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Intuitionistic Fuzzy Q-Ideals in Q-Algebras

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Abstract

We consider the Intuitionistic fuzzification of the concept Q-ideal and the image (preimage) of Q-ideal in Q-algebra, and investigate some of there properties. Moreover, we introduce the notion of product of intuitionistic fuzzy Q-ideal in Q-algebras, and investigate some related properties.

Keywords: Q-algebra, Q-subalgebra, fuzzy Q-ideal, fuzzy Q-subalgebra, intuitionistic fuzzy Q-ideal, intuitionistic fuzzy image (preimage) of Q-ideal .

2000 Mathematics subjects classification: 03B52, 06F35, 03G25, 94D05.

Introduction

In 1965, L. A. Zadeh [11] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty in real physical world. As generalization of intuitionistic fuzzy subset was defined by K. T. Atanassov [1, 2] and it was extended to intuitionistic fuzzy ideal by Basnet and Benerjee [3, 4]. Intuitionistic fuzzy sets have also been defined by G. Takeuti and S. Titanti in [9]. G. Takeuti and S. Titanti however considered intuitionistic fuzzy logic in the narrow sense and derived a set theory from logic which they called intuitionistic fuzzy set theory. In 1991 O. G. Xi [10] applied the concept of fuzzy set to BCK-algebras which are introduced by Y. Imai and K. Iséki. It was known that the class of BCK-algebras is proper subclass of the class of BCI-algebras. In [5, 6] Q. P. Hu and X. Li introduced a wide class of abstract algebras BCH-algebra. they demonstrated that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. J. Neggers, S. S. Ahn and H. S. Kim [8] introduced a notion, called Q-algebras, which is generalization of BCH / BCI / BCK-algebras and generalized some theorems discussed in BCI-algebras. In this paper, we introduce the notion of intuitionistic fuzzy Q-ideals in Q-algebras and fuzzy intuitionistic image (preimage) of Q-ideals in Q-algebras. We also introduce the Cartesian product of two intuitionistic fuzzy Q-ideals in Q-algebras and investigate some results.

Preliminaries

Definition 2.1 [7]

An algebraic system $(X,*,0)$ of type (2, 0) is called a BCK-algebra if it satisfying the following conditions:

(BCI-1) $((x*y)*(x*z))*(z*y)=0,$

(BCI-2) $(x*(x*y))*y=0,$

(BCI-3) $x*x=0,$

(BCI-4) $x*y=y*x=0$ implies $x=y,$

(BCK-5) $0*x=0,$ for all $x,y,z\in X$.

In a BCK-algebra *X*, we can define a partial ordering"$\leq$" by: $x\leq y$ if and only if $x*y=0$.

Definition 2. 2 [8]

An algebraic system $(X,*,0)$ of type (2, 0) is called a Q-algebra if it satisfying the following axioms:

(1) $x*x=0$,

(2) $x*0=x$,

(3) $(x*y)*z=(x*z)*y$ for all x,y and $z\in X$.

we can define a partial ordering"$\leq$" by $x\leq y$ if and only if $x*y=0$.

Definition 2. 3 [8]

A non empty subset *S* of a Q-algebra *X* is said to be Q-subalgebra of *X*, if $x,y\in S$ Implies $x*y\in S$.

Definition 2. 4 [7]

A non empty subset *I* of a BCK-algebra *X* is said to be a BCK-ideal of *X* if it satisfies:

(I_1) $0\in I$,

(I_2) $x*y\in I$ and $y\in I$ implies $x\in I$ for all $x,y\in X$.

Definition 2. 5

A non empty subset *I* of a Q-algebra *X* is said to be a Q-ideal of *X* if it satisfies:

(I_1) $0\in I$,

(Q_1) $(x*y)*z\in I$ and $y\in I$ imply $x*z\in I$ for all x,y and $z\in X$.

Proposition 2.6
Let $(X,*,0)$ be a Q-algebra and *I* is a Q-ideal of *X*, then *I* is a BCK-ideal of *X*.
Proof. I_1 is satisfied.
Put in (Q_1) $z=0$, we have $x*y\in I$ and $y\in I$ imply $x*0=x\in I$, for all x, y and $z\in X$ i.e. *I* is a BCK-ideal of *X*.

Example 2.7
Let *X* = {0,1,2,3,4} be a set with a binary operation $*$ defined by the following table:

$*$	0	1	2	3	4
0	0	0	0	0	4
1	1	0	0	1	4
2	2	2	0	0	4
3	3	0	3	0	4
4	4	4	4	4	0

Then $(X,*,0)$ is a Q-algebra, Let A = {0,1,2,3} be a subset of X. Then A is a Q-ideal, but it is not BCK-subalgebra, since $((2*1)*(2*3))*(3*1)=2*0=2\neq 0$. On the other hand $A^\backslash$ ={0,1,2} is a Q-subalgebra, which is not BCK-ideal because $0=3*1\in A^\backslash$, $1\in A^\backslash$ but $3\notin A^\backslash$. Thus in Q-algebras BCK-ideals and Q-subalgebras are independent concepts.

Fuzzy Q-ideal

Definition 3.1 [10]
Let *X* be a BCK-algebra. a fuzzy set μ in *X* is called a fuzzy BCK-ideal of *X* if it satisfies:
(FI_1) $\mu(0)\geq\mu(x)$,
(FI_2) $\mu(x)\geq\min\{\mu(x*y),\mu(y)\}$, for all x, y and $z\in X$.

Definition 3.2
Let *X* be a Q-algebra. A fuzzy set μ in *X* is called a fuzzy Q-ideal of *X* if it satisfies:
(FI_1) $\mu(0)\geq\mu(x)$,
(FQ) $\mu(x*z)\geq\min\{\mu((x*y)*z),\mu(y)\}$, for all x, y and $z\in X$.

Lemma 3.3
Any fuzzy Q-ideal of Q-algebra is a fuzzy BCK-ideal of *X*.

Proof. In definition 3.2. Put $z=0$, then (FQ) gives that

$$\mu(x*0)\geq\min\{\mu((x*y)*0),\mu(y)\}=\min\{\mu(x*y),\mu(y)\}, \text{ since } x*0=x.$$

Intuitionistic fuzzy Q-ideal in Q-algebra

Definition 4.1

An Intuitionistic fuzzy set (briefly IFS) A in a nonempty set X is an object having the form $A=\{(x,\alpha_A(x),\beta(x))\,|\,x\in X\}$, where the function α_A: $X\to[0,1]$ and β_A: $X\to[0,1]$ denote the degree of membership and degree of non membership, respectively and $0\le\alpha_A(x)+\beta_A(x)\le 1$ for all $x\in X$. An intuitionistic fuzzy set $A=\{(x,\alpha_A(x),\beta(x))\,|\,x\in X\}$ in X can be identified to an order pair (α_A,β_B) in $I^X\times I^X$.

We shall use the symbol $A=(\alpha_A,\beta_A)$ for IFS $A=\{(x,\alpha_A(x),\beta(x))\,|\,x\in X\}$.

Definition 4.2

An IFS $A=(\alpha_A,\beta_A)$ in a Q-algebra X is called an intuitionistic fuzzy Q-subalgebra of X if it satisfies the following

(I S_1) $\alpha_A(x*y)\ge\min\{\alpha_A(x),\alpha_A(y)\}$

(I S_2) $\beta_A(x*y)\le\max\{\beta_A(x),\beta_A(y)\}$, for all $x,y\in X$.

Example 4.3

Let $X=\{0,1,2,3,4\}$ as in example 2.7, and $A=(\alpha_A,\beta_A)$ be an IFS in X defined by $\alpha_A(0)=\alpha_A(2)=\alpha_A(3)=\alpha_A(4)=0.7<0.3=\alpha_A(1)$, and $\beta_A(0)=\beta_A(2)=\beta_A(3)=\beta_A(4)=0.2<0.5=\beta_A(1)$. Then $A=(\alpha_A,\beta_A)$ is an intuitionistic fuzzy Q-subalgebra of X.

Lemma 4.4

Every, intuitionistic fuzzy subalgebra $A=(\alpha_A,\beta_A)$ of X satisfies the inequalities

$\alpha_A(0)\ge\alpha_A(x)$, and $\beta_A(0)\le\beta_A(x)$ for all $x\in X$.

Proof: $\alpha_A(0)=\alpha_A(x*x)\ge\min\{\alpha_A(x),\alpha_A(x)\}=\alpha_A(x)$, and $\beta_A(0)=\beta_A(x*x)\le\max\{\beta_A(x),\beta_A(x)\}=\beta_A(x)$.

Definition 4.5

An IFS $A=(\alpha_A,\beta_A)$ in X is called an intuitionistic fuzzy Q-ideal of X if it satisfies the following inequalities:

(IFQ_1) $\alpha_A(0)\ge\alpha_A(x)$ and $\beta_A(0)\le\beta_A(x)$,

(IFQ_2) $\alpha_A(x*z)\ge\min\{\alpha_A((x*y)*z),\alpha_A(y)\}$,

(IFQ_3) $\beta_A(x*z)\le\max\{\beta_A((x*y)*z),\beta_A(y)\}$, for all $x,y,z\in X$.

Example 4.6

Let $X=\{0,1,2,3,4\}$ be a set with a binary operation $*$ define by the following table:

$*$	0	1	2	3	4
0	0	0	0	0	4
1	1	0	0	1	4
2	2	2	0	0	4
3	3	0	3	0	4
4	4	4	4	4	0

Define IFS $A=(\alpha_A,\beta_A)$ in X as follows $\alpha_A(0)=\alpha_A(2)=1$, $\alpha_A(1)=\alpha_A(3)=\alpha_A(3)=t$. $\beta_A(0)=\beta_A(2)=0$, $\beta_A(1)=\beta_A(3)=\beta_A(4)=s$. Where $t,s\in[0,1]$ and $t+s\le 1$. By routine calculations we know that $A=(\alpha_A,\beta_A)$ is an intuitionistic fuzzy Q-ideal of *X*.

Lemma 4.7
Let $A=(\alpha_A,\beta_A)$ be an intuitionistic fuzzy Q-ideal of *X*. if the inequality $x*y\le z$ hold in *X*, then

$$\alpha_A(x)\ge\min\{\alpha_A(y),\alpha_A(z)\},\ \beta_A(x)\ge\max\{\beta_A(y),\beta_A(z)\}.$$

Proof: Let $x,y,z\in X$ be such that $x*y\le z$, then $(x*y)*z=0$ and in (FIQ2) put $z=0$, we get $\alpha_A(x*0)=\alpha_A(x)\ge\min\{\alpha_A(x*y),\alpha_A(y)\}\ge$ $\min\{\alpha_A((x*z)*y),\alpha_A(z),\alpha_A(y)\}=\min\{\alpha_A((x*y)*z),\alpha_A(z),\alpha_A(y)\}=$ $\min\{\alpha_A(0),\alpha_A(z),\alpha_A(y)\}=\min\{\alpha_A(y),\alpha_A(z)\}$. Similarly for $\beta_A(x)$.

Lemma 4.8
Let $A=(\alpha_A,\beta_A)$ be an intuitionistic fuzzy Q-ideal of *X*. If $x\le y$ in *X*, then

$$\alpha_A(x)\ge\alpha_A(y),\ \beta_A(x)\le\beta_A(y).$$

Proof: Let $x,y\in X$ be such that $x\le y$, then $x*y=0$ and so (IFQ2)

$\alpha_A(x*z)\ge\min\{\alpha_A((x*z)*y),\alpha_A(y)\}$ put $z=0$ in (IFQ2), we get

$\alpha_A(x*0)=\alpha_A(x)\ge\min\{\alpha_A((x*y)*0),\alpha_A(y)\}=\min\{\alpha_A((x*y),\alpha_A(y)\}=$ $\min\{\alpha_A(0),\alpha_A(x)\}=\alpha_A(y)$.

Similarly for (IFQ3) $\beta_A(x*z)\le\max\{\beta_A((x*y)*z),\beta_A(y)\}$, put $z=0$ in (IFQ3), we get

$\beta_A(x*0)=\beta_A(x)\le\max\{\beta_A((x*y)*0),\beta_A(y)\}=\max\{\beta_A(x*y),\beta_A(y)\}=$
$\max\{\beta_A(0),\beta_A(y)\}=\beta_A(y)$.

Definition 4.9
For any $t\in[0,1]$ and a fuzzy set μ in a nonempty set *X*, the set

$U(\mu,t) := \{x \in X \mid \mu(x) \geq t\}$ is called an upper t-level cut of μ, and the set $L(\mu,t) := \{x \in X \mid \mu(x) \leq t\}$ is called a lower t-level cut of μ.

Theorem 4.10
An IFS $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy Q-ideal of *X* if and only if for all $s,t \in [0,1]$, the set $U(\alpha_A, t)$ and $L(\beta_A, s)$ are either empty or Q-ideal of *X*.

Proof: Let $A = (\alpha_A, \beta_A)$ be an intuitionistic fuzzy Q-ideal of *X* and $U(\alpha_A, t) \neq \phi \neq L(\beta_A, s)$.

Since $\alpha_A(0) \geq t$ and $\beta_A(0) \leq s$, let $x, y, z \in X$ be such that $(x * y) * z \in U(\alpha_A, t)$. $y \in U(\alpha_A, t)$, then $\alpha_A((x * y) * z) \geq t$ and $\alpha_A(y) \geq t$, it follows that $\alpha_A(x * z) \geq \min\{\alpha_A((x * y) * z), \alpha_A(y)\} \geq t$, so that $x * z \in U(\alpha_A, t)$. Hence $U(\alpha_A, t)$ is an Q-ideal of *X*. Let $x, y, z \in X$ be such that $(x * y) * z \in L(\beta_A, s)$ and $y \in L(\beta_A, s)$, then $\beta_A((x * y) * z) \leq s$ and $\beta_A(y) \leq s$ which imply that $\beta_A(x * z) \leq \max\{\beta_A((x * y) * z), \beta_A(y)\} \leq s$. Thus $x * z \in L(\beta_A, s)$ and therefore $L(\beta_A, s)$ is a Q-ideal of *X*.

Conversely, assume that for each $s,t \in [0,1]$, the sets $U(\alpha_A, t)$ and $L(\beta_A, s)$ are either empty or Q-ideal of *X*. For any $x \in X$, let $\alpha_A(x) = t$ and $\beta_A(x) = s$. Then $x \in U(\alpha_A, t) \cap L(\beta_A, s)$ and so $U(\alpha_A, t) \neq \phi \neq L(\beta_A, s)$. Since $U(\alpha_A, t)$ and $L(\beta_A, s)$ are Q-ideal of *X*, therefore $0 \in U(\alpha_A, t) \cap L(\beta_A, s)$. Hence $\alpha_A(0) \geq t = \alpha_A(x)$ and $\beta_A(0) \leq s = \beta_A(x)$ for all $x \in X$.

If there exist $x', y', z' \in X$ be such that $\alpha_A(x' * z') \geq \min\{\alpha_A((x' * y') * z'), \alpha_A(y')\}$. Then by taking $t_0 := \frac{1}{2}\{\alpha_A(x' * z') + \min\{\alpha_A((x' * y') * z'), \alpha_A(y')\}\}$, we get $\alpha_A(x' * z') < t_0 < \min\{\alpha_A((x' * y') * z'), \alpha_A(y')\}$ and hence $(x' * y') \notin U(\alpha_A, t_0)$, $(x' * y') * z' \in U(\alpha_A, t_0)$ and $y' \in U(\alpha_A, t_0)$, i.e. $U(\alpha_A, t_0)$ is not an Q-ideal of *X*, which make a contradiction. Finally assume that there exist $a, b, c \in X$ such that $\beta_A(a * c) > \max\{\beta_A((a * b) * c), \beta_A(b)\}$. Then by taking $s_0 := \frac{1}{2}\{\beta_A(a * c) + \max\{\alpha_A((a * b) * c), \beta_A(b)\}\}$, we get $\max\{\beta_A((a * b) * c), \beta_A(b)\} < s_0 < \beta_A(a * c)$ therefore $((a * b) * c) \in L(\beta_A, s_0)$ and $b \in L(\beta_A, s_0)$ but $(a * c) \notin L(\beta_A, s_0)$, which make a contradiction. This completes the proof.

Homomorphism of Q-algebra

Definition 5.1

Let $(X,*,0)$ and $(Y,*',0')$ be Q-algebras. A mapping $f: X \to Y$ is said to be a homomorphism if $f(x*y) = f(x)*' f(y)$ for all $x, y \in X$. Note that if $f: X \to Y$ is a homomorphism of Q-algebras, then $f(0) = 0'$.

Let $f: X \to Y$ be a homomorphism of Q-algebras for any IFS $A = (\alpha_A, \beta_A)$ in *Y*, we define new IFS $A^f = (\alpha_A^f, \beta_A^f)$ in *X* by $\alpha_A^f(x) := \alpha_A(f(x))$, and $\beta_A^f(x) := \beta_A(f(x))$ for all $x \in X$.

Theorem 5.2

Let $f: X \to Y$ be a homomorphism of Q-algebras. If the IFS $A = (\alpha_A, \beta_A)$, is an intuitionistic fuzzy Q-ideal of *Y*, then the IFS $A^f = (\alpha_A^f, \beta_A^f)$ in *X* is an intuitionistic fuzzy Q-ideal of *X*.

Proof: $\alpha_A^f(x) := \alpha_A(f(x)) \le \alpha_A(0) = \alpha_A(f(0)) = \alpha_A^f(0)$, and $\beta_A^f(x) := \beta_A(f(x)) \ge \beta_A(0) = \beta_A(f(0)) = \beta_A^f(0)$, for all $x, y \in X$.

And

$$\alpha_A^f(x*z) := \alpha_A(f(x*z)) = \alpha_A(f(x)*' f(z)) \ge \min\{\alpha_A((f(x)*' f(y))*' f(z)), \alpha_A(f(y))\}$$
$$= \min\{\alpha_A(f(x*y)*' f(z)), \alpha_A(f(y))\} = \min\{\alpha_A(f((x*y)*z), \alpha_A(f(y))\}$$
$$= \min\{\alpha_A^f((x*y)*z), \alpha_A^f(y)\},$$

and

$$\beta_A^f(x*z) := \beta_A(f(x*z)) = \beta_A(f(x)*' f(z)) \le \max\{\beta_A((f(x)*' f(y))*' f(z)), \beta_A(f(y))\}$$
$$= \max\{\beta_A(f(x*y)* f(z)), \beta_A(f(y))\} = \max\{\beta_A(f((x*y)*z), \beta_A(f(y))\}$$
$$= \max\{\beta_A^f((x*y)*z), \beta_A^f(y)\}.$$

Hence $A^f = (\alpha_A^f, \beta_A^f)$ is an intuitionistic fuzzy Q-ideal in *X*.

Product of intuitionistic fuzzy Q-ideal

Definition 6.1

Let μ and λ be are two fuzzy sets in the set *X*. the Cartesian product $\lambda \times \mu : X \times X \to [0,1]$ is defined by, $(\lambda \times \mu)(x, y) = \min\{\lambda(x), \mu(y)\}$, and $\lambda_A \times \lambda_B : X \times X \to [0,1]$ is defined by $(\lambda_A \times \lambda_B)(x, y) = \max\{\lambda_A(x), \lambda_B(y)\}$ for all $x, y \in X$.

Definition 6.2

Let $A = (X, \lambda_A, \mu_A)$ and $B = (X, \lambda_B, \mu_B)$ are two IFS of *X*, the Cartesian product $A \times B = (X \times X, \mu_A \times \mu_B, \lambda_A \times \lambda_B)$ is defined by

$(\mu_A \times \mu_B)(x,y) = \min\{\mu_A(x), \mu_B(y)\}$ and $(\lambda_A \times \lambda_B)(x,y) = \max\{\lambda_A(x), \lambda_B(y)\}$ where $\mu_A \times \mu_B : X \times X \to [0,1]$, for all $x, y \in X$.

Remark 6.3
Let *X* and *Y* be a Q-algebras, we define $*$ on $X \times Y$ by: For every $(x,y),(u,v) \in X \times Y$ $(x,y)*(u,v) = (x*u, y*v)$ then clearly $(X * Y ; *, (0,0))$ is Q-algebra.

Proposition 6.4
Let $A = (X, \lambda_A, \mu_A)$, $B = (X, \lambda_B, \mu_B)$ are intuitionistic fuzzy Q-ideal of *X*, then $A \times B$ is intuitionistic fuzzy Q-ideal of $X \times X$.

Proof: $(\mu_A \times \mu_B)(0,0) = \min\{\mu_A(0), \mu_B(0)\} \geq \min\{\mu_A(x), \mu_B(y)\} = (\mu_A \times \mu_B)(x,y)$, for all $x, y \in X$ .And $(\lambda_A \times \lambda_B)(0,0) = \max\{\lambda_A(0), \lambda_B(0)\} \leq \max\{\lambda_A(x), \lambda_B(y)\} = (\lambda_A \times \lambda_B)(x,y)$, for all $x, y \in X$. Now let $(x_1, x_2),(y_1, y_2),(z_1, z_2) \in X \times X$,
then

$$
\begin{aligned}
&\min\{(\mu_A \times \mu_B)(((x_1,x_2)*(y_1,y_2))*(z_1,z_2)),(\mu_A \times \mu_B)(y_1,y_2)\}\\
&= \min\{(\mu_A \times \mu_B)((x_1*y_1, x_2*y_2)*(z_1,z_2)),(\mu_A \times \mu_B)(y_1,y_2)\}\\
&= \min\{(\mu_A \times \mu_B)((x_1*y_1)*z_1,(x_2*y_2)*z_2),(\mu_A \times \mu_B)(y_1,y_2)\}\\
&= \min\{\{\mu_A((x_1*y_1)*z_1), \mu_B((x_2*y_2)*z_2)\},\{\mu_A(y_1), \mu_B(y_2)\}\}\\
&= \min\{\min\{\mu_A((x_1*y_1)*z_1), \mu_A(y_1)\}, \min\{\mu_B((x_2*y_2)*z_2), \mu_B(y_2)\}\\
&= \min\{\min\{\mu_A((x_1*y_1)*z_1), \mu_A(y_1)\}, \min\{\mu_B((x_2*y_2)*z_2), \mu_B(y_2)\}\\
&\leq \min\{\mu_A(x_1*z_1), \mu_B(x_2*z_2)\}\\
&= (\mu_A \times \mu_B)(x_1*z_1, x_2*z_2) .
\end{aligned}
$$

And

$$
\begin{aligned}
&\max\{(\lambda_A \times \lambda_B)(((x_1,x_2)*(y_1,y_2))*(z_1,z_2)),(\lambda_A \times \lambda_B)(y_1,y_2)\}\\
&= \max\{(\lambda_A \times \lambda_B)((x_1*y_1, x_2*y_2)*(z_1,z_2)),(\lambda_A \times \lambda_B)(y_1,y_2)\}\\
&= \max\{(\lambda_A \times \lambda_B)((x_1*y_1)*z_1,(x_2*y_2)*z_2),(\lambda_A \times \lambda_B)(y_1,y_2)\}\\
&= \max\{\{\lambda_A((x_1*y_1)*z_1), \lambda_B((x_2*y_2)*z_2)\},\{\lambda_A(y_1), \lambda_B(y_2)\}\}\\
&= \max\{\max\{\lambda_A((x_1*y_1)*z_1), \lambda_A(y_1)\}, \max\{\lambda_B((x_2*y_2)*z_2), \lambda_B(y_2)\}\\
&= \max\{\max\{\lambda_A((x_1*y_1)*z_1), \lambda_A(y_1)\}, \max\{\lambda_B((x_2*y_2)*z_2), \lambda_B(y_2)\} =\\
&\geq \max\{\lambda_A(x_1*z_1), \lambda_B(x_2*z_2)\}\ (\lambda_A \times \lambda_B)(x_1*z_1, x_2*z_2) .
\end{aligned}
$$

This completes the proof.

Definition 6.5
Let $A = (X, \lambda_A, \mu_A)$ and $B = (X, \lambda_B, \mu_B)$ are intuitionistic fuzzy subset of Q-algebras *X*. for $s,t \in [0,1]$ the set $U(\mu_A \times \mu_B, s) := \{(x,y) \in X \times X \mid (\mu_A \times \mu_B)(x,y) \geq s\}$ is

called upper level of $(\mu_A \times \mu_B)(x,y)$ and $L(\lambda_A \times \lambda_B, t) := \{(x,y) \in X \times X \mid (\lambda_A \times \lambda_B)(x,y) \le t\}$ is called lower level of $(\lambda_A \times \lambda_B)(x,y)$.

Theorem 6.6
An intuitionistic fuzzy set $A = (X, \lambda_A, \mu_A)$ and $B = (X, \lambda_B, \mu_B)$ are intuitionistic fuzzy Q-ideal of *X* if and only if the non empty set upper *s*-level cut $U(\mu_A \times \mu_B, s)$ and the non empty *t*-level cut $L(\lambda_A \times \lambda_B, t)$ are Q-ideal of $X \times X$ for any $s, t \in [0,1]$.

Proof: Let $A = (X, \lambda_A, \mu_A)$ and $B = (X, \lambda_B, \mu_B)$ are intuitionistic fuzzy Q-ideal of *X*, therefore for any $(x,y) \in X \times X$, $\mu_A \times \mu_B(0,0) = \min\{\mu_A(0), \mu_B(0)\} \ge \min\{\mu_A(x), \mu_B(y)\} = \mu_A \times \mu_B(x,y)$ and for $s \in [0,1]$, if $(\mu_A \times \mu_B)(x_1 * x_2, z_1 * z_2) = (\mu_A \times \mu_B)(x_1 * z_1, x_2 * z_2) \ge s$, therefore $(x_1 * z_1, x_2 * z_2) \in U(\mu_A \times \mu_B, s)$ .Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$ be such that $(((x_1, x_2) * (y_1, y_2)) * (z_1, z_2)) \in U(\mu_A \times \mu_B, s)$, and $(y_1, y_2) \in U(\mu_A \times \mu_B, s)$.

Now $(\mu_A \times \mu_B)((x_1, x_2) * (z_1, z_2)) = (\mu_A \times \mu_B)(x_1 * z_1, x_2 * z_2) \ge$
$\min\{(\mu_A \times \mu_B)(((x_1, x_2) * (y_1, y_2)) * (z_1, z_2)), (\mu_A \times \mu_B)(y_1, y_2)\}$
$= \min\{(\mu_A \times \mu_B)((x_1 * y_1) * z_1, (x_2 * y_2) * z_2), (\mu_A \times \mu_B)(y_1, y_2)\} \ge \min\{s, s\} = s$,

therefore $(x_1, x_2) * (z_1, z_2) \in U((\mu_A \times \mu_B)(x,y), s)$ is Q-ideal of $X \times X$. In a similar way, we can prove that $L((\lambda_A \times \lambda_B)(x,y), t)$ is a Q-ideal of $X \times X$. This completes the proof.

References

[1] K. Atanassov, intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1) (1986), 87-96

[2] K. Atanassov, New operations defined over the intuitionistic fuzzy sets, fuzzy sets and systems, 61 (2) 1994, 137-142.

[3] D. Basnet, intuitionistic fuzzy ideals, J. of fuzzy mathematics, 15 (4) 2007, 811-819.

[4] B. Benerjee and D. Basnet, intuitionistic fuzzy subrings and ideals, J. of fuzzy mathematics, 11 (1) 2003, 139-155.

[5] Q. P. Hu and X. Li, On BCH-algebras, Math. Sem. Notes Kobe Univ. 2 (1983), 313-320.

[6] Q. P. Hu and X. Li, On proper BCH-algebras, Math. Japon. 30 (1985), 659-661.

[7] K. Iséki and S. Tanaka, An introduction to the theory of BCK-algebras, Math. Japonica 23 (1) (1978),1-26.

[8] J. Neggers, S. S. Ahn and H. S. Kim, On Q-algebras, Int. J. Math. Math. Sci. 27(12) (2001), 749- 757.
[9] G. TaKeuti and S. Titants, Intuitionistic fuzzy Logic and Intuitionistic fuzzy set theory, Journal of Symbolic Logic, 49 (1984), 851-866.
[10] O. G. Xi, Fuzzy BCK-algebra, Math. Japon. 36 (1991) 935-942.
[11] L. A. Zadeh, Fuzzy sets, Inform. And Control 8 (1965) 338-353.