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Quantum Algorithm for Eulerian Graph Problem

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Abstract

A quantum algorithm for the Eulerian graph problem and its example are reported. When nodes are connected by edges, a route that traverses each edge just for once is obtained. When it is assumed that a computational complexity of a classical computation is Z, a computational complexity of the quantum algorithm by using quantum phase inversion gates and quantum inversion about mean gates is about $Z^{1/2}$. Therefore, a high-speed process becomes possible.

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1. Introduction

A quantum computer can compute speedily by a parallel computation that uses quantum entangled states. Deutsch-Jozsa's algorithm for the rapid solution [1–3], Shor's algorithm for the factorization [2–4], Grover's algorithms for the database search [2,5–7], Durr-Hoyer's algorithm for finding a minimum [8], Brassard-Hoyer-Tapp's algorithm for counting [9], and so on are known. A quantum algorithm for the knapsack problem has recently been reported by Fujimura [10]. The Eulerian graph problem [11] is examined this time to expand the application range of the quantum computation. Therefore, its result is reported.

2. Eulerian graph problem

When nodes are connected by edges, a route that traverses each edge just for once is obtained. There are a unicursal route and a traversable route.

3. Quantum algorithm

3.1. Premise

A graph is assumed that k nodes connected by n edges. There are $h_{u,v}$ edges between the u-th node and the v-th node $[0 \leq u < v \leq k-1]$. Where, there aren't any edges from a node to same node directly. It is assumed that t is $k + \sum_{v=1,u<v}^{k-1} \sum_{u=0}^{k-2} (h_{u,v} - 1)$, and m is $n + \sum_{v=1,u<v}^{k-1} \sum_{u=0}^{k-2} (h_{u,v} - 1)$. When an the w-th node $[w = 0, \ldots, t-1]$ has Q_w edges, R_w is a minimum integer that is $Q_w/2$ or more. Where, $\sum_{w=0}^{t-1} R_w$ is a necessary number of nodes that the graph changes a line that may be contained same nodes. Therefore, the graph has 3 cases.

(I) A case of $\sum_{w=0}^{t-1} R_w = m$. This case is a unicursal route.

(II) A case of $\sum_{w=0}^{t-1} R_w = m + 1$. This case is a traversable route.

(III) A case of $\sum_{w=0}^{t-1} R_w > m + 1$. This case isn't the Eulerian graph. Therefore, it isn't necessary that we calculate this case.

A length of the δ-th edge $[\delta = 0, \ldots, h_{u,v} - 1]$ between the i-th node and the j-th node $[0 \leq i < j \leq t-1]$ is $L_{i,j}^{(\delta)}$. Where, $L_{(p,q)}^{(0)}$ [p and q are integers.] is $L_{p,q}^{(0)}$ at $p < q$, or $L_{q,p}^{(0)}$ at $q < p$. A total length of edges on the Eulerian graph is L^*. At (I), L^* is L_{uni}^*, and at (II), it is L_{tra}^*. One edge of $h_{u,v}$ is $L_{u,v}^{(0)}$, and remains $L_{u,v}^{(1)}, \ldots, L_{u,v}^{(h_{u,v}-2)}$, and $L_{u,v}^{(h_{u,v}-1)}$ are $L_{u,g_{u,v}^{(1)}}^{(0)} + L_{v,g_{u,v}^{(1)}}^{(0)}, \ldots, L_{u,g_{u,v}^{(h_{u,v}-2)}}^{(0)} + L_{v,g_{u,v}^{(h_{u,v}-2)}}^{(0)}$, and $L_{u,g_{u,v}^{(h_{u,v}-1)}}^{(0)} + L_{v,g_{u,v}^{(h_{u,v}-1)}}^{(0)}$, respectively. Where, $g_{u,v}^{(1)}, \ldots, g_{u,v}^{(h_{u,v}-2)}$, and $g_{u,v}^{(h_{u,v}-1)}$ are middle nodes. When there aren't edges between the i-th node and the j-th node, $L_{i,j}^{(0)}$ is assumed $2L^*$. Moreover, $L_{i,i}^{(0)}$ and $L_{j,j}^{(0)}$ are assumed $2L^*$. These edges that are assumed $2L^*$ are excluded by gate actions. The cases of (I) and (II) are as follows.

3.2. Case of $\sum_{w=0}^{t-1} R_w = m$ [Unicursal route]

It is assumed that k nodes are $P_0(x_0, y_0, z_0)$, $P_1(x_1, y_1, z_1)$, ..., $P_{k-2}(x_{k-2}, y_{k-2}, z_{k-2})$, and $P_{k-1}(x_{k-1}, y_{k-1}, z_{k-1})$. There are $h_{u,v}$ edges between the u-th node and the v-th node. New nodes are assumed $P_k(x_k, y_k, z_k)$, ..., $P_{t-2}(x_{t-2}, y_{t-2}, z_{t-2})$, and $P_{t-1}(x_{t-1}, y_{t-1}, z_{t-1})$. These nodes are $g_{u,v}^{(1)}, \ldots, g_{u,v}^{(h_{u,v}-1)}$, and so on. When P_0 starts and ends, we consider that combinations of t nodes that may be contained same nodes. An answer contains a counter course for a course. Therefore, the final answer has 2 routes at least. Therefore, a computational complexity of a classical computation of a unicursal route [$= Z_{uni}$] is $(1/2)(m-1)!/((R_0-1)!R_1!\ldots R_{t-1}!)$.

First of all, quantum registers $|a_0>, |a_1>, \ldots, |a_{m-2}>$, and $|b>$ are prepared. When α is a minimum integer that is $log_2 t$ or more, each of $|a_f>$ that f is an integer from 0 to $m-2$ is consisted of α quantum bits [=qubits]. States of $|a_f>$ and $|b>$ are a_f and b, respectively.

Step 1: Each qubit of $|a_f>$ and $|b>$ is set $|0>$.

Step 2: The Hadamard gate $\boxed{\mathrm{H}}$ [2, 3] acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^{\alpha})^{m-1}$.

Step 3: It is assumed that a quantum gate (A) changes $|b>$ for $|b+1>$ at $a_f \geq t$, or it doesn't change $|b>$ at $a_f < t$. As a target state for $|b>$ is 0, quantum phase inversion gates (PI) and quantum inversion about mean gates (IM) [2, 5–7] act on $|b>$. When β is a minimum even integer that is $(2^{\alpha}/t)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is β because they are a couple. Next, an observation gate (OB) observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_{m-2}>$. Therefore, each state of $|a_f>$ is $0, 1, \ldots, t-2$, or $t-1$, and the total states become $t^{m-1}[= W_0]$.

Step 4: It is assumed that a quantum gate (B_0) changes $|b>$ for $|b+1>$ at $a_0 = 0$, or it doesn't change $|b>$ at $a_0 \neq 0$. Similarly, these actions are repeated sequentially from $|a_1>$ to $|a_{m-2}>$. As a target state for $|b>$ is R_0-1, (PI) and (IM) act on $|b>$. By these actions, states of $W_1 = (m-1)!(t-1)^{(m-1)-(R_0-1)}/(((m-1)-(R_0-1))!(R_0-1)!)$ are selected. When γ_1 is a minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is γ_1. Next, (OB) observes $|b>$. By these actions, states of W_1 remain. And $|b>$ is set $|0>$. Similarly, the quantum gate $(B_s)[s = 1, \ldots, t-2$. s is an integer.] changes $|b>$ for $|b+1>$ at $a_0 = s$, or it doesn't change $|b>$ at $a_0 \neq s$. These actions are repeated sequentially from $|a_1>$ to $|a_{m-2}>$. As a target state for $|b>$ is R_s, (PI) and (IM) act on $|b>$. By these actions, states of $W_{s+1} = (m-1)!(t-(s+1))^{(m-1)-((R_0-1)+R_1+\cdots+R_s)}/(((m-1)-((R_0-1)+R_1+\cdots+R_s))!(R_0-1)!R_1!\cdots R_s!)$ are selected. When γ_{s+1} is a minimum even integer that is $(W_s/W_{s+1})^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is γ_{s+1}. Next, (OB) observes $|b>$. By these actions, states of W_{s+1} remain. And $|b>$ is set $|0>$. These actions are repeated sequentially from 1 to $t-2$ at s. After all, states of

$W_{t-1} = (m-1)!(t-(t-1))^{(m-1)-((R_0-1)+R_1+\cdots+R_{t-2})}/(((m-1)-((R_0-1)+R_1+\cdots+R_{t-2}))!(R_0-1)!R_1!\cdots R_{t-2}!)$ remain.

Step 5: It is assumed that a quantum gate (C_1) obtains states of $|a_0>$ and $|a_1>$, and it changes $|b>$ for $|b+L^{(0)}_{(0,a_0)}+L^{(0)}_{(a_0,a_1)}>$. Next, (C_2) obtains states of $|a_1>$ and $|a_2>$, and it changes $|b>$ for $|b+L^{(0)}_{(a_1,a_2)}>$. Similarly, these actions are repeated sequentially from $|a_2>$ to $|a_{m-2}>$. Where, (C_{m-2}) obtains states of $|a_{m-3}>$ and $|a_{m-2}>$, and it changes $|b>$ for $|b+L^{(0)}_{(a_{m-3},a_{m-2})}+L^{(0)}_{(a_{m-2},0)}>$. As a target state for $|b>$ is L^*_{uni}, (PI) and (IM) act on $|b>$. By these actions, states of $2[=W_t]$ are selected. They contains a counter course for a course. When γ_t is a minimum even integer that is $(W_{t-1}/W_t)^{1/2}[=Z^{1/2}_{uni}]$ or more, the total number that (PI) and (IM) act on $|b>$ is γ_t. Next, (OB) observes $|a_0>, |a_1>, \ldots, |a_{m-2}>$, and $|b>$. By these actions, $a_0, a_1, \ldots, a_{m-2}$, and b are obtained. Therefore, one final answer is a permutation of $P_0 \to P_{a_0} \to P_{a_1} \to \cdots \to P_{a_{m-2}} \to P_0$. And the counter course for this course is another final answer.

3.3. Case of $\sum_{w=0}^{t-1} R_w = m+1$ [Traversable route]

It is assumed that k nodes are $P_0(x_0, y_0, z_0)$, $P_1(x_1, y_1, z_1), \ldots, P_{k-2}(x_{k-2}, y_{k-2}, z_{k-2})$, and $P_{k-1}(x_{k-1}, y_{k-1}, z_{k-1})$. There are $h_{u,v}$ edges between the u-th node and the v-th node. New nodes are assumed $P_k(x_k, y_k, z_k), \ldots, P_{t-2}(x_{t-2}, y_{t-2}, z_{t-2})$, and $P_{t-1}(x_{t-1}, y_{t-1}, z_{t-1})$. These nodes are $g^{(1)}_{u,v}, \ldots, g^{(h_{u,v}-1)}_{u,v}$, and so on. When a node starts and other node ends, we consider that combinations of t nodes that may be contained same nodes. An answer contains a counter course for a course. Therefore, the final answer has 2 routes at least. Therefore, a computational complexity of a classical computation of a traversable route $[=Z_{tra}]$ is $(1/2)(m+1)!/(R_0!R_1!\cdots R_{t-1}!)$.

First of all, $|a_0>, |a_1>, \ldots, |a_m>$, and $|b>$ are prepared. When α is a minimum integer that is $log_2 t$ or more, each of $|a_f>$ that f is an integer from 0 to m is consisted of α qubits. States of $|a_f>$ and $|b>$ are a_f and b, respectively.

Step 1: Each qubit of $|a_f>$ and $|b>$ is set $|0>$.

Step 2: $\boxed{\text{H}}$ acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^\alpha)^{m+1}$.

Step 3: (A) changes $|b>$ for $|b+1>$ at $a_f \geq t$, or it doesn't change $|b>$ at $a_f < t$. As a target state for $|b>$ is 0, (PI) and (IM) act on $|b>$. When β is a minimum even integer that is $(2^\alpha/t)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is β. Next, (OB) observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_m>$. Therefore, each state of $|a_f>$ is $0, 1, \ldots, t-2$, or $t-1$, and the total states become $t^{m+1}[=W_0]$.

Step 4: (B_0) changes $|b>$ for $|b+1>$ at $a_0 = 0$, or it doesn't change $|b>$ at $a_0 \neq 0$. Similarly, these actions are repeated sequentially from $|a_1>$ to $|a_m>$. As a target state for $|b>$ is R_0, (PI) and (IM) act on $|b>$. By these actions, states of $W_1 = (m+1)!(t-1)^{(m+1)-R_0}/(((m+1)-R_0)!R_0!)$ are selected. When γ_1 is a minimum even integer that is $(W_0/W_1)^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is γ_1. Next, (OB) observes $|b>$. By these actions, states of W_1 remain. And $|b>$ is set $|0>$. Similarly, $(B_s)[s = 1, \dots, t-2]$ changes $|b>$ for $|b+1>$ at $a_0 = s$, or it doesn't change $|b>$ at $a_0 \neq s$. These actions are repeated sequentially from $|a_1>$ to $|a_m>$. As a target state for $|b>$ is R_s, (PI) and (IM) act on $|b>$. By these actions, states of $W_{s+1} = (m+1)!(t-(s+1))^{(m+1)-(R_0+R_1+\cdots+R_s)}/(((m+1)-(R_0+R_1+\cdots+R_s))!$ $R_0!R_1!\cdots R_s!)$ are selected. When γ_{s+1} is a minimum even integer that is $(W_s/W_{s+1})^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is γ_{s+1}. Next, (OB) observes $|b>$. By these actions, states of W_{s+1} remain. And $|b>$ is set $|0>$. These actions are repeated sequentially from 1 to $t-2$ at s. After all, states of $W_{t-1} = (m+1)!(t-(t-1))^{(m+1)-(R_0+R_1+\cdots+R_{t-2})}/(((m+1)-(R_0+R_1+\cdots+R_{t-2}))!R_0!R_1!\cdots R_{t-2}!)$ remain.

Step 5: (C_1) obtains states of $|a_0>$ and $|a_1>$, and it changes $|b>$ for $|b+L^{(0)}_{(a_0,a_1)}>$. Next, (C_2) obtains states of $|a_1>$ and $|a_2>$, and it changes $|b>$ for $|b+L^{(0)}_{(a_1,a_2)}>$. Similarly, these actions are repeated sequentially from $|a_2>$ to $|a_m>$. Finally, (C_m) obtains states of $|a_{m-1}>$ and $|a_m>$, and it changes $|b>$ for $|b+L^{(0)}_{(a_{m-1},a_m)}>$. As a target state for $|b>$ is L^*_{tra}, (PI) and (IM) act on $|b>$. By these actions, states of $2[=W_t]$ are selected. They contains a counter course for a course. When γ_t is a minimum even integer that is $(W_{t-1}/W_t)^{1/2}[=Z^{1/2}_{tra}]$ or more, the total number that (PI) and (IM) act on $|b>$ is γ_t. Next, (OB) observes $|a_0>, |a_1>, \dots, |a_m>$, and $|b>$. By these actions, $a_0, a_1, \dots, a_m$, and b are obtained. Therefore, one final answer is a permutation of $P_{a_0} \to P_{a_1} \to \cdots \to P_{a_m}$. And the counter course for this course is another final answer.

4. Numerical calculation

4.1. Case of $\sum_{w=0}^{t-1} R_w$ [Unicursal route]

$6[=k]$ nodes on a graph are $P_0(0,0,0)$, $P_1(2,0,0)$, $P_2(2,2,1)$, $P_3(1,2,0)$, $P_4(0,2,2)$, and $P_5(1,1,0)$. As for a unicursal route, the length of $12[=n]$ edges on the graph are $L^{(0)}_{0,1} = 2$, $L^{(0)}_{1,5} = L^{(0)}_{0,5} = 2^{1/2} \approx 1.4$, $L^{(0)}_{0,2} = 3$, $L^{(0)}_{2,3} = 2^{1/2} \approx 1.4$, $L^{(1)}_{2,3} = L^{(2)}_{2,3} = 2$, $L^{(0)}_{3,4} = 5^{1/2} \approx 2.2$, $L^{(1)}_{3,4} = 6^{1/2} + 1 \approx 3.4$, $L^{(0)}_{1,4} = 12^{1/2} \approx 3.5$, $L^{(0)}_{1,3} = 5^{1/2} \approx 2.2$, and $L^{(0)}_{0,4} = 2 \cdot 2^{1/2} \approx 2.8$. L^*_{uni} is about 27.3. P_2 and P_3 have 3 edges, and P_3 and P_4 have 2 edges. $h_{2,3}$ is 3, and $h_{3,4}$ is 2. Therefore, t is $k + \sum_{v=1,u<v}^{k-1} \sum_{u=0}^{k-2} (h_{u,v} - 1) =$

$6+2+1=9$, and m is $n+\sum_{v=1,u<v}^{k-1}\sum_{u=0}^{k-2}(h_{u,v}-1)=12+2+1=15$. Added 3 nodes are $P_6(2,2,0)[=g^{(1)}_{2,3}]$, $P_7(1,2,1)[=g^{(2)}_{2,3}]$, and $P_8(0,1,2)[=g^{(1)}_{3,4}]$. $L^{(1)}_{2,3}$, $L^{(2)}_{2,3}$, and $L^{(1)}_{3,4}$ become $L^{(0)}_{2,6}+L^{(0)}_{3,6}=1+1=2$, $L^{(0)}_{2,7}+L^{(0)}_{3,7}=1+1=2$, and $L^{(0)}_{3,8}+L^{(0)}_{4,8}\approx 2.4+1=3.4$, respectively. When there aren't edges between the i-th node and the j-th node, $L^{(0)}_{i,j}$ is assumed $2L^*_{uni}$. Moreover, $L^{(0)}_{i,i}$ and $L^{(0)}_{j,j}$ are assumed $2L^*_{uni}$. These edges that are assumed $2L^*_{uni}$ are excluded by gate actions. The values of $Q_0/2, Q_1/2, \ldots, Q_7/2$, and $Q_8/2$ are $4/2\le 2=R_0=R_1=R_2, 6/2\le 3=R_3, 4/2\le 2=R_4$, and $2/2\le 1=R_5=R_6=R_7=R_8$, respectively. Therefore, $\sum_{w=0}^{8}R_w$ is $15[=m]$, and it is checked this graph is a unicursal route.

It is assumed that P_0 starts and ends. An answer contains a counter course for a course. Therefore, the final answer has 2 routes at least. Z_{uni} is $(1/2)(m-1)!/((R_0-1)!R_1!\cdots!R_8!)=(1/2)(15-1)!/((2-1)!2!2!3!2!1!1!1!1!)\approx 9.1\times 10^8$.

First of all, $|a_0>, |a_1>, \ldots, |a_{13}>$, and $|b>$ are prepared. As $log_2 t$ is $log_2 9\approx 3.2\le 4=\alpha$, each of $|a_f>$ that f is an integer from 0 to 13 is consisted of 4 qubits.

Step 1: Each qubit of $|a_f>$ and $|b>$ is set $|0>$.

Step 2: $\boxed{\text{H}}$ acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^\alpha)^{m-1}=(2^4)^{14}$.

Step 3: (A) changes $|b>$ for $|b+1>$ at $a_f\ge 9$, or it doesn't change $|b>$ at $a_f<9$. As a target state for $|b>$ is 0, (PI) and (IM) act on $|b>$. When β is a minimum even integer that is $(2^\alpha/t)^{1/2}=(2^4/9)^{1/2}\approx 1.3\le 2=\beta$, the total number that (PI) and (IM) act on $|b>$ is 2. Next, (OB) observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_{13}>$. Therefore, each state of $|a_f>$ is $0, 1, \ldots, 7$, or 8, and the total states become $t^{m-1}=9^{14}[=W_0]$.

Step 4: (B_0) changes $|b>$ for $|b+1>$ at $a_0=0$, or it is doesn't change $|b>$ at $a_0\ne 0$. Similarly, these actions are repeated sequentially from $|a_1>$ to $|a_{13}>$. As a target state for $|b>$ is $1[=R_0-1]$, (PI) and (IM) act on $|b>$. By these actions, states of $W_1=(15-1)!(9-1)^{(15-1)-(2-1)}/(((15-1)-(2-1))!(2-1)!)=14\cdot 8^{13}$ are selected. When γ_1 is a minimum even integer that is $(W_0/W_1)^{1/2}\approx 1.7\le 2=\gamma_1$, the total number that (PI) and (IM) act on $|b>$ is 2. Next, (OB) observes $|b>$. By these actions, states of W_1 remain. And $|b>$ is set $|0>$. Similarly, $(B_s)[s=1,\ldots,7]$ changes $|b>$ for $|b+1>$ at $a_0=s$, or it doesn't change $|b>$ at $a_0\ne s$. These actions are repeated sequentially from $|a_1>$ to $|a_{13}>$. As a target state for $|b>$ is R_s, (PI) and (IM) act on $|b>$. By these actions, states of $W_{s+1}=14!(9-(s+1))^{14-(1+R_1+\cdots+R_s)}/((14-(1+R_1+\cdots+R_s))!1!R_1!\cdots R_s!)$ are selected. When γ_{s+1} is a minimum even integer that is $(W_s/W_{s+1})^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is γ_{s+1}. Next, (OB) observes $|b>$. By these actions, states of W_{s+1} remain. And $|b>$ is set $|0>$. These actions are repeated sequentially from

1 to 7 at s. They are $(W_1/W_2)^{1/2} \approx 1.9 \leq 2 = \gamma_2$, $(W_2/W_3)^{1/2} \approx 1.9 \leq 2 = \gamma_3$, $(W_3/W_4)^{1/2} \approx 2.8 \leq 4 = \gamma_4$, $(W_4/W_5)^{1/2} \approx 2.0 \leq 2 = \gamma_5$, $(W_5/W_6)^{1/2} \approx 1.5 \leq 2 = \gamma_6$, $(W_6/W_7)^{1/2} \approx 1.5 \leq 2 = \gamma_7$, and $(W_7/W_8)^{1/2} \approx 1.4 \leq 2 = \gamma_8$. Where, the values of $W_2, \ldots, W_7$, and W_8 are $14!7^{11}/(11!1!2!), \ldots, 14!2^2/(2!1!2!2!3!2!1!1!)$, and $14!1^1/(1!1!2!2!3!2!1!1!1!)$, respectively. After all, states of W_8 remain.

Step 5: (C_1) obtains states of $|a_0>$ and $|a_1>$, and it changes $|b>$ for $|b + L^{(0)}_{(0,a_0)} + L^{(0)}_{(a_0,a_1)}>$. Next, (C_2) obtains states of $|a_1>$ and $|a_2>$, and it changes $|b>$ for $|b + L^{(0)}_{(a_1,a_2)}>$. Similarly, these actions are repeated sequentially from $|a_2>$ to $|a_{13}>$. Where, (C_{13}) obtains states of $|a_{12}>$ and $|a_{13}>$, and it changes $|b>$ for $|b+L^{(0)}_{(a_{12},a_{13})}+L^{(0)}_{(a_{13},0)}>$. As a target state for $|b>$ is $27.3[= L^*_{uni}]$, (PI) and (IM) act on $|b>$. By these actions, states of $2[= W_9]$ are selected. They contains a counter course for a course. When γ_9 is a minimum even integer that is $(W_8/W_9)^{1/2}[= Z^{1/2}_{uni}] \approx 3.0 \times 10^4 = \gamma_9$, the total number that (PI) and (IM) act on $|b>$ is γ_9. Next, (OB) observes $|a_0>, |a_1>, \ldots, |a_{13}>$, and $|b>$. By these actions, for example, $a_0, a_1, \ldots, a_{13}$, and b are $1, 5, 0, 2, 3, 4, 1, 3, 6, 2, 7, 3, 8, 4$, and 27.3, respectively. Therefore, one final answer is a permutation of $P_0 \to P_1 \to P_5 \to P_0 \to P_2 \to P_3 \to P_4 \to P_1 \to P_3 \to P_6 \to P_2 \to P_7 \to P_3 \to P_8 \to P_4 \to P_0$. And the counter course for this course is another final answer.

4.2. Case of $\sum_{w=0}^{t-1} R_w = m + 1$ [Traversable route]

$6[= k]$ nodes on a graph are $P_0(0, 0, 0)$, $P_1(2, 0, 0)$, $P_2(2, 2, 1)$, $P_3(1, 2, 0)$, $P_4(0, 2, 2)$, and $P_5(1, 1, 0)$. As for a traversable route, the length of $11[= n]$ edges on the graph are $L^{(0)}_{0,1} = 2$, $L^{(0)}_{1,5} = L^{(0)}_{0,5} = 2^{1/2} \approx 1.4$, $L^{(0)}_{0,2} = 3$, $L^{(0)}_{2,3} = 2^{1/2} \approx 1.4$, $L^{(1)}_{2,3} = L^{(2)}_{2,3} = 2$, $L^{(0)}_{3,4} = 5^{1/2} \approx 2.2$, $L^{(1)}_{3,4} = 6^{1/2} + 1 \approx 3.4$, $L^{(0)}_{1,4} = 12^{1/2} \approx 3.5$, and $L^{(0)}_{1,3} = 5^{1/2} \approx 2.2$. L^*_{tra} is about 24.5. P_2 and P_3 have 3 edges, and P_3 and P_4 have 2 edges. $h_{2,3}$ is 3, and $h_{3,4}$ is 2. Therefore, t is $k + \sum_{v=1,u<v}^{k-1} \sum_{u=0}^{k-2}(h_{u,v} - 1) = 6+2+1 = 9$, and m is $n + \sum_{v=1,u<v}^{k-1} \sum_{u=0}^{k-2}(h_{u,v} - 1) = 11+2+1 = 14$. Added 3 nodes are $P_6(2, 2, 0)[= g^{(1)}_{2,3}]$, $P_7(1, 2, 1)[= g^{(2)}_{2,3}]$, and $P_8(0, 1, 2)[= g^{(1)}_{3,4}]$. $L^{(1)}_{2,3}$, $L^{(2)}_{2,3}$, and $L^{(1)}_{3,4}$ become $L^{(0)}_{2,6}+L^{(0)}_{3,6} = 1+1 = 2$, $L^{(0)}_{2,7}+L^{(0)}_{3,7} = 1+1 = 2$, and $L^{(0)}_{3,8}+L^{(0)}_{4,8} \approx 2.4+1 = 3.4$, respectively. When there aren't edges between the i-th node and the j-th node, $L^{(0)}_{i,j}$ is assumed $2L^*_{tra}$. Moreover, $L^{(0)}_{i,i}$ and $L^{(0)}_{j,j}$ are assumed $2L^*_{tra}$. These edges that are assumed $2L^*_{tra}$ are excluded by gate actions. The values of $Q_0/2, Q_1/2, \ldots, Q_7/2$, and $Q_8/2$ are $3/2 \leq 2 = R_0$, $4/2 \leq 2 = R_1 = R_2$, $6/2 \leq 3 = R_3$, $3/2 \leq 2 = R_4$, and

$2/2 \leq 1 = R_5 = R_6 = R_7 = R_8$, respectively. Therefore, $\sum_{w=0}^{8} R_w$ is $15[= m+1]$, and it is checked this graph is a traversable route.

An answer contains a counter course for a course. Therefore, the final answer has 2 routes at least.
Z_{tra} is $(1/2)(m+1)!/(R_0!R_1!\cdots R_8!) = (1/2)(14+1)!/(2!2!2!3!2!1!1!1!1!) \approx 6.8 \times 10^9$.

First of all, $|a_0>, |a_1>, \ldots, |a_{14}>$, and $|b>$ are prepared. As $log_2 t$ is $log_2 9 \approx 3.2 \leq 4 = \alpha$, each of $|a_f>$ that f is an integer from 0 to 14 is consisted of 4 qubits.

Step 1: Each qubit of $|a_f>$ and $|b>$ is set $|0>$.

Step 2: $\boxed{\text{H}}$ acts on each qubit of $|a_f>$. It changes them for entangled states. The total states are $(2^\alpha)^{m+1} = (2^4)^{15}$.

Step 3: (A) changes $|b>$ for $|b+1>$ at $a_f \geq 9$, or it doesn't change $|b>$ at $a_f < 9$. As a target state for $|b>$ is 0, (PI) and (IM) act on $|b>$. When β is a minimum even integer that is $(2^\alpha/t)^{1/2} = (2^4/9)^{1/2} \approx 1.3 \leq 2 = \beta$, the total number that (PI) and (IM) act on $|b>$ is 2. Next, (OB) observes $|b>$. These actions are repeated sequentially from $|a_0>$ to $|a_{14}>$. Therefore, each state of $|a_f>$ is $0, 1, \ldots, 7$, or 8, and the total states become $t^{m+1} = 9^{15}[= W_0]$.

Step 4: (B_0) changes $|b>$ for $|b+1>$ at $a_0 = 0$, or it is doesn't change $|b>$ at $a_0 \neq 0$. Similarly, these actions are repeated sequentially from $|a_1>$ to $|a_{14}>$. As a target state for $|b>$ is $2[= R_0]$, (PI) and (IM) act on $|b>$. By these actions, states of $W_1 = (14+1)!(9-1)^{(14+1)-2}/(((14+1)-2)!2!) = 15 \cdot 7 \cdot 8^{13}$ are selected. When γ_1 is a minimum even integer that is $(W_0/W_1)^{1/2} \approx 1.9 \leq 2 = \gamma_1$, the total number that (PI) and (IM) act on $|b>$ is 2. Next, (OB) observes $|b>$. By these actions, states of W_1 remain. And $|b>$ is set $|0>$. Similarly, $(B_s)[s = 1, \ldots, 7]$ changes $|b>$ for $|b+1>$ at $a_0 = s$, or it doesn't change $|b>$ at $a_0 \neq s$. These actions are repeated sequentially from $|a_1>$ to $|a_{14}>$. As a target state for $|b>$ is R_s, (PI) and (IM) act on $|b>$. By these actions, states of $W_{s+1} = 15!(9-(s+1))^{15-(2+R_1+\cdots+R_s)}/((15-(2+R_1+\cdots+R_s))!2!R_1!\cdots R_s!)$ are selected. When γ_{s+1} is a minimum even integer that is $(W_s/W_{s+1})^{1/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is γ_{s+1}. Next, (OB) observes $|b>$. By these actions, states of W_{s+1} remain. And $|b>$ is set $|0>$. These actions are repeated sequentially from 1 to 7 at s. They are $(W_1/W_2)^{1/2} \approx 1.9 \leq 2 = \gamma_2$, $(W_2/W_3)^{1/2} \approx 1.9 \leq 2 = \gamma_3$, $(W_3/W_4)^{1/2} \approx 2.8 \leq 4 = \gamma_4$, $(W_4/W_5)^{1/2} \approx 1.9 \leq 2 = \gamma_5$, $(W_5/W_6)^{1/2} \approx 1.6 \leq 2 = \gamma_6$, $(W_6/W_7)^{1/2} \approx 1.5 \leq 2 = \gamma_7$, and $(W_7/W_8)^{1/2} \approx 1.4 \leq 2 = \gamma_8$. Where, the values of $W_2, \ldots, W_7$, and W_8 are $15!7^{11}/(11!2!2!), \ldots, 15!2^2/(2!2!2!2!3!2!1!1!)$, and $15!1^1/(1!2!2!2!3!2!1!1!1!)$, respectively. After all, states of W_8 remain.

Step 5: (C_1) obtains states of $|a_0>$ and $|a_1>$, and it changes $|b>$ for $|b + L^{(0)}_{(a_0,a_1)}>$. Next, (C_2) obtains states of $|a_1>$ and $|a_2>$, and it changes $|b>$ for $|b + L^{(0)}_{(a_1,a_2)}>$. Similarly, these actions are repeated sequentially from $|a_2>$ to $|a_{14}>$. Finally, (C_{14})

obtains states of $|a_{13}>$ and $|a_{14}>$, and it changes $|b>$ for $|b + L^{(0)}_{(a_{13},a_{14})}>$. As a target state for $|b>$ is 24.5[$= L^{*}_{tra}$], (PI) and (IM) act on $|b>$. By these actions, states of 2[$= W_9$] are selected. They contains a counter course for a course. When γ_9 is a minimum even integer that is $(W_8/W_9)^{1/2}$[$= Z^{1/2}_{tra}$] $\approx 8.4 \times 10^4 = \gamma_9$, the total number that (PI) and (IM) act on $|b>$ is γ_9. Next, (OB) observes $|a_0>$, $|a_1>, \ldots, |a_{14}>$, and $|b>$. By these actions, for example, $a_0, a_1, \ldots, a_{14}$, and b are 0, 1, 5, 0, 2, 3, 4, 1, 3, 6, 2, 7, 3, 8, 4, and 24.5, respectively. Therefore, one final answer is a permutation of $P_0 \rightarrow P_1 \rightarrow P_5 \rightarrow P_0 \rightarrow P_2 \rightarrow P_3 \rightarrow P_4 \rightarrow P_1 \rightarrow P_3 \rightarrow P_6 \rightarrow P_2 \rightarrow P_7 \rightarrow P_3 \rightarrow P_8 \rightarrow P_4$. And the counter course for this course is another final answer.

5. Discussion and Summary

The computational complexity of this quantum algorithm [$= S$] becomes the following.

(I) The cace of $\sum_{w=0}^{t-1} R_w = m$. [a unicursal route] In the order of the actions by the gates, the number of them is $(m-1)\alpha$ at $\boxed{\text{H}}$, $(m-1)$ at (A), $2(m-1)$ at (PI) and (IM), $(m-1)$ at (OB), $(m-1)(t-1)$ at $(B_s)[s = 0, \ldots, t-2]$, $\sum_{e=1}^{t-1} \gamma_e$ at (PI) and (IM), $t-1$ at (OB), $2(m-2)$ at $(C_d)[d = 1, \ldots, m-2$. d is an integer.], γ_t at (IP) and (IM), and 1 at (OB). Therefore, S_{uni} that is S of a unicursal route becomes $(\alpha+4)(m-1)+m(t-1)+2(m-2)+1+\sum_{e=1}^{t} \gamma_e$. In the example of the section 4.1, S_{uni} is about 3.0×10^4. S_{uni}/Z_{uni} is about $1/Z^{1/2}_{uni} \approx 1/(3.0 \times 10^4)$ because Z_{uni} is about 9.1×10^8.

(II) The cace of $\sum_{w=0}^{t-1} R_w = m + 1$. [a traversable route] In the order of the actions by the gates, the number of them is $(m+1)\alpha$ at $\boxed{\text{H}}$, $(m+1)$ at (A), $2(m+1)$ at (PI) and (IM), $(m+1)$ at (OB), $(m+1)(t-1)$ at $(B_s)[s = 0, \ldots, t-2]$, $\sum_{e=1}^{t-1} \gamma_e$ at (PI) and (IM), $t-1$ at (OB), $2m$ at $(C_d)[d = 1, \ldots, m]$, γ_t at (IP) and (IM), and 1 at (OB). Therefore, S_{tra} that is S of a traversable route becomes $(\alpha+4)(m+1)+(m+2)(t-1)+2m+1+\sum_{e=1}^{t} \gamma_e$. In the example of the section 4.2, S_{tra} is about 8.4×10^4. S_{tra}/Z_{tra} is about $1/Z^{1/2}_{tra} \approx 1/(8.4 \times 10^4)$ because Z_{tra} is about 6.8×10^9.

When k and n [in other words, t and m] are large enough, there are $S_{uni}/Z_{nui} \approx 1/Z_{uni}^{1/2}$ and $S_{tra}/Z_{tra} \approx 1/Z_{tra}^{1/2}$ because of $S_{uni} \approx \gamma_t \approx Z_{uni}^{1/2}$ and $S_{tra} \approx \gamma_t \approx Z_{tra}^{1/2}$. For example, as for $k = t = n = m = 100$, $R_0 = R_2 = \cdots = R_{99} = 1$, and $R_1 = 2$, S/Z is about $1/(4.9 \times 10^{79})$ because of $\sum_{w=0}^{t-1} R_w = n + 1 = m + 1$ [in other words, a traversable route] and $Z \approx 2.4 \times 10^{159}$.

Further development of the quantum computation in the future is expected.

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