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Common Fixed Point Theorem of Mappings without Continuity in Intuitionistic Fuzzy Metric Space

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Introduction

Since the concept of the introduction of fuzzy sets by Zadeh in 1965, many authors have investigated and generalized the result of fuzzy metric in different ways . Many authors obtained common fixed point theorems for weakly commuting map and R-weakly commuting mappings.

Alaca, Turkoglu and Yildiz defined the notion of intuitionistic fuzzy metric space as Park with the help of continuous t-norm and continuous t-conorm as a generalization of fuzzy metric spaces due to kramosil and Michalek .Further they introduced the notion of Cauchy sequences in intuitionistic fuzzy metric spaces.
We want to genaralize the results for multi-valued mappings .

Preliminaries

Definition 1 : A binary operation $*: [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-norm if $*$ is satisfying the following conditions:
(i) $*$ is commutative and associative,
(ii) $*$ is continuous,
(iii) $a * 1 = a$ for all $a \in [0, 1]$,
(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, $a, b, c, d \in [0, 1]$.

Definition 2 : A binary operation $\lozenge: [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-conorm if $\lozenge$ is satisfying the following conditions:
(i) $\lozenge$ is commutative and associative,
(ii) $\lozenge$ is continuous,
(iii) $a \lozenge 0 = a$ for all $a \in [0, 1]$,
(iv) $a \lozenge b \le c \lozenge d$ whenever $a \le c$ and $b \le d$, $a, b, c, d \in [0, 1]$.

Definition 3 : A 5-tuple $(X, M, N, *, \lozenge)$ is said to be an intuitionistic fuzzy metric spaces if X is an arbitrary set, $*$ is a continuous t-norm, $\lozenge$ is a continuous t-conorm and M, N are fuzzy sets on X^2 x $[0, \infty)$ satisfying the following conditions for all x, y, $z \in X$ and t, s > 0,

i. $M(x, y, t) + N(x, y, t) \leq 1$,
ii. $M(x, y, 0) = 0$,
iii. $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
iv. $M(x, y, t) = M(y, x, t)$,
v. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
vi. $M(x, y, .): [0, \infty) \to [0, 1]$ is left continuous,
vii. $\lim_{t\to\infty} M(x, y, t) = 1$ for all x, y in X,
viii. $N(x, y, 0) = 1$,
ix. $N(x, y, t) = 0$ for all $t > 0$ if and only if $x = y$,
x. $N(x, y, t) = N(y, x, t)$,
xi. $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$,
xii. $N(x, y, .): [0, \infty) \to [0, 1]$ is right continuous,
xiii. $\lim_{t\to\infty} N(x, y, t) = 0$ for all x, y in X.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Example 1 : Let (X, d) be a metric space. Define t-norm $a * b = \min \{a, b\}$ and t-conorm $a \lozenge b = \max \{a, b\}$ and for all $x, y \in X$ and $t > 0$,

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}, \quad N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)} \qquad \text{(a)}$$

Then (X, M, N, *, ◊) is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d the standard intuitionistic fuzzy metric.

Definition 4 : Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. Then

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ (denoted by $\lim_{n\to\infty} x_n = x$) if, for all $t > 0$,

$\lim_{n\to\infty} M(x_n, x, t) = 1, \quad \lim_{n\to\infty} N(x_n, x, t) = 0.$

(ii) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1, \quad \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0.$

Definition 5 : An intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be complete if and only if every Cauchy sequence in X is convergent.
It is called compact if every sequence contains a convergent sub sequence.

Lemma 1 : Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exists a number $k \in (0, 1)$ such that

(I) $M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$.

(II) $N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$,

for all $t > 0$ and $n = 1, 2, \ldots..$, then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 2 : Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and for all $x, y \in X$, $t > 0$ and if for a number $k \in (0, 1)$
$M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$
then $x = y$.

Definition 6 : Let A and B be maps from an intuitionistic fuzzy metric space (IFM-space) (X, M, N, *, ◊) into itself. The maps A and B are said to be compatible if for all $t > 0$,
$\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$ and
$\lim_{n\to\infty} N(ABx_n, BAx_n, t) = 0$
whenever $\{x_n\}$ is a sequence in X such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$.

Definition 7 : Let A and B be maps from an intuitionistic fuzzy metric space (IFM-space) (X, M, N, *, ◊) into itself. The maps A and B are said to be compatible of type (α) if for all $t > 0$,
$\lim_{n\to\infty} M(ABx_n, BBx_n, t) = 1$ and $\lim_{n\to\infty} N(ABx_n, BBx_n, t) = 0$,
$\lim_{n\to\infty} M(BAx_n, AAx_n, t) = 1$ and $\lim_{n\to\infty} N(BAx_n, AAx_n, t) = 0$
whenever $\{x_n\}$ is a sequence in X such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$.

Definition 8 : Let A and B be maps from an intuitionistic fuzzy metric space (IFM-space) (X, M, N, *, ◊) into itself. The maps A and B are said to be compatible of type (β) if for all $t > 0$,
$\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1$ and $\lim_{n\to\infty} N(AAx_n, BBx_n, t) = 0$,
whenever $\{x_n\}$ is a sequence in X such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some $z \in X$.

Definition 9 : Two self maps A and B on a set X are said to be weakly compatible if they commute at coincidence points ; i.e., if $Au = Bu$ for some $u \in X$, then $ABu = BAu$.

Definition 10 : Let (X, M, N, $*$, ◊) be an intuitionistic fuzzy metric space and consider $I : X \rightarrow X$ and $T : X \rightarrow CB(X)$. A point $z \in X$ is called a coincidence point of I and T if and only if $Iz \in Tz$.

Main Results

We prove the following theorems.

Theorem 1 : Let (X, M, N, *, ◊) be a complete intuitionistic fuzzy metric space and F, G : X→X be mappings satisfying the following conditions:

(1.1) G (X) ⊆ F (X);

(1.2) F and G are weak compatible;

(1.3) There exits 0 < k < 1 such that for all x, y ∈ X

$M(Gx, Gy, kt) \geq M(Fx, Fy, t)$

and

$N(Gx, Gy, kt) \leq N(Fx, Fy, t)$.

Then F and G have a unique common fixed point in X.

Proof : Let $x_0 \in X$ by (1.1) we can find x_0 such that $Fx_1 = Gx_0$ By induction we can define a sequence $\{x_n\}$ in X such that $Fx_n = Gx_{n-1}$ By induction again.

$$M(Fx_n, Fx_{n+1}, t) = M(Gx_{n-1}, Gx_n, t) \geq M(Fx_{n-1}, Fx_n, t/k)$$
$$\geq \ldots\ldots\ldots\ldots$$
$$\geq M(Fx_0, Fx_1, t/k^n)$$
$$N(Fx_n, Fx_{n+1}, t) = N(Gx_{n-1}, Gx_n, t) \leq N(Fx_{n-1}, Fx_n, t/k)$$
$$\leq \ldots\ldots\ldots\ldots$$
$$\leq N(Fx_0, Fx_1, t/k^n)$$

Hence for any positive integer p

$$M(Fx_n, Fx_{n+p}, t) \geq M(Fx_n, Fx_{n+1}, t/k)* \ldots^{(p\text{-times})} \ldots *M(Fx_{n+p-1}, Fx_{n+p}, t/k)$$
$$\geq M(Fx_0, Fx_1, t/pk^n)* \ldots^{(p\text{-times})} \ldots * M(Fx_0, Fx_1, t/pk^{n+p-1})$$
$$N(Fx_n, Fx_{n+p}, t) \leq N(Fx_n, Fx_{n+1}, t/k)\Diamond \ldots^{(p\text{-times})} \ldots \Diamond N(Fx_{n+p-1}, Fx_{n+p}, t/k)$$
$$\leq N(Fx_0, Fx_1, t/pk^n\}) \Diamond \ldots^{(p\text{-times})} \ldots \Diamond N(Fx_0, Fx_1, t/pk^{n+p-1})$$

By (VII) and (XIII) of Definition 3, since

$\lim_{n\to\infty} M(Fx_0, Fx_1, t/pk^n) = 1$;

$\lim_{n\to\infty} N(Fx_0, Fx_1, t/pk^n) = 0$,

tt follows that

$$\lim_{n\to\infty} M(Fx_n, Fx_{n+p}, t) \geq 1 *\text{-------------}* \geq 1 ,$$

and

$$\lim_{n\to\infty} N(Fx_n, Fx_{n+p}, t) \leq 0 \Diamond\text{-----------}\Diamond \leq 0 .$$

Thus $\{Fx_n\}$ is a Cauchy sequence and so by completeness of X, $\{Fx_n\}$ converges to a point z and $Gx_{n-1} = Fx_n$ converges also to a point z.

Since there exists a point $u \in X$ such that fu = z then using (1.3) we write

$M(Gu, Gx_{n+1}, kt) \geq M(Fu, Fx_{n+1}, t)$,

and $N(Gu, Gx_{n+1}, kt) \leq N(Fu, Fx_{n+1}, t)$.

Taking the limit $n \to \infty$, we have

$M(Gu, z, kt) \geq M(z, z, t) \to 1$,

andN $(Gu, z, kt) \leq N(z, z, t) \to 0$.

Therefore by Definition 3 we have Gu = z, therefore Gu = Fu = z

Since F and G are Weak compatible therefore F and G commute at their coincidence point i.e.
FGu = GFu i.e. Fz = Gz
Now we prove that Gz = z by (III) We have
$M(Gz, Gx_{n+1}, kt) \geq M(Fz, Fx_{n+1}, t)$,
and
$N(Gz, Gx_{n+1}, kt) \leq N(Fz, Fx_{n+1}, t)$.
Taking the limit $n \to \infty$, we obtain
$M(Gz\ z, kt) \geq M(Gz, z, t)$,
and $N(Gz, z, kt) \leq N(Gz, z, t)$.
Therefore by Lemma 2, we have Gz = z and therefore Gz = z = Fz
Thus z is a common fixed point of G and F .
This completes the proof of the theorem.

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