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Application of the Spectral Method Galerkin to the Numerical Resolution of the Energy Equation in a Fluid Flow to Axisymmetric Geometry

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Abstract

The object of this article consists with the application of the spectral method Galerkin to solve the equation of energy in a flow with axisymmetric geometry where the main axis of the flow coincides with the axis of pipe.
By giving itself the equation of energy in absence of the external sources of heat and a dissipation of the mechanical energy, this equation associated with the boundary conditions suitable undergoes transformations necessary for the applicability of the method itself. The examination of this flow is focused particularly in the stationary case, i.e. there is no explicit dependence with the time of the sizes such speed or the temperature.

Keywords: energy equation, axisymmetric flow, spectral method of Galerkin, polynomials of Chebyshev

Introduction

The undertaken study, in this research task relates to the numerical resolution of the energy equation by the spectral method. Let us announce that the spectral method is one of the numerical methods of resolution of a problem given in the form of partial derivative equations. However, the choice of a numerical method for the resolution of a problem is an delicate subject owing to the fact that this one must meet certain requirements such: stability, convergence, the taking into account of the physical parameters of the problem.

In the work suggested in this article, the duct is considered axisymmetric and periodical in space. It is shown that the spectral method is appropriate for this type of situation (Batchi, 2005; Batina et al., 1989) from where interest of the justification of the choice of the method. Moreover, the boundary conditions of the thermal problem also enable us to apply the spectral method.

This one requires a wise choice of functions basis. Thus he is proposed the use of the developments according to the polynomial Chebyshev basis in the two directions axial and radial of channel then in other situations the polynomials of Legendre prove more adapted (Shen and Temam,1993). As the examination of the heat transfer in the duct takes place thanks to a forced convection, one considers the formulation stream function-vorticity of the dynamic problem.

This paper is subdivided in three great parts: the first part is concerning to the formulation of the problem of energy and the characterization of the duct geometry. Taking into consideration this geometry, one gives oneself boundary conditions relating to the temperature such as the conditions to the entry, the exit and the walls

Then, one proceeds to introduce reference variables which give us a new formulation of the thermal problem and also of the new boundary conditions.

The last section relates to the application of the Galerkinspectral method for the resolution of the energy equation. Indeed, by energy, it is a question of examining the change of the temperature in this type of flow when one gives oneself certain conditions upstream and downstream but the way in which the heat transfer takes place.

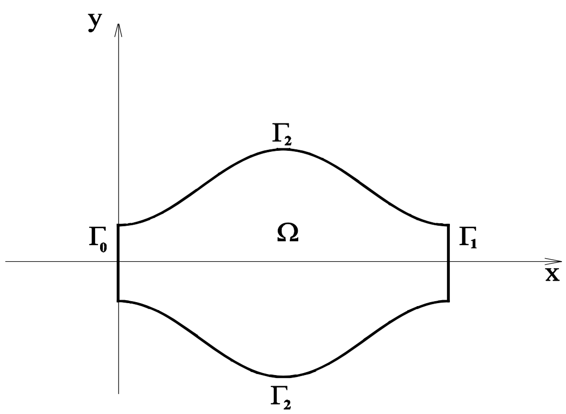
The Energy Equation formulation and Characterization of the Geometry

In the absence of external sources of heat and a dissipation of the mechanical energy, the energy equation is given by:

$$\frac{\partial T}{\partial t} + (\mathbf{v}.\nabla)T = a\Delta T \text{ in } \Omega\text{x}]0,\text{T}[\quad (3.1)$$

Where *T* represents the fluid temperature of the fluid and T being time. The coefficient a = k/ρC_P is thermal diffusivity where in this relation k is thermal conductivity and C_P the thermal capacity.

Geometry domain study



Let be Ω the open and bounded of R^2 such as it is indicated on the figure above and $\Gamma = \overline{\Gamma}_0 \cup \overline{\Gamma}_1 \cup \overline{\Gamma}_2$ being the border of the domain Ω .

Boundary conditions

- Entry of channel: it is supposed that the fluid enters channel with the conditions of noted room temperature $T\infty$, one thus has

T_0 (z = 0, r) = $T\infty$

- Exit of channel: it is considered that one has a null variation in temperature. What results in a homogeneous condition of Neumann

$$\frac{\partial T_0}{\partial z} = 0$$

- Walls of channel: one considers a heated wall at temperature constant T_w from where it comes that T = T_w

New Formulation of the Thermal Problem

Setting in dimensionless form

To characterize the flow, it is physically important to introduce sizes or numbers without dimension. In our case, one establishes the dimensionless equations of assessment whose reference variables are the following ones: the L_0 length, U_0 speed, the P_0 pressure, time t_0 and the T_0 temperature.

Considering a temperature of constant wall, one poses

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{3.2}$$

A dimensionless temperature is thus defined θ based on the variation in temperature enters the T_w wall and the infinite temperature T_∞ upstream.

The various dimensionless sizes obtained are: the Reynolds number Re, the number of Prandtl Pr, the Peclet number Pe = Pr. Re

Thus, the dimensionless writing of the equation of energy is

$$\beta \frac{\partial \theta}{\partial t} + (\mathbf{v}.\nabla)\theta = \frac{1}{\Pr .Re} \Delta \theta \tag{3. 3}$$

where β is a parameter related to the geometry of channel.

After the dimensionless formulation takes place, the equation of the energy which takes account of the new geometry of channel is given by:

$$\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+(\frac{\lambda}{h}v-\frac{\rho}{\lambda}\frac{h'}{h}u)\frac{\partial\theta}{\partial\rho}= \frac{1}{\Pr.\hat{Re}}\left[\frac{\partial^2\theta}{\partial x^2}-2\rho\frac{h'}{h}\frac{\partial^2\theta}{\partial\rho\partial x} +\left(\rho\frac{2h'^2-h''h}{h^2}+\frac{\lambda^2}{\rho h^2}\right)\frac{\partial\theta}{\partial\rho}+ \frac{1}{h^2}(\lambda^2+\rho^2 h'^2)\frac{\partial^2\theta}{\partial\rho^2}\right] \tag{3.4}$$

where Ř $_e$ = λ R$_e$ .The wall function $h(x)=1+\frac{e}{2}(1-\cos(\pi n(x+1)))$ where n indicates the number of geometrical periods and e represents the reduced amplitude.

While multiplying (3. 4) by h^2 and taking account of the dimensionless frame of reference (x,r), one leads to:

$$h^2\frac{\partial\theta}{\partial t}+h^2u\frac{\partial\theta}{\partial x}+(\lambda hv-rh'hu)\frac{\partial\theta}{\partial r}=\frac{1}{\mathrm{Pe}}\Delta_F\theta \tag{3. 5}$$

where Δ_F is the operator defined by :

$$\Delta_F\theta=h^2\frac{\partial^2\theta}{\partial x^2}-2\rho h'h\frac{\partial^2\theta}{\partial\rho\partial x}+\left(\rho(2h'^2-h''h)+\frac{\lambda^2}{\rho}\right)\frac{\partial\theta}{\partial\rho}+(\lambda^2+\rho^2h'^2)\frac{\partial^2\theta}{\partial\rho^2} \tag{3.6}$$

Taking account of the transformations related to the geometry of duct, the domain Ω is brought back to a square unit. Thus, boundary conditions associated relating to θ who are written then:

- wall of channel : $\theta=1$ at $r=\pm 1\ \forall\ x\in[-1, 1]$
- entry of channel corresponding to x = -1, one gives oneself a crenel of temperature regularized of the type $\theta=r^{2m}$ where m indicates the degree of the polynomials which must be regulated in a data-processing way.
- exit of channel: one imposes

$$\frac{\partial\theta}{\partial x}=0 \text{ at } x=1,\forall r\in[-1,+1]$$

The Stationary Thermal Problem

By expressing the velocity field v= (u, v) according to φ and ω , finally the stationary thermal problem consists in solving the equation

$$\left(\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{1}{r}\frac{\partial\varphi}{\partial r}\right)\frac{\partial\theta}{\partial x}-\frac{1}{r}\frac{\partial\psi}{\partial x}.\frac{\partial\theta}{\partial r}-\frac{1}{Re\Pr}\Delta_F\theta=S_\varphi \tag{3.7}$$

S φ being the source term.

The temperature θ is then approximate as follows:

$$\theta(x,r)=\sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}q_k(x)p_{2l}(r) \tag{3.8}$$

where θ_{kl} are the unknown coefficients to determine, Mx and Mr being the orders of truncation of the developments in series according to the directions axial and radial respectively.

I Taking account of the function formulation stream function-vorticity(ψ- ω) for the Navier-Stokes equations in which the incompressibility condition is automatically satisfied, $\psi$is developed as follows:

$$\psi(x,r) = \sum_{k=0}^{N_x}\sum_{l=0}^{N_r}\psi_{kl} Q_k(x) P_{2l}(r) \qquad (3.9)$$

with $\omega(x,r) = -\Delta_f \psi(x,r)$ and whereQk(x) determine the polynomial basis depending on the axial direction x and P_{2l} (r) the polynomial basis depending on radial coordinate r. The boundary conditions imposed onψthen allow to write:

$$Q_k(x) = T_k(x) + \frac{(k+3)^2(k+1)}{(k+2)^2(k+2)} T_{k+1}(x) - \frac{k^2}{(k+2)^2} T_{k+2}$$
$$(x) - \frac{(k+3)^2(k+1)^3}{(k+2)^2(k+2)(k+3)^2} T_{k+3}(x) \qquad (3\ .10)$$

with k= 0,1, 2,…,N_x

$$P_{2l}(r) = T_{2l}(r) - \frac{(l+1)}{(l+2)} T_{2(l+1)}(r) - T_{2(l+2)}(r) + \frac{(l+1)}{(l+2)} T_{2(l+3)}(r) \qquad (3.11)$$

with l=0, 1, 2,…., N_r

These basic functions were developed in a way detailed in work of thesis ofBatchi (2005).

Note: The method of Galerkin applied to the equation (3.7) consists in projecting this one according to the basis (q_k(x) p_{2l} (r)) (Canuto et al.,1988). One then obtains a system of equations which must lead us to the determination of the unknown coefficients.

Determination of The Basic Functions in the Relation (3.7)

The polynomialsbasis(q_k(x) p_{2l} (r)) is selected so that the boundary conditions of the problem are satisfied(Shen (1997), Gelfgat, 2004).

Determination of the base q_k (x)

We seek the functions q_k (x) like a linear combination of the polynomials of Chebyshev in the following way:

$q_k (x) = T_k(x)+ \alpha_1 T_{k+1} (x) + \alpha_2 T_{k+2} (x)$

The new function θ(x,r) must check the boundary conditions established in section 2.

Subtitling (3.8) in the expressions giving the boundary conditions of section 2, the constants α_1 and α_2 check the system of equation:

$$\begin{cases} T_k(-1) + \alpha_1 T_{k+1}(-1) + \alpha_2 T_{k+2}(-1) = 0 \\ T'_k(+1) + \alpha_1 T'_{k+1}(+1) + \alpha_2 T'_{k+2}(+1) = 0 \end{cases}$$

While taking of account the properties on the Chebyshev polynomials (Boyd, 1989) by their respective values, it comes whereas this system with two equations gives us:

$$\alpha_2 = -\frac{(k+1)^2 + k^2}{(k+1)^2 + (k+2)^2}; \alpha_1 = \frac{(k+2)^2 - k^2}{(k+1)^2 + (k+2)^2}$$

From where one can write

$$q_k(x) = \quad T_k(x) + \frac{(k+2)^2 - k^2}{(k+1)^2 + (k+2)^2} T_{k+1}(x) - \frac{(k+1)^2 + k^2}{(k+1)^2 + (k+2)^2} T_{k+2}(x)$$

with k = 0, 1, …, M_x

Determination of the base p_{2l} (r)

In a similar way, the approximate solution (3.8) satisfying the condition given in section 2, involves that p_{2l} (r) checks the condition established for θ (x, r) = 0 at r = 1. One writes then p_{2l} (r)= T_{2l}(r) + $\beta_1 T_{2(l+1)}$ (r)

One finds thus without difficulty β_1 = -1 andthus p_{2l} (r) is written as follows:

p_{2l} (r) = T_{2l}(r) - $T_{2\,(l+1)}$ (r)

with l= 0, 1, 2, …, Mr

Resolution of the equation (3.7) by the spectral method of Galerkin

The spectral method Galerkin is based on a variational formulation which incorporates the conditions in the poles and takes account the singularity in r=0. However in the case of the axisymmetric problems, there are no conditions with the poles but the singularity remains present.

Variational formulation of the problem

Problem :

Setting $W_{M_x \times M_r} = vect\{q_k(x) p_{2l}(r).;0 \le k \le M_x; 0 \le l \le M_r\}$ the subspace generated by the base $q_k(x) p_{2l}(r)$.

The approximation of Galerkin consists in posing the problem as follows :

$$\begin{cases} \text{Found} & \theta^M \in W_{M_x \times M_r} & \text{such that} \\ (L\theta^M, \phi)_\varpi & = (S, \phi)_\varpi & \forall \phi \in W_{M_x \times M_r} \end{cases} \qquad (4.1)$$

where $\theta^M = \sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl} q_k(x) p_{2l}(r)$ is the orthogonal projection of the function θ on space $W_{M_x \times M_r}$ and the linear operator L is defined by :

$$L_\theta = \left(\frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r}\frac{\partial\varphi}{\partial r}\right)\frac{\partial\theta}{\partial x} - \frac{1}{r}\frac{\partial\psi}{\partial x}.\frac{\partial\theta}{\partial r} - \frac{1}{Re\,\mathrm{Pr}}\Delta_F\theta$$

with

$$\Delta_F\theta = h^2\frac{\partial^2\theta}{\partial x^2} + (\lambda^2 + r^2h'^2)\frac{\partial^2\theta}{\partial r^2} - 2rhh'\frac{\partial^2\theta}{\partial x\partial r} + \left[r(2h'^2 - hh'') + \frac{\lambda^2}{r}\right]\frac{\partial\theta}{\partial r} \tag{4.2}$$

By considering the development of $\theta^M \in W_{M_x \times M_r}$ in the basis ($q_k(x)$ p_{2l} (r)), and when one takes as function test the functions q_i (x) p_{2j}(r)dans (3.7), it comes whereas the equation (3.7) is equivalent to:

$$\left(h^2u\theta^M,\phi\right)_\varpi + \left(h(\lambda v - rh'u)\theta^M,\phi\right)_\varpi$$
$$-\frac{1}{Re\,\mathrm{Pr}}\left(\left(h^2\frac{\partial\theta^M}{\partial x},\frac{\partial\phi}{\partial x}\right)_\varpi + \left((\lambda^2 + r^2h'^2)\frac{\partial\theta^M}{\partial r},\frac{\partial\phi}{\partial r}\right)_\varpi\right.$$
$$\left.-2\left(rhh'\frac{\partial^2\theta^M}{\partial x\partial r},\phi\right)_\varpi + \left(\left[r(2h'^2 - hh'') + \frac{\lambda^2}{r}\right]\theta^M,\phi\right)_\varpi\right) = \left(S_\varphi,\phi\right)_\varpi \tag{4.3}$$

Matrix writing of the energyequation

The relation (4.3) can be put in matrix form

$$A\Theta = B \tag{4.4}$$

i.e. while writing according to the elements of each matrix

$$\sum_{k=0}^{M_x}\sum_{l=0}^{M_r}A_{ij,kl}\theta_{kl} = B_{ij} \tag{4.5}$$

where the matrix B is written : $B = (B_{ij}) = \left\langle S_\varphi, q_i(x)p_{2j}(r)\right\rangle$ with 0 $\le k \le M_x$; $0 \le l \le M_r$.

The unknown factor θ is written : $\Theta = (\theta_{kl}) = \left(\theta_{00},...,\theta_{1l},...,\theta_{k1},...,\theta_{M_xM_r}\right)^T$ with 0 $\le k \le M_x$; $0 \le l \le M_r$ et $A = (A_{ij,kl})$ avec 0 $\le k,i \le M_x$; $0 \le l, j \le M_r$ est la matrice du système.

It is pointed out that the matrix A is the discrete writing of the variational form $\left(L_\theta,\phi\right)$in which $\theta^M = \sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}q_k(x)p_{2l}(r)$ is the projection of θ on the basis

$\left(q_k(x)p_{2l}(r)\right)$and the trial function ϕ is taken equalizes with $\phi = q_i(x)p_{2j}(r)$. One obtains thus :

$$\left\langle L_\theta, q_i(x)p_{2j}(r)\right\rangle = \left\langle S_\varphi, q_i(x)p_{2j}(r)\right\rangle \tag{4.6}$$

with

$$\begin{cases} L_\theta = & \left(\dfrac{1}{r}\dfrac{\partial\psi}{\partial r} + \dfrac{1}{r}\dfrac{\partial\varphi}{\partial r}\right)\dfrac{\partial\theta}{\partial x} - \dfrac{1}{r}\dfrac{\partial\psi}{\partial x}\dfrac{\partial\theta}{\partial r} - \dfrac{1}{Re\,\mathrm{Pr}}\Delta_F\theta \\ S_\varphi = & \dfrac{1}{r}\dfrac{\partial\psi}{\partial x}\dfrac{\partial\theta_\varphi}{\partial r} + \dfrac{1}{Re\,\mathrm{Pr}}\Delta_F\theta_\varphi \end{cases} \tag{4.7}$$

The writing of has will be broken up into two parts: a part which gathers all the scalar products coming from the convective term and a part gathering the terms of diffusion.

Development of the convective terms of the member of (4.3)

One obtains the following expressions thus representing the terms of convection

$$C1 = \sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\left(\sum_{m=0}^{N_x}\sum_{n=0}^{N_r}\psi_{mn}\left\langle Q_m q_k', q_i\right\rangle . \left\langle \frac{P_{2n}'}{r}p_{2l}, p_{2j}\right\rangle\right) \tag{4.8}$$

$$C2 = \sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\left(\left\langle q_k', q_i\right\rangle . \left\langle \frac{1}{r}\frac{\partial\varphi}{\partial r}p_{2l}, p_{2j}\right\rangle\right) \tag{4.9}$$

$$C3 = \sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\left(\sum_{m=0}^{N_x}\sum_{n=0}^{N_r}\psi_{mn}\left\langle Q_m' q_k, q_i\right\rangle . \left\langle P_{2n}\frac{p_{2l}'}{r}, p_{2j}\right\rangle\right) \tag{4.10}$$

Development of the terms of diffusion

$$\begin{aligned} D1 &= -\frac{1}{Re\,\mathrm{Pr}}\sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\left\langle h^2 q_k''(x)p_{2l}(r), q_i(x)p_{2j}(r)\right\rangle \\ &= -\frac{1}{Re\,\mathrm{Pr}}\sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\left\langle h^2 q_k'', q_i\right\rangle . \left\langle p_{2l}, p_{2j}\right\rangle; \end{aligned}$$

$$\begin{aligned} D42 &= -\frac{1}{Re\,\mathrm{Pr}}\sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\left\langle (\lambda^2 + r^2 h'^2)q_k(x)p_{2l}''(r), q_i(x)p_{2j}(r)\right\rangle \\ &= -\frac{1}{Re\,\mathrm{Pr}}\lambda^2\sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\left\langle q_k, q_i\right\rangle\left\langle p_{2l}'', p_{2j}\right\rangle - \frac{1}{Re\,\mathrm{Pr}}\sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\left\langle h'^2 q_k, q_i\right\rangle\left\langle r^2 p_{2l}'', p_{2j}\right\rangle; \end{aligned}$$

$$\begin{aligned} D3 &= \frac{2}{Re\,\mathrm{Pr}}\sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\left\langle rhh' q_k'(x)p_{2l}'(r), q_i(x)p_{2j}(r)\right\rangle \\ &= \frac{2}{Re\,\mathrm{Pr}}\sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\left\langle hh' q_k', q_i\right\rangle\left\langle r p_{2l}', p_{2j}\right\rangle; \end{aligned}$$

$$D4 = -\frac{1}{Re\,\Pr}\sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\left\langle\left[r(2h'^2-hh'')+\frac{\lambda^2}{r}\right]q_k(x)p'_{2l}(r), q_i(x)p_{2j}(r)\right\rangle$$
$$= -\frac{1}{Re\,\Pr}\sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\left\langle(2h'^2-hh'')q_k,q_i\right\rangle.\left\langle rp'_{2l},p_{2j}\right\rangle$$
$$-\frac{1}{Re\,\Pr}\lambda^2\sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\left\langle q_k,q_i\right\rangle\left\langle\frac{p'_{2l}}{r},p_{2j}\right\rangle$$

From where we have finally a more elegant writing for the diffusion term of D4:

$$D4 = -\frac{1}{Re\,\Pr}\sum_{k=0}^{M_x}\sum_{l=0}^{M_r}\theta_{kl}\begin{pmatrix}\left\langle h^2q''_k,q_i\right\rangle.\left\langle p_{2l},p_{2j}\right\rangle+\lambda^2\left\langle q_k,q_i\right\rangle\left\langle p''_{2l},p_{2j}\right\rangle+\\ +\left\langle h'^2q_k,q_i\right\rangle\left\langle r^2p''_{2l},p_{2j}\right\rangle\\ -2\left\langle hh'q'_k,q_i\right\rangle\left\langle rp'_{2l},p_{2j}\right\rangle+\left\langle(2h'^2-hh'')q_k,q_i\right\rangle.\left\langle rp'_{2l},p_{2j}\right\rangle\\ +\lambda^2\left\langle q_k,q_i\right\rangle\left\langle\frac{p'_{2l}}{r},p_{2j}\right\rangle\end{pmatrix} \quad (4.11)$$

Remark:

The constitution of the elements of the matrix has is formed of the scalar products which require a linearization for their exploitation. Coefficients of the matrix A are thus given by :

$$A_{ij,kl} = \left(\sum_{m=0}^{N_x}\sum_{n=0}^{N_r}\psi_{mn}\left[\left\langle Q_mq'_k,q_i\right\rangle.\left\langle\frac{P'_{2n}}{r}p_{2l},p_{2j}\right\rangle-\left\langle Q'_mq_k,q_i\right\rangle.\left\langle P_{2n}\frac{p'_{2l}}{r},p_{2j}\right\rangle\right]\right.$$
$$\left.+\left\langle q'_k,q_i\right\rangle.\left\langle\frac{1}{r}\frac{\partial\varphi}{\partial r}p_{2l},p_{2j}\right\rangle\right) -\frac{1}{Re\,\Pr}\left(\left\langle h^2q''_k,q_i\right\rangle.\left\langle p_{2l},p_{2j}\right\rangle+\lambda^2\left\langle q_k,q_i\right\rangle\left\langle p''_{2l},p_{2j}\right\rangle \quad (4.12)\right.$$
$$+\left\langle h'^2q_k,q_i\right\rangle\left\langle r^2p''_{2l},p_{2j}\right\rangle +\left\langle(2h'^2-hh'')q_k,q_i\right\rangle.\left\langle rp'_{2l},p_{2j}\right\rangle+\lambda^2\left\langle q_k,q_i\right\rangle\left\langle\frac{p'_{2l}}{r},p_{2j}\right\rangle$$
$$\left.-2\left\langle hh'q'_k,q_i\right\rangle\left\langle rp'_{2l},p_{2j}\right\rangle\right)$$

Development of the second member of (4.3)

That is to say the source term $B_{ij} = \left\langle S_\varphi, q_i(x)p_{2j}(r)\right\rangle$ who can then be written:

$$\left\langle S_\varphi,q_i(x)p_{2j}(r)\right\rangle = \sum_{k=0}^{N_x}\sum_{l=0}^{N_r}\psi_{kl}\left\langle Q'_k,q_i\right\rangle\left\langle\frac{1}{r}\frac{\partial\theta_\varphi}{\partial r}P_{2l},p_{2j}\right\rangle + \lambda^2\left\langle 1,q_i(x)\right\rangle\left\langle\frac{\partial^2\theta_\varphi}{\partial r^2},p_{2j}(r)\right\rangle$$
$$+\left\langle h'^2,q_i(x)\right\rangle\left\langle r^2\frac{\partial^2\theta_\varphi}{\partial r^2},p_{2j}(r)\right\rangle \quad +\left\langle(2h'^2-hh''),q_i(x)\right\rangle\left\langle r\frac{\partial\theta_\varphi}{\partial r},p_{2j}(r)\right\rangle$$
$$+\lambda^2\left\langle 1,q_i(x)\right\rangle\left\langle\frac{1}{r}\frac{\partial\theta_\varphi}{\partial r},p_{2j}(r)\right\rangle \quad (4.13)$$

Remark

The resolution of the equation (4.5) requires as a preliminary the calculation of the various scalar products.

With this intention, the treatment of the various scalar products contained in (4.12) and (4.13) must take account of the properties of the polynomials of Chebyshev of first species. It is a question of checking that these products of polynomials are also linear combinations of the polynomials of Chebyshev for the implementation in the data-processing plan.

Conclusion

The study carried out in this work primarily concerned the numerical resolution of the equation of energy by the spectral method Galerkin. Indeed, the spectral method thus consists in building starting from a vector space of the polynomial functions to a variable and degree lower or equal to N, a space of approximation which uses the boundary conditions of the problem. To solve the problem by the aforementioned method, then amounts finding a solution approximate of the solution exact of the problem in the space of approximation so that the residue is small.

A work in the course of implementation will produce some results of the data processing of the problem but especially to examine the non stationary case. It will be a question of looking at the way in which the heat transfer is done by calculating the number of Nusselt.

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