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Quantum Algorithm for Bin-packing Problem

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Abstract

Quantum algorithm for the bin-packing problem and its example are reported. It is decided whether n pieces of luggage are packed into k boxes, where each weight of the luggage is W or less, and a maximum storage weight of the box is W. A computational complexity of a classical calculation is k^n. The computational complexity becomes about $k^{n/2}$ by the quantum algorithm that uses quantum phase inversion gates and quantum inversion about mean gates.

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1. Introduction

A quantum computer can solve a problem at high speed by a parallel computation that uses quantum entangled states. Deutsch-Jozsa's algorithm for the rapid solution [1–3], Shor's algorithm for the factorization [2–4], Grover's algorithms for the database search [2,5–7], and so on are known. A quantum algorithm for the knapsack problem has recently been reported by Fujimura [8]. The bin-packing problem [9] is examined this time to expand the application range of the quantum computation. Therefore, its result is reported.

2. Bin-packing problem

It is decided whether n pieces of luggage are packed into k boxes, where each weight of the luggage is W or less, and a maximum storage weight of the box is W [9]. A computational complexity of a classical calculation is k^n because there are k choices about each luggage.

3. Quantum algorithm

It is assumed that there are n pieces of luggage and k boxes, where each weight of the luggage is W or less, and the maximum storage weight of the box is W. Each weight of the luggage is $x_0, x_1, \ldots, x_{n-2}$, and x_{n-1}. First of all, the quantum registers $|a_0>, |a_1>, \ldots, |a_{n-1}>, |b>$, and $|c>$ are prepared. When α is a minimum integer that is $log_2 k$ or more, each of $|a_i>$ that i is an integer from 0 to $n-1$ is consisted of α quantum bits (=qubits). $|c>$ is consisted of qubits for the total weight of the luggage enough. States of $|a_i>$ and $|c>$ are a_i and c, respectively.

Step 1: Each qubit of $|a_i>, |b>$, and $|c>$ is set $|0>$.

Step 2: The Hadamard gate $\boxed{\text{H}}$ [2, 3] acts on each qubit of $|a_i>$. It changes them for entangled states. The total states are $(2^{\alpha})^n$.

Step 3: It is assumed that a quantum gate (A) changes $|b>$ for $|b+1>$ at $a_i \geq k$, or it doesn't change $|b>$ at $a_i < k$. The action of (A) is repeated sequentially from $|a_0>$ to $|a_{n-1}>$. As a target state for $|b>$ is 0, quantum phase inversion gates (PI) and quantum inversion about mean gates (IM) [2, 5–7] act on $|b>$. When β is a minimum even number that is $(2^{\alpha}/k)^{n/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is β because they are a couple. Next, an observation gate (OB) observes $|b>$. By these actions, each state of $|a_i>$ is $0, 1, \ldots, k-2$, or $k-1$. Therefore, the total states become k^n.

Step 4: Quantum gates of $(B_{i,j})$ and (C_j) are prepared, where i is an integer from 0 to $n-1$, and j is an integer from 0 to $k-1$. At $j = a_i$, $(B_{i,j})$ changes $|c>$ for $|c+x_i>$, or at $j \neq a_i$, $(B_{i,j})$ doesn't change $|c>$. i of $(B_{i,j})$ is repeated from 0 to $n-1$. Therefore, the state of $|c>$ becomes the total weight of the luggage in j-th box. At $c \leq W$, (C_j) doesn't change $|b>$, or at $c > W$, (C_j) changes $|b>$ for $|b+1>$. Then $|c>$ is set $|0>$. j of $(B_{i,j})$ and (C_j) is repeated from 0 to $k-1$. As the target state for $|b>$ is 0, (PI) and (IM) act on $|b>$. When γ is a minimum even number that is $k^{n/2}$ or more, the total number that (PI) and (IM) act on $|b>$ is γ. Next, (OB) observes $|a_i>, |b>$, and $|c>$. As a result, each state of them is obtained.

4. Numerical calculation

One example is given here. It is assumed that there are seven pieces of luggage and three boxes, where each weight of the luggage is $x_0 = 7kg, x_1 = 6kg, x_2 = 5kg, x_3 = 4kg, x_4 = 3kg, x_5 = 3kg$, and $x_6 = 2kg$, and the maximum storage weight of the box is $10kg$. Quantum registers $|a_0>, |a_1>, \ldots, |a_6>, |b>$, and $|c>$ are prepared. Because of $log_2 3 \approx 1.6' \leq 2 = \alpha$, each of $|a_0>, |a_1>, \ldots, |a_5>$, and $|a_6>$ is consisted of two qubits. $|c>$ is consisted of qubits for the total weight of the luggage enough.

Step 1: Each qubit of $|a_0>, |a_1>, \ldots, |a_6>, |b>$, and $|c>$ is set $|0>$.

Step 2: $\boxed{\text{H}}$ acts on each qubit of $|a_0>, |a_1>, \ldots, |a_5>$, and $|a_6>$. It changes them for entangled states. The total states are $(2^2)^7 = 16384$.

Step 3: The action of (A) is repeated sequentially from $|a_0>$ to $|a_6>$. As the target state for $|b>$ is 0, (PI) and (IM) act on $|b>$. Because of $(2^2/3)^{7/2} \approx 2.7 \leq 4 = \beta$, the total number that they act on $|b>$ is 4. Next, (OB) observes $|b>$. By these actions, each state of $|a_0>, |a_1>, \ldots, |a_5>$, and $|a_6>$ is 0, 1, or 2. Therefore, the total states become $3^7 = 2187$.

Step 4: It is assumed that $|a_0>, |a_1>, |a_2>, |a_3>, |a_4>, |a_5>$, and $|a_6>$ are $|0>, |1>, |2>, |1>, |0>, |2>$, and $|2>$, respectively. $(B_{0,0})$ changes $|c>$ for $|c + x_0 = 7>$. $(B_{1,0})$ doesn't change $|c>$. Moreover, i of $(B_{i,0})$ is repeated to 6. Therefore, c becomes $7 + 3 = 10$. Because of $c \leq 10$, (C_0) doesn't change $|b>$. Then $|c>$ is set $|0>$. Next, $(B_{0,1})$ doesn't change $|c>$. $(B_{1,1})$ changes $|c>$ for $|c + x_1 = 6>$. Moreover i of $(B_{i,1})$ is repeated to 6. Therefore, c becomes $6 + 4 = 10$. Because of $c \leq 10$, (C_1) doesn't change $|b>$. Then $|c>$ is set $|0>$. Next, $(B_{0,2})$ doesn't change $|c>$. $(B_{1,2})$ doesn't change $|c>$. $(B_{2,2})$ changes $|c>$ for $|c + x_2 = 5>$. Moreover, i of $(B_{i,2})$ is repeated to 6. Therefore, c becomes $5 + 3 + 2 = 10$. Because of $c \leq 10$, (C_2) doesn't change $|b>$. Then $|c>$ is set $|0>$. Other combinations of $|a_0>, |a_1>, \ldots, |a_5>$, and $|a_6>$ are similarly calculated. When a combination doesn't satisfy the condition, (C_j) changes $|b>$ for $|b + 1>$. As the target state for $|b>$ is 0, (PI) and (IM) act on $|b>$. Because of $3^{7/2} \approx 46.8 \leq 48 = \gamma$, the total number that two gates made to act on $|b>$ becomes 48. Next, (OB) observes $|a_0>, |a_1>, \ldots, |a_6>, |b>$, and $|c>$. As a result, the states of them become 0, 1, 2, 1, 0, 2, 2, 0, and 0, respectively.

5. Discussion and Summary

The computational complexity of this quantum algorithm ($= S$) becomes the following. In the order of the actions by the gates, the number of them is $n\alpha$ times at $\boxed{\text{H}}$, n times at (A), β times at (PI) and (IM), once at (OB), nk times at $(B_{i,j})$, k times at (C_j), γ times at (PI) and (IM), and once at (OB). Therefore, S becomes $(\alpha + k + 1)n + \beta + k + \gamma + 2$. In the example of the section 4, S is 99. It corresponds to about $1/20$ of the computational complexity of the classical calculation ($= Z$) because Z is $k^n = 3^7 = 2187$. When n is large enough, S becomes about $\gamma \approx k^{n/2}$. For example, when n and k are 100 and 5 respectively, S becomes about $8.9' \times 10^{34}$ and Z is about $7.9' \times 10^{69}$. Therefore, S/Z is about $1/10^{35}$.

Further development of the quantum computation in the future is expected.

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