\right)\right|+\left|f'\left(\frac{b}{m}\right)\right|\right)\right].
\end{aligned}$$

Proof. From Lemma 3.1 and using the Hölder inequality for $p=\frac{q}{q-1}$ and $q>1$ and inequality (4), we have

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \\
&\leq \frac{b-a}{2}\left[\int_0^1 t\,|f'(ta+(1-t)b)|\mathrm{d}t + \int_0^1 t\,|f'(tb+(1-t)a)|\mathrm{d}t\right] \\
&\leq \frac{b-a}{2}\left\{\left(\int_0^1 t^p\,\mathrm{d}t\right)^{\frac{1}{p}}\left(\int_0^1 |f'(ta+(1-t)b)|^q\mathrm{d}t\right)^{\frac{1}{q}}\right. \\
&\quad \left. + \left(\int_0^1 t^p\,\mathrm{d}t\right)^{\frac{1}{p}}\left(\int_0^1 |f'(tb+(1-t)a)|^q\mathrm{d}t\right)^{\frac{1}{q}}\right\} \\
&\leq \frac{b-a}{2}\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left\{\left[\int_0^1\left(t\,|f'(a)|^q + m(1-t)\left|f'\left(\frac{b}{m}\right)\right|^q\right)\mathrm{d}t\right]^{\frac{1}{q}}\right. \\
&\quad \left. + \left[\int_0^1\left(t\,|f'(b)|^q + m(1-t)\left|f'\left(\frac{a}{m}\right)\right|^q\right)\mathrm{d}t\right]^{\frac{1}{q}}\right\}
\end{aligned}$$

$$= \left(\frac{b-a}{2}\right)\left(\frac{1}{2}\right)^{\frac{1}{q}}\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left\{\left[|f'(a)|^q + m\left|f'\left(\frac{b}{m}\right)\right|^q\right]^{\frac{1}{q}}\right.$$

$$\left. + \left[|f'(b)|^q + m\left|f'\left(\frac{a}{m}\right)\right|^q\right]^{\frac{1}{q}}\right\}$$

$$\leq \left(\frac{b-a}{4}\right)\left(\frac{2q-2}{2q-1}\right)^{\frac{q-1}{q}}\left[|f'(a)| + |f'(b)| + m^{\frac{1}{q}}\left(\left|f'\left(\frac{a}{m}\right)\right| + \left|f'\left(\frac{b}{m}\right)\right|\right)\right],$$

which is required. ■

Remark 3.4. From Theorems 3.2, 3.3, we have

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \leq \min\{E_1, E_2\}$$

where

$$E_1 = \left(\frac{b-a}{4}\right)\left(\frac{2}{3}\right)^{\frac{1}{q}}\left[|f'(a)| + |f'(b)| + \left(\frac{m}{2}\right)^{\frac{1}{q}}\left(\left|f'\left(\frac{a}{m}\right)\right| + \left|f'\left(\frac{b}{m}\right)\right|\right)\right],$$

$$E_2 = \left(\frac{b-a}{4}\right)\left(\frac{2q-2}{2q-1}\right)^{\frac{q-1}{q}}\left[|f'(a)| + |f'(b)| + m^{\frac{1}{q}}\left(\left|f'\left(\frac{a}{m}\right)\right| + \left|f'\left(\frac{b}{m}\right)\right|\right)\right].$$

4. Applications to special means

Now using the results of Sections 2 and 3, we obtain some applications to special means of positive real numbers.

(1) The arithmetic mean: $A(a,b) = \dfrac{a+b}{2}, \qquad a,b \in \mathbb{R}, \quad a,b > 0.$

(2) The logarithmic mean: $L(a,b) = \dfrac{b-a}{\ln b - \ln a}, \quad a,b \in \mathbb{R}, \quad a \neq b,\ a,b > 0.$

(3) The generalized logarithmic mean:

$$L_n(a,b) = \left[\frac{b^{n+1} - a^{n+1}}{(n+1)(b-a)}\right]^{\frac{1}{n}}, \ n \in \mathbb{R} \setminus \{-1, 0\},\ a,b \in \mathbb{R},\ a \neq b,\ a,b > 0.$$

The following propositions hold.

Proposition 4.1. Let $n \in (-\infty, 0) \cup [1, \infty) \setminus \{-1\}$ and $[a, b] \subset (0, \infty)$ and $q > 1$. Then we have the following inequality:

$$\left|A(a^n, b^n) - L_n^n(a, b)\right| \leq n \left(\frac{b-a}{2}\right) \left(\frac{2}{3}\right)^{\frac{1}{q}-1} A(a^{n-1}, b^{n-1}).$$

Proof. The assertion follows from Theorem 3.2 for $f(x) = x^n$ and n as specified and $m = 1$. ■

Proposition 4.2. Let $q > 1$ and $[a, b] \subset (0, \infty)$. Then we have the following inequality:

$$\left|A(a^{-1}, b^{-1}) - L^{-1}(a, b)\right| \leq \left(\frac{b-a}{2}\right) \left(\frac{2q-2}{2q-1}\right)^{\frac{q-1}{q}} A(a^{-2}, b^{-2}).$$

Proof. The assertion follows from Theorem 2.4 for $f(x) = \frac{1}{x}$ and $m = 1$. ■

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