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Quantum Algorithm for Modified Three-dimensional Random Walk Problem by Central Limit Theorem

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Abstract

A quantum algorithm for a modified three-dimensional random walk problem by the central limit theorem and its example are reported. When a random variable V_i [$1 \leq i \leq n$. i and n are integers.] becomes $(1, 1, 1)$, $(-1, -1, -1)$, $(-1, 1, 1)$, $(1, -1, -1)$, $(-1, -1, 1)$, $(1, 1, -1)$, $(1, -1, 1)$ and $(-1, 1, -1)$ in an $x - y - z$ space as each probability $1/8$, one example in paths that arrive at a point in the space is obtained. A computational complexity of a classical calculation is 8^n. The computational complexity becomes about $10n$ by the quantum algorithm that uses quantum phase inversion gates, quantum inversion about mean gates and the standard normal distribution. Therefore, a polynomial time process becomes possible.

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Keywords: Quantum algorithm, modified three-dimensional random walk problem, central limit theorem, computational complexity, standard normal distribution, polynomial time.

1. Introduction

The quantum algorithm that had been started by Deutsch-Jozsa [1–3] was found the method of solving the factoring in a polynomial time by Shor [2–4]. And then, the algorithm for the database search was shown by Grover [2,5,6]. A quantum algorithm for the vertex coloring problem by the central limit theorem has recently been reported by Fujimura [7]. Its computational complexity becomes a polynomial time. In the three-dimensional random walk problem [8], its modified problem that is set up is examined this time. Therefore, its result is reported.

2. Modified Three-dimensional Random Walk Problem

When a random variable V_i $[1 \le i \le n.\ i$ and n are integers.] becomes $(1,1,1), (-1,-1,-1), (-1,1,1), (1,-1,-1), (-1,-1,1), (1,1,-1), (1,-1,1)$ and $(-1,1,-1)$ in an $x-y-z$ space as each probability 1/8, one example in paths that arrive at a point in the space is searched.

3. Quantum Algorithm

It is assumed that a person in an $x-y-z$ space walks from a point to one direction of $(1,1,1), (-1,-1,-1), (-1,1,1), (1,-1,-1), (-1,-1,1), (1,1,-1),$ $(1,-1,1)$ and $(-1,1,-1)$ as each probability 1/8, and the person arrives at $(x,y,z) = (M_x, M_y, M_z)$ after n times [n is a positive integer] of this process, where a start point is $(0,0,0)$. When n is the odd number, each distribution of x, y and z becomes $-n, -(n-2), \cdots, -3, -1, 1, 3, \cdots, n-2, n$. When n is the even number, it becomes $-n, -(n-2), \cdots, -4, -2, 0, 2, 4, \cdots, n-2, n$.

In the x-axis, it is assumed that the random variable X_i $[1 \le i \le n.\ i$ and n are integers.], X_i becomes 1 and -1 as each probability 1/2, a mean is 0, and a dispersion is 1. Therefore, when a total mean is 0 and a total dispersion is $\sigma_x^2 = n$, $(\sum_{i=1}^{n} X_i)/\sigma_x$ follows the normal distribution from the central limit theorem. When the standard normal distribution $f(w)$ is $\int_0^w (e^{-w^2/2}/(2\pi)^{1/2})dw$, and values of $\int_{u_{xp}}^{v_{xp}} (e^{-w^2/2}/(2\pi)^{1/2})dw$ are $1/2^2, 1/2^4, 1/2^6, 1/2^8, \cdots$ and $1/2^{2p}$ [p is a positive integer], each value of w is assumed u_{xp} and v_{xp} that these range are contained a value x $[= M_x]$ of a point in the $x-y-z$ space that is searched. u_{xp} and v_{xp} are obtained from the table of $f(w)$. Each total number of the data between $u_{xp}\sigma_x$ and $v_{xp}\sigma_x$ is $8^n/2^2, 8^n/2^4, 8^n/2^6, 8^n/2^8, \cdots$, respectively. A height at $w\sigma_x = m_x$ is $8^n e^{-(m_x/\sigma_x)^2/2}/((2\pi)^{1/2}\sigma_x)[= H_x(m_x)]$.

Therefore, the number of a value m of x is $T_x(m_x) = H_x(m_x) + (H_x(m_x-1) + H_x(m_x+1))/2$.

In the y-axis, it is assumed that the random variable Y_i $[1 \le i \le n.\ i$ and n are integers.], Y_i becomes 1 and -1 as each probability 1/2, a mean is 0, and a dispersion is 1. Therefore, when a total mean is 0 and a total dispersion is $\sigma_y^2 = n$, $(\sum_{i=1}^{n} Y_i)/\sigma_y$ follows the normal distribution from the central limit theorem. When values of $\int_{u_{yp}}^{v_{yp}} (e^{-w^2/2}/(2\pi)^{1/2})dw$ are $1/2^2, 1/2^4, 1/2^6, 1/2^8, \cdots$ and $1/2^{2p}$ [p is a positive integer], each value of w is assumed u_{yp} and v_{yp} that these range are contained a value y $[= M_y]$ of a point in the $x-y-z$ space that is searched. u_{yp} and v_{yp} are obtained from the table of $f(w)$. Each total number of the data between $u_{yp}\sigma_y$ and $v_{yp}\sigma_y$ is $T_x(M_x)/2^2, T_x(M_x)/2^4, T_x(M_x)/2^6, T_x(M_x)/2^8, \cdots$, respectively. The height at $w\sigma_y = m_y$ is $T_x(M_x)e^{-(m_y/\sigma_y)^2/2}/((2\pi)^{1/2}\sigma_y)[= H_y(m_y)]$.

Therefore, the number of a value m of y is $T_y(m_y) = H_y(m_y) + (H_y(m_y - 1) + H_y(m_y + 1))/2$.

In the z-axis, it is assumed that the random variable Z_i [$1 \leq i \leq n$. i and n are integers.], Z_i becomes 1 and -1 as each probability 1/2, a mean is 0, and a dispersion is 1. Therefore, when a total mean is 0 and a total dispersion is $\sigma_z^2 = n$, $(\sum_{i=1}^{n} Z_i)/\sigma_z$ follows the normal distribution from the central limit theorem. When values of $\int_{u_{zp}}^{v_{zp}} (e^{-w^2/2}/(2\pi)^{1/2})dw$ are $1/2^2, 1/2^4, 1/2^6, 1/2^8, \cdots$ and $1/2^{2p}$ [p is a positive integer], each value of w is assumed u_{zp} and v_{zp} that these range are contained a value z [$= M_z$] of a point in the $x - y - z$ space that is searched. u_{zp} and v_{zp} are obtained from the table of $f(w)$. Each total number of the data between $u_{zp}\sigma_z$ and $v_{zp}\sigma_z$ is $T_y(M_y)/2^2, T_y(M_y)/2^4, T_y(M_y)/2^6, T_y(M_y)/2^8, \cdots$, respectively. The height at $w\sigma_z = m_z$ is $T_y(M_y)e^{-(m_z/\sigma_z)^2/2}/((2\pi)^{1/2}\sigma_z)[= H_z(m_z)]$.

Therefore, the number of a value m of z is $T_z(m_z) = H_z(m_z) + (H_z(m_z - 1) + H_z(m_z + 1))/2$.

Next, a quantum algorithm is shown as the following.

First of all, quantum registers $|a_1>, |a_2>, \cdots, |a_n>, |b>, |c>, |d>$ and $|h>$ are prepared. Each of $|a_f>$ that f is an integer from 1 to n is consisted of 3 quantum bits [= qubits]. States of $|a_1>, |a_2>, \cdots, |a_n>, |b>, |c>, |d>$ and $|h>$ are $a_1, a_2, \cdots, a_n, b, c, d$ and h, respectively.

Step 1: Each qubit of $|a_1>, |a_2>, \cdots, |a_n>, |b>, |c>, |d>$ and $|h>$ is set $|0>$.
Step 2: The Hadamard gate $\boxed{\text{H}}$ [2, 3] acts on each qubit of $|a_1>, |a_2>, \cdots, |a_{n-1}>$ and $|a_n>$. It changes them for entangled states. The total states are 8^n [$= W_0$].
Step 3: It is assumed that a quantum gate (A) changes ($|b>, |c>, |d>$) for ($|b+1>, |c+1>, |d+1>$), ($|b-1>, |c-1>, |d-1>$), ($|b-1>, |c+1>, |d+1>$), ($|b+1>, |c-1>, |d-1>$), ($|b-1>, |c-1>, |d+1>$), ($|b+1>, |c+1>, |d-1>$), ($|b+1>, |c-1>, |d+1>$) and ($|b-1>, |c+1>, |d-1>$) at $a_1 = 0, 1, 2, 3, 4, 5, 6$ and 7, respectively. This action repeats to $|a_n>$. Therefore, $|b>, |c>$ and $|d>$ become from $|-n>$ to $|n>$.
Step 4: It is assumed that a quantum gate (B_1) doesn't change $|h>$ in $u_{x1}\sigma_x \leq b \leq v_{x1}\sigma_x$, or it changes $|h>$ for $|h+1>$ in the others of b. As a target state for $|h>$ is 0, quantum phase inversion gates (PI) and quantum inversion about mean gates (IM) [2,5,6] act on $|h>$. The number of the data that is included in $u_{x1}\sigma_x \leq b \leq v_{x1}\sigma_x$ is $W_1 \approx 8^n/2^2$. When γ_1 is a minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|h>$ is $\gamma_1 \approx 2$, because they are a couple. Next, an observation gate (OB) observes $|h>$, and the data of W_1 remain.

Similarly, (B_{R_x}) [$2 \leq R_x \leq g_x - 1$. R_x is an integer. g_x that is an integer follows $W_0/T_x(M_x) = 1/(e^{-(M_x/\sigma_x)^2/2}/((2\pi)^{1/2}\sigma_x)) \approx 2^{2g_x}$.] doesn't change $|h>$ in $u_{xR_x}\sigma_x \leq$

$b \leq v_{xR_x}\sigma_x$, or it changes $|h>$ for $|h+1>$ in the others of b. As the target state for $|h>$ is 0, (PI) and (IM) act on $|h>$. The number of the data that is included in $u_{xR_x}\sigma_x \leq b \leq v_{xR_x}\sigma_x$ is $W_{R_x} \approx 8^n/2^{2R_x}$. When γ_{R_x} is the minimum even integer that is $(W_{R_x-1}/\ W_{R_x})^{1/2}$ or more, the total number that (PI) and (IM) act on $|h>$ is $\gamma_{R_x} \approx 2$. Next, (OB) observes $|h>$, and the data of W_{R_x} remain. These actions are repeated sequentially from 2 to $g_x - 1$ at R_x.

(B_{g_x}) doesn't change $|h>$ at $b = M_x$ $[u_{xg_x}\sigma_x \approx M_x \leq b \leq v_{xg_x}\sigma_x \approx M_x]$, or it changes $|h>$ for $|h+1>$ in $b \neq M_x$. As the target state for $|h>$ is 0, (PI) and (IM) act on $|h>$. The number of the data that is included at $b = M_x$ is $W_{g_x} \approx 8^n e^{-(M_x/\sigma_x)^2/2}/((2\pi)^{1/2}\sigma_x)[\ = T_x(M_x)] \approx 8^n/2^{2g_x}$. When γ_{g_x} is the minimum even integer that is $(W_{g_x-1}/\ W_{g_x})^{1/2} \approx ((8^n/2^{2(g_x-1)})/(8^n/2^{2g_x}))^{1/2} = 2 \leq 2 = \gamma_{g_x}$, the total number that (PI) and (IM) act on $|h>$ is $\gamma_{g_x} \approx 2$. Next, (OB) observes $|h>$, and the data of W_{g_x} remain.

Step 5: It is assumed that a quantum gate (C_1) doesn't change $|h>$ in $u_{y1}\sigma_y \leq c \leq v_{y1}\sigma_y$, or it changes $|h>$ for $|h+1>$ in the others of c. As the target state for $|h>$ is 0, (PI) and (IM) act on $|h>$. The number of the data that is included in $u_{y1}\sigma_y \leq c \leq v_{y1}\sigma_y$ is $W_{g_x+1} \approx 8^n/2^{2(g_x+1)}$. When γ_{g_x+1} is the minimum even integer that is $(W_{g_x}/\ W_{g_x+1})^{1/2}$ or more, the total number that (PI) and (IM) act on $|h>$ is $\gamma_{g_x+1} \approx 2$. Next, (OB) observes $|h>$, and the data of W_{g_x+1} remain.

Similarly, (C_{R_y}) $[2 \leq R_y \leq g_y - 1$. R_y is an integer. g_y that is an integer follows $W_{g_x}/T_y(M_y) \approx 1/(e^{-(M_y/\sigma_y)^2/2}/((2\pi)^{1/2}\sigma_y)) \approx 2^{2g_y}$.] doesn't change $|h>$ in $u_{yR_y}\sigma_y \leq c \leq v_{yR_y}\sigma_y$, or it changes $|h>$ for $|h+1>$ in the others of c. As the target state for $|h>$ is 0, (PI) and (IM) act on $|h>$. The number of the data that is included in $u_{yR_y}\sigma_y \leq c \leq v_{yR_y}\sigma_y$ is $W_{g_x+R_y} \approx T_x(M_x)/2^{2R_y} \approx (8^n/2^{2g_x})/2^{2R_y} = 8^n/2^{2(g_x+R_y)}$. When $\gamma_{g_x+R_y}$ is the minimum even integer that is $(W_{g_x+R_y-1}/W_{g_x+R_y})^{1/2}$ or more, the total number that (PI) and (IM) act on $|h>$ is $\gamma_{g_x+R_y} \approx 2$. Next, (OB) observes $|h>$, and the data of $W_{g_x+R_y}$ remain. These actions are repeated sequentially from 2 to $g_y - 1$ at R_y.

(C_{g_y}) doesn't change $|h>$ at $c = M_y$ $[u_{yg_y}\sigma_y \approx M_y \leq c \leq v_{yg_y}\sigma_y \approx M_y]$, or it changes $|h>$ for $|h+1>$ in $c \neq M_y$. As the target state for $|h>$ is 0, (PI) and (IM) act on $|h>$. The number of the data that is included at $c = M_y$ is $W_{g_x+g_y} \approx 8^n/2^{2(g_x+g_y)}$. When $\gamma_{g_x+g_y}$ is the minimum even integer that is $(W_{g_x+g_y-1}/W_{g_x+g_y})^{1/2} \approx ((8^n/2^{2(g_x+g_y-1)})/(8^n/2^{2(g_x+g_y)}))^{1/2} = 2 \leq 2 = \gamma_{g_x+g_y}$, the total number that (PI) and (IM) act on $|h>$ is $\gamma_{g_x+g_y} \approx 2$. Next, (OB) observes $|h>$, and the data of $W_{g_x+g_y}$ remain.

Step 6: It is assumed that a quantum gate (D_1) doesn't change $|h>$ in $u_{z1}\sigma_z \leq d \leq v_{z1}\sigma_z$, or it changes $|h>$ for $|h+1>$ in the others of d. As the target state for $|h>$ is 0, (PI) and (IM) act on $|h>$. The number of the data that is included in $u_{z1}\sigma_z \leq d \leq v_{z1}\sigma_z$ is $W_{g_x+g_y+1} \approx 8^n/2^{2(g_x+g_y+1)}$. When $\gamma_{g_x+g_y+1}$ is the minimum even integer that is $(W_{g_x+g_y}/\ W_{g_x+g_y+1})^{1/2}$ or more, the total number that (PI) and (IM) act on $|h>$ is $\gamma_{g_x+g_y+1} \approx 2$. Next, (OB) observes $|h>$, and the data of $W_{g_x+g_y+1}$ remain.

Similarly, (D_{R_z}) $[2 \leq R_z \leq g_z - 1$. R_z is an integer. g_z that is an integer follows $W_{g_x+g_y}/T_z(M_z) \approx 1/(e^{-(M_z/\sigma_z)^2/2}/((2\pi)^{1/2}\sigma_z)) \approx 2^{2g_z}$.] doesn't change $|h>$ in $u_{zR_z}\sigma_z \le d \le v_{zR_z}\sigma_z$, or it changes $|h>$ for $|h+1>$ in the others of d. As the target state for $|h>$ is 0, (PI) and (IM) act on $|h>$. The number of the data that is included in $u_{zR_z}\sigma_z \le d \le v_{zR_z}\sigma_z$ is $W_{g_x+g_y+R_z} \approx T_y(M_y)/2^{2R_z} \approx (8^n/2^{2(g_x+g_y)})/2^{2R_z} = 8^n/2^{2(g_x+g_y+R_z)}$. When $\gamma_{g_x+g_y+R_z}$ is the minimum even integer that is $(W_{g_x+g_y+R_z-1}/W_{g_x+g_y+R_z})^{1/2}$ or more, the total number that (PI) and (IM) act on $|h>$ is $\gamma_{g_x+g_y+R_z} \approx 2$. Next, (OB) observes $|h>$, and the data of $W_{g_x+g_y+R_z}$ remain. These actions are repeated sequentially from 2 to g_z-1 at R_z.

(D_{g_z}) doesn't change $|h>$ at $d = M_z$ $[u_{zg_z}\sigma_z \approx M_z \le d \le v_{zg_z}\sigma_z \approx M_z]$, or it changes $|h>$ for $|h+1>$ in $d \ne M_z$. As the target state for $|h>$ is 0, (PI) and (IM) act on $|h>$. The number of the data that is included at $d = M_z$ is $W_{g_x+g_y+g_z} \approx 8^n/2^{2(g_x+g_y+g_z)}$. When $\gamma_{g_x+g_y+g_z}$ is the minimum even integer that is $(W_{g_x+g_y+g_z-1}/W_{g_x+g_y+g_z})^{1/2} \approx ((8^n/2^{2(g_x+g_y+g_z-1)})/(8^n/2^{2(g_x+g_y+g_z)}))^{1/2} = 2 \le 2 = \gamma_{g_x+g_y+g_z}$, the total number that (PI) and (IM) act on $|h>$ is $\gamma_{g_x+g_y+g_z} \approx 2$. Next, (OB) observes $|a_1>, |a_2>, \cdots, |a_n>, |b>, |c>, |d>$ and $|h>$, and one of the data of $W_{g_x+g_y+g_z}$ remains. Therefore, one example of paths that arrive at the point (M_x, M_y, M_z) in the $x-y-z$ space is obtained.

4. Numerical Computation

It is assumed that there are $n = 3, M_x = 3, M_y = 3, M_z = 1, \sigma_x = \sigma_y = \sigma_z = n^{1/2} \approx 1.732, 8^n = 8^3 = 512, H_x(0) \approx 117.9, H_x(1) = H_x(-1) \approx 99.8, H_x(2) = H_x(-2) \approx 60.5, H_x(3) = H_x(-3) \approx 26.3, H_x(4) = H_x(-4) \approx 8.2, T_x(1) = T_x(-1) \approx 189.0, T_x(3) = T_x(-3) \approx 60.7, g_x = 2, u_{x1} \approx 0.5488, u_{x2} \approx 1.258, v_{x1} = v_{x2} \approx 1.732, H_y(0) \approx 14.0, H_y(1) = H_y(-1) \approx 11.9,$
$H_y(2) = H_y(-2) \approx 7.2, H_y(3) = H_y(-3) \approx 3.1, H_y(4) = H_y(-4) \approx 1.0,$
$T_y(1) = T_y(-1) \approx 22.5, T_y(3) = T_y(-3) \approx 7.2, g_y = 2, u_{y1} \approx 0.5488, u_{y2} \approx 1.258, v_{y1} = v_{y2} \approx 1.732, H_z(0) \approx 1.6, H_z(1) = H_z(-1) \approx 1.4, H_z(2) = H_z(-2) \approx 0.8, H_z(3) = H_z(-3) \approx 0.4, H_z(4) = H_z(-4) \approx 0.1, T_z(1) = T_z(-1) \approx 2.6, T_z(3) = T_z(-3) \approx 0.9, g_z = 1, u_{z1} = 0$ and $v_{z1} \approx 0.6745$.

First of all, $|a_1>, |a_2>, |a_3>, |b>, |c>, |d>$ and $|h>$ are prepared. Each of $|a_1>, |a_2>$ and $|a_3>$ is consisted of 3 qubits. States of $|a_1>, |a_2>, |a_3>, |b>, |c>, |d>$ and $|h>$ are a_1, a_2, a_3, b, c, d and h, respectively.

Step 1: Each qubit of $|a_1>, |a_2>, |a_3>, |b>, |c>, |d>$ and $|h>$ is set $|0>$.
Step 2: $\boxed{\text{H}}$ acts on each qubit of $|a_1>, |a_2>$ and $|a_3>$. It changes them for entangled states. The total states are $8^3 = 512$ $[= W_0]$.
Step 3: (A) changes $(|b>, |c>, |d>)$ for $(|b+1>, |c+1>, |d+1>)$, $(|b-1>, |c-1>, |d-1>)$, $(|b-1>, |c+1>, |d+1>)$, $(|b+1>, |c-1>, |d-1>)$, $(|b-1>, |c-1>, |d+1>)$, $(|b+1>, |c+1>, |d-1>)$, $(|b+1>, |c-1>, |d+1>)$ and $(|b-1>, |c+1>, |d-1>)$ at $a_1 = 0, 1, 2, 3, 4, 5, 6$ and 7, respectively. This action repeats to $|a_3>$. Therefore, $|b>, |c>$ and $|d>$ become from $|-3>$ to $|3>$.

Step 4: (B_1) doesn't change $|h>$ in $u_{x1}\sigma_x \approx 1 \le b \le v_{x1}\sigma_x \approx 3$, or it changes $|h>$ for $|h+1>$ in the others of b. As the target state for $|h>$ is 0, (PI) and (IM) act on $|h>$. The number of the data that is included in $1 \le b \le 3$ is $W_1 = T_x(1)+T_x(3) \approx 250$. When γ_1 is the minimum even integer that is $(W_0/W_1)^{1/2} = (512/250)^{1/2} \approx 1.4 \le 2 = \gamma_1$, the total number that (PI) and (IM) act on $|h>$ is $\gamma_1 \approx 2$. Next, (OB) observes $|h>$, and the data of W_1 remain.

(B_2) doesn't change $|h>$ at $b = 3$ $[u_{x2}\sigma_x \approx 2 \le b \le v_{x2}\sigma_x \approx 3]$, or it changes $|h>$ for $|h+1>$ in $b \ne 3$. As the target state for $|h>$ is 0, (PI) and (IM) act on $|h>$. The number of the data that is included at $b = 3$ is $W_2 = T_x(3) \approx 61$. When γ_2 is the minimum even integer that is $(W_1/W_2)^{1/2} \approx (250/61)^{1/2} \approx 2.0 \le 2 = \gamma_2$, the total number that (PI) and (IM) act on $|h>$ is $\gamma_2 \approx 2$. Next, (OB) observes $|h>$, and the data of W_2 remain.

Step 5: (C_1) doesn't change $|h>$ in $u_{y1}\sigma_y \approx 1 \le c \le v_{y1}\sigma_y \approx 3$, or it changes $|h>$ for $|h+1>$ in the others of c. As the target state for $|h>$ is 0, (PI) and (IM) act on $|h>$. The number of the data that is included in $1 \le c \le 3$ is $W_3 = T_y(1)+T_y(3) \approx 30$. When γ_3 is the minimum even integer that is $(W_2/W_3)^{1/2} \approx (61/30)^{1/2} \approx 1.4 \le 2 = \gamma_3$, the total number that (PI) and (IM) act on $|h>$ is $\gamma_3 \approx 2$. Next, (OB) observes $|h>$, and the data of W_3 remain.

(C_2) doesn't change $|h>$ at $c = 3$ $[u_{y2}\sigma_y \approx 2 \le c \le v_{y2}\sigma_y \approx 3]$, or it changes $|h>$ for $|h+1>$ in $c \ne 3$. As the target state for $|h>$ is 0, (PI) and (IM) act on $|h>$. The number of the data that is included at $c = 3$ is $W_4 = T_y(3) \approx 7$. When γ_4 is the minimum even integer that is $(W_3/W_4)^{1/2} \approx (30/7)^{1/2} \approx 2.1 \approx 2 \le 2 = \gamma_4$, the total number that (PI) and (IM) act on $|h>$ is $\gamma_4 \approx 2$. Next, (OB) observes $|h>$, and the data of W_4 remain.

Step 6: (D_1) doesn't change $|h>$ at $d = 1$ $[u_{z1}\sigma_z = 0 \le d \le v_{z1}\sigma_z \approx 1$, or it changes $|h>$ for $|h+1>$ in $d \ne 1$. As the target state for $|h>$ is 0, (PI) and (IM) act on $|h>$. The number of the data that is included at $d = 1$ is $W_5 = T_z(1) \approx 3$. When γ_5 is the minimum even integer that is $(W_4/W_5)^{1/2} \approx (7/3)^{1/2} \approx 1.5 \le 2 = \gamma_5$, the total number that (PI) and (IM) act on $|h>$ is $\gamma_5 \approx 2$. Next, (OB) observes $|a_1>, |a_2>, |a_3>, |b>, |c>, |d>$ and $|h>$, and one of the data of W_5 remains. For example, when a_1, a_2, a_3, b, c, d and h are 0, 0, 5, 3, 3, 1 and 0, respectively, it is obtained that one example of directions of paths is $(1,1,1) \to (1,1,1) \to (1,1,-1)$. Therefore, its transfer is $(1,1,1) \to (2,2,2) \to (3,3,1)$.

5. Discussion and Summary

The computational complexity of this quantum algorithm $[=S]$ becomes the following. In the order of the actions by the gates, the number of them is $3n$ at $\boxed{\mathrm{H}}$, n at (A), about g_x at (B_{R_x}) $[1 \le R_x \le g_x$. R_x is the integer.], about $2g_x$ at (PI) and (IM), about g_x at (OB), about g_y at (C_{R_y}) $[1 \le R_y \le g_y$. R_y is the integer.], about $2g_y$ at (PI) and (IM), about g_y at (OB), about g_z at (D_{R_z}) $[1 \le R_z \le g_z$. R_z is the integer.], about $2g_z$ at (PI) and (IM) and about g_z at (OB). Therefore, S becomes about $4(n+g_x+g_y+g_z)$. In the example of the section 4, S is 32. The computational complexity of the classical computation [$= Z$] is $8^n = 8^3 = 512$. After all, S/Z becomes about $1/16$. When n is large enough, S becomes about $4(n + g_x + g_y + g_z) \approx 10n$, where the maximum value of $g_x + g_y + g_z$ is about $3n/2$, and S/Z is about $10n/8^n$. For example, as for $n = 100$, S/Z is about $1000/8^{100} \approx 1/10^{87}$.

Therefore, a polynomial time process becomes possible.

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